Influence of structural parameters of the masonry on effective elastic properties and strength

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Abstract

The masonry model, representing an elastic mortar and elastic bricks, is analyzed with aim to evaluate effective elastic moduli and strength properties. By means of the direct finite element simulation and homogenization procedure the analysis of variation influence in the heterogeneous material microstructure characteristics (influence of brick aspect ratio and orientation angle) on the local stress-strain state and mechanical properties of the representative volume element of considered composite has been fulfilled.

1 Introduction

The masonry is two phase composite material consisting of brick and mortar joints, generally arranged periodically. Studying influence of the masonry structural parameters on it deformation characteristics is important for designing and retrofitting masonry structures. The computer simulation of masonry deformation and failure processes can be used as a viable alternative to otherwise expensive and time-consuming laboratory and field experiments.

The most common approach is to model each brick and each mortar joint in the assembly, where linear and nonlinear constitutive behaviors of bricks and mortar can be considered. But these methods require intensive computational efforts [1–4].

On the other hand, masonry can be treated as an effectively elastic continuum. A way of modeling a structure made of heterogeneous materials without treating all of the heterogeneities individually consists in trying to replace the heterogeneous medium by an equivalent homogeneous medium (EHM) endowed with so-called effective properties.

Pande et al. [5] have proposed multilayer model to estimate the effective elastic properties of masonry. The effective properties of a multilayered system with alternating joints are obtained in closed form on the basis of equality of the strain energy. The multilayer solution is first applied to homogenize the horizontal strip, and then used again to integrate the above homogenized strips with the horizontal bed joints (Fig. 1b). Bati et al. [6] proposed using aligned elliptical cylinders to approximate the rectangular bricks (Fig. 1c). Pietruszczak and Niu [7] also proposed a two-step homogenization scheme. In the first stage, they considered homogeneous matrix formed from bricks with aligned head joints as inclusions. The equivalent elastic properties of the medium can be found using Eshelby's solution for an elliptic cylinder inclusion in combination with the Mori–Tanaka mean field theory [8]. Effective properties are found using mechanics of the laminate material, formed homogenized medium from the first step and the continuous bed joints. The scheme is also illustrated in Fig. 1d.

Not one of the above methods have explicitly taken into account the specific pattern of brick and mortar, nor have fully exploited the periodicity of geometry, stress, strains, as well as other field quantities. In this paper, authors propose implementing periodic homogenization method, to model masonry structures. Gang Wang et al. also proposed implementing periodic homogenization method, to model masonry structures [9]. Periodic model as shown in Fig. 1e.

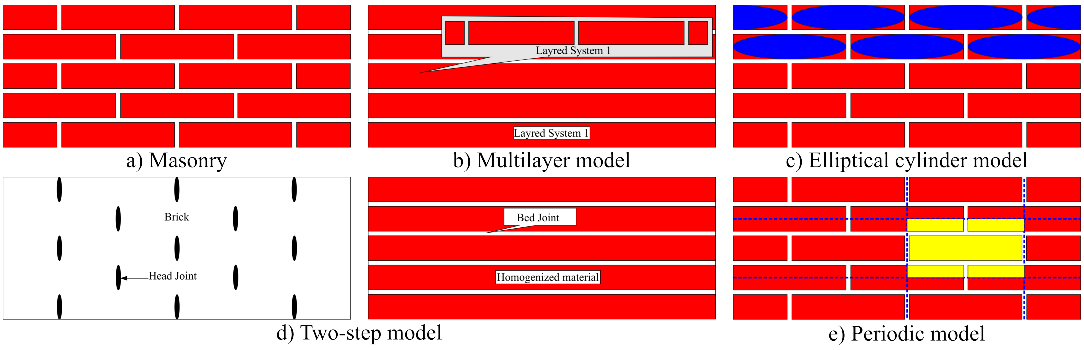

Figure 1: Homogenization models for masonry structure.

2 Representative volume element

The effective properties are calculated on the base of the spatial averaging of stress-strain state within the representative volume element (RVE) of material. The RVE can be introduced for a material with a statistically uniform distribution (ergodic hypothesis) taking into account the scale-separability of heterogeneities. In this case the least volume containing all the a priori statistical information on the distribution and morphology of the material heterogeneities can be correctly introduced.

The simplest two-dimensional variant of RVE (unit cell) (Fig. 2) is presented by the central rectangular inclusion and four fragments of neighboring undeflected inclusions (the area of each).

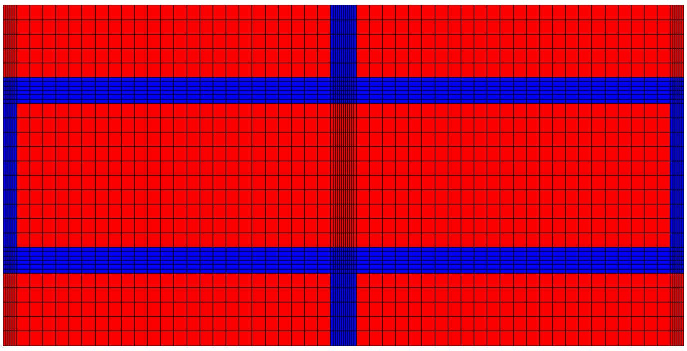

Figure 2: FE model of masonry RVE.

3 Results of FE modeling of deformation processes of material RVE

Finite element simulations of the deformation processes of RVE are aimed to solve two main subtasks:

1. to identify effective mechanical properties of the material RVE (homogenization problem);
2. to obtain the extreme values of stress fields within heterogeneous RVE for subsequent strength analysis (heterogenization problem).

3.1 Effective elastic properties

The effective elastic properties of the masonry composite RVE are found by means of methods of the FE homogenization. It is supposed that the effective properties of homogenized masonry material correspond to orthotropic elastic material, for which Hooke's law can be written as:

$$\bar{\boldsymbol{\varepsilon}} = {}^4\bar{\boldsymbol{C}} \cdot\cdot \bar{\boldsymbol{\sigma}}, \tag{1}$$

where $\bar{\boldsymbol{\varepsilon}} = \frac{1}{V_{RVE}} \int\limits_{V_{RVE}} \boldsymbol{\varepsilon} dV$ is the spatial averaged strain tensor, $\bar{\boldsymbol{\sigma}} = \frac{1}{V_{RVE}} \int\limits_{V_{RVE}} \boldsymbol{\sigma} dV$ is the spatial averaged stress tensor, ${}^4\bar{\boldsymbol{C}}$ is tensor of effective elastic compliances of 4th rank. The bar over tensors indicates a correspondence to the homogenized material.

The elastic moduli are determined on the basis of relations $\bar{E}_i = \frac{\bar{\sigma}_{ii}}{\bar{\varepsilon}_{ii}}$, $\bar{v}_{ij} = -\frac{\bar{\varepsilon}_{jj}}{\bar{\varepsilon}_{ii}}$. 2D boundary value problems are solved assuming the plane strain condition. The determination of the two elastic moduli $\bar{E}_1$ and $\bar{E}_2$ requires to solve only two boundary value problems with boundary conditions for tension (or compression) in the directions of the anisotropy axes (in the vertical and in the horizontal directions for the RVE in Fig. 2)

The mechanical properties of the individual components correspond to the isotropic material. The elastic moduli of mansory components are taken from the literature [9, 10].

Table 1. Normal elastic moduli of the individual components of mansory.

	E, GPa	ν
Brick	11	0.2
Mortar	2.2	0.25

The normal brick dimensions are l = 250 mm (length), h = 55 mm (height) (bricks aspect ratio l/h =4.54). The normal mortar joints are 10 mm thick.

The evaluation of the accuracy of numerical solutions is based on the analysis of practical convergence of the effective elastic moduli with increasing number of unit cells and number of finite elements. The upper and lower boundaries for moduli are obtained using boundary conditions for the displacements and for the tractions. Three types of boundary conditions are applied and compared:

kinematic uniform boundary condition

$$\boldsymbol{u}|_{S_u} = \bar{\boldsymbol{\varepsilon}}^* \cdot \boldsymbol{r}, \tag{2}$$

static uniform boundary condition

$$\boldsymbol{n} \cdot \boldsymbol{\sigma}|_{S_\sigma} = \boldsymbol{n} \cdot \bar{\boldsymbol{\sigma}}^*, \tag{3}$$

periodicity condition

$$\boldsymbol{u}|_{S_{u_1}} = \boldsymbol{u}|_{S_{u2}} + \bar{\boldsymbol{\varepsilon}}^* \cdot (\boldsymbol{r}_1 - \boldsymbol{r}_2), \tag{4}$$

where $\bar{\boldsymbol{\varepsilon}}^*$ and $\bar{\boldsymbol{\sigma}}^*$ are prescribed constant symmetric tensors corresponding to the different possible states (axial tensions or shears), $\boldsymbol{r}$ is the radius-vector. The periodicity condition (4) can be rewritten in the form $\boldsymbol{u}|_{S_u} = \bar{\boldsymbol{\varepsilon}}^* \cdot \boldsymbol{r} + \tilde{\boldsymbol{w}}$, where a fluctuation $\tilde{\boldsymbol{w}}$ is periodic, i.e., $\tilde{\boldsymbol{w}}$ takes the same values on opposite sides of RVE. In this case also the traction $\boldsymbol{n} \cdot \boldsymbol{\sigma}$ takes opposite values on opposite sides. The boundary conditions (2)-(4) satisfy to Hill homogeneity condition and provide the existence and uniqueness of the solution of the corresponding boundary value problems.

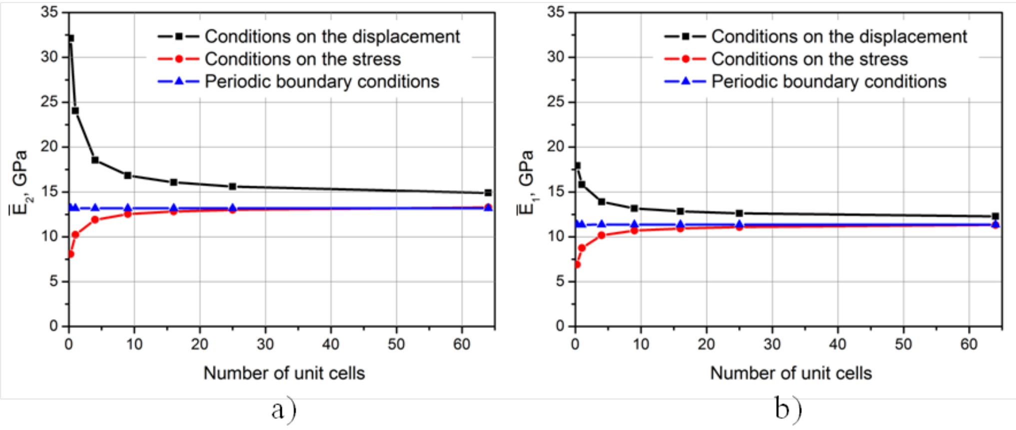

Figure 3: Dependence of the effective elastic moduli a) $\bar{E}_2$ (in the vertical direction) and b) $\bar{E}_1$ (in the horizontal direction) on the number of unit cells.

As a result of multivariant computational experiments, it is found that by using periodicity conditions (4) satisfactory accuracy (close enough to the asymptotic value) is achieved (Fig. 3) even by using of RVE including one bridge ($\frac{1}{4}$ of the unit cell in Fig. 2). Whereas with the boundary conditions (2) and (3) the convergence is achieved (Fig. 3) only if RVE includes 64 bridges (4 × 4 unit cells) or more. The using of (4) allows to reduce significantly the dimension of FE model and the computation time. The finite element models are shown in Fig. 4.

The computations were performed using the finite element software package PANTOCRATOR [11], which allows to automatically generate discrete models of RVE arbitrary geometry, to obtain solutions of boundary problems, to determine the effective elastic moduli and strength properties of RVE, to analyze the distribution of stress and strain fields.

3.2 Influence of Young's modulus of mortar

In order to evaluate the influence of the Young's modulus of mortar on the effective elastic moduli of RVE the FE computations are carried out with different Young's modulus of mortar: 1, 2, 4, 6, 8, 10 GPa. The results of computational experiments have shown that the Young's modulus of mortar exerts considerable influence on the effective elastic moduli (Fig. 5). The increase of the Young's modulus of mortar from 2 GPa up to 10 GPa leads to increases of $\bar{E}_1$ on 21.6% and to increases of $\bar{E}_2$ on 68,6%.

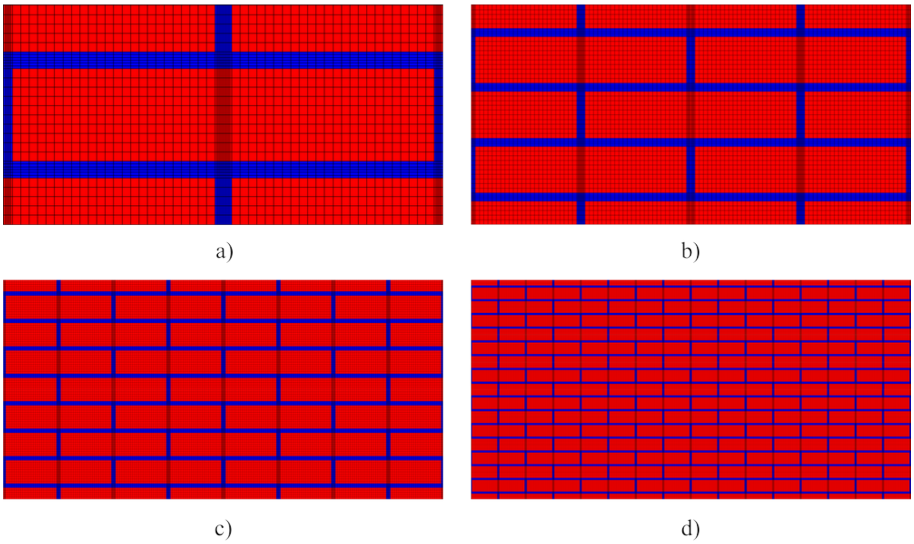

Figure 4: Finite element model with different number of unit cells: a) 1x1 unit cells, b) 2x2 unit cells, c) 4x4 unit cells, d)8x8 unit cells.

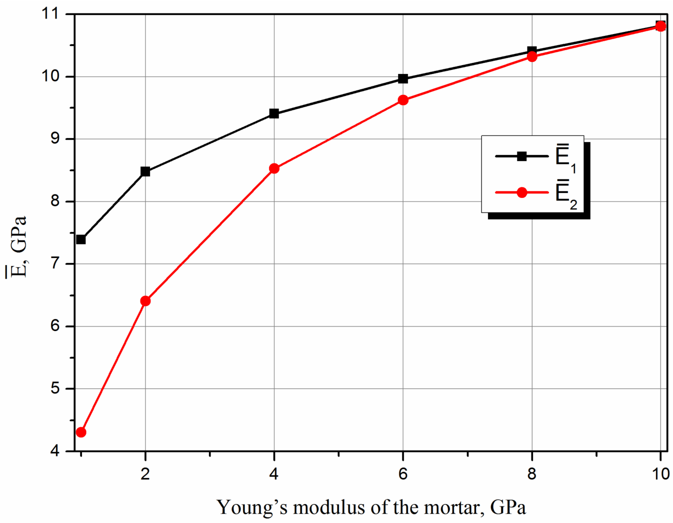

Figure 5: Dependence of the elastic moduli of masonry RVE on the Young's modulus.

3.3 Influence of brick orientation

With aim to evaluate the effect of the angular orientation of the brick on the stress-strain state and on the effective elastic moduli of RVE the FE computations are carried out for three different deviation angles of the bricks from horizontal direction: 0°, 2° and 4° (Fig. 6). The results of computational experiments have shown that the rotation of a single brick does not practically affect on the effective elastic moduli $\bar{E}_1, \bar{E}_2$ (difference is less than 1%), but it has considerable influence on the level of maximum stress intensity (Fig. 7).

The obtained results allow us to establish that the disorder of brick leads to an increase of the stresses in the masonry RVE. Ideal structure without brick disorientation has maximum strength. The extremal stresses are observed at brick corners.

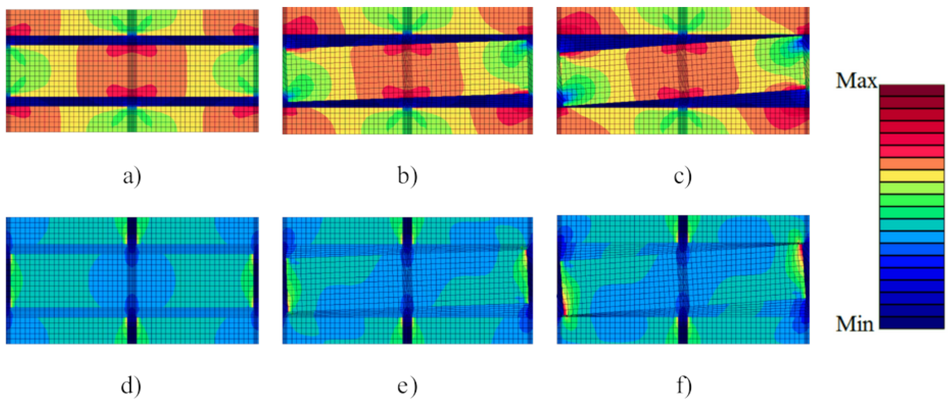

Figure 6: Distribution of the von Mises stress intensity fields in the masonry RVE under the *horizontal* compression with rotation of single brick on: a) 0°, b) 2°, b) 4°; under the *vertical* compression with rotation of single brick on: d) 0°, e) 2°, f) 4°.

3.4 Influence of the mortar thickness

In order to evaluate the influence of the thickness of mortar on the stress-strain state and on the effective elastic moduli of RVE the FE calculations are carried out with different thickness of mortar: 5, 10, 20, ... ,70 mm. The results of computational experiments have shown that the thickness of mortar exerts considerable influence on the effective elastic moduli (Fig. 8) and on the von Mises stress intensity averaged over the whole volume of RVE and over its individual components. The increase of the mortar thickness 10 mm up to 30 mm leads to decrease of $\bar{E}_1$ on 40.2% and of $\bar{E}_2$ on 54%.

The same increase of the mortar thickness under the compression in *horizontal* direction leads to the decreasing of von Mises stress intensity:

- on 39.8% for averaging over the whole volume of RVE;
- on 39.8% for averaging over the whole volume of brick;
- on 12.9% for averaging over the whole volume of mortar.

The same increase of the mortar thickness under the compression in *vertical* direction leads to the decreasing of von Mises stress intensity:

- on 53.8% for averaging over the whole volume of RVE;
- on 48.8% for averaging over the whole volume of brick;
- on 46.6% for averaging over the whole volume of mortar.

3.5 Influence of brick aspect ratio

In order to evaluate the influence of the bricks aspect ratio on the stress-strain state and on the effective elastic moduli of RVE the FE calculations are carried out with different bricks aspect ratio: 1, 2, 3, ... , 10 (FE models are shown in Fig. 9). The results of computational experiments have shown that the bricks aspect ratio affects considerable

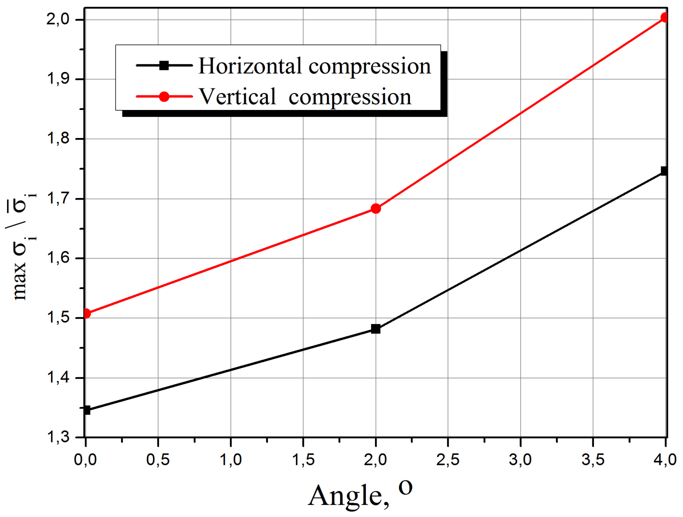

Figure 7: Dependence of the maximum von Mises stress intensity on the central brick rotation angle.

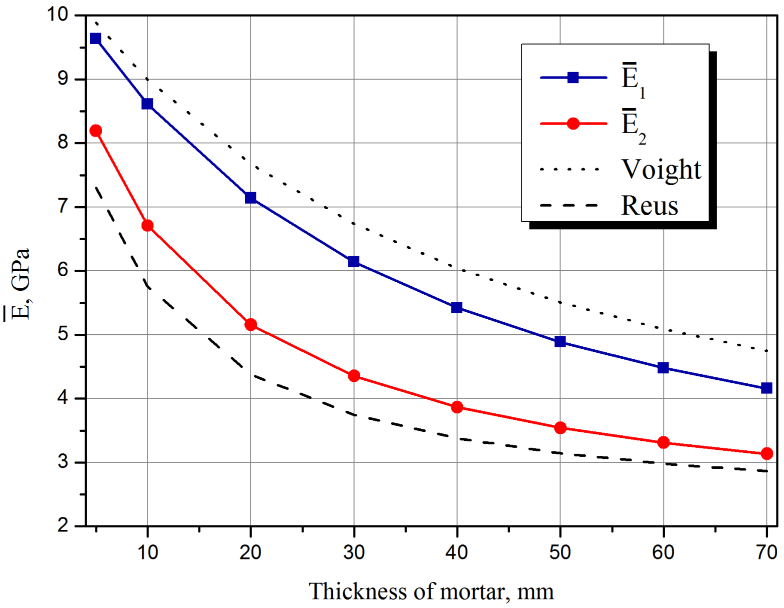

Figure 8: Influence of the thickness of mortar on the effective elastic moduli (upper and lower curves correspond to the analytical estimations by Voigt (averaging of component stiffnesses) and by Reuss (averaging of component compliances)).

influence on the effective moduli (Fig. 10) and on the von Mises stress intensity averaged over the whole volume of RVE and over its individual components.

The results of computational experiments have shown that the bricks aspect ratio does not practically affect on the $\bar{E}_1$ (change in the elastic modulus is less by 7%) (Fig. 10), but it has considerable influence on the effective elastic moduli $\bar{E}_2$ (Fig. 10). The increase of the bricks aspect ratio from 1 up to 10 leads to decreases of $\bar{E}_2$ on 35%.

The same increase of the bricks aspect ratio under the compression in *horizontal* direction leads to the varying of von Mises stress intensity:

- increase on 6.6% for averaging over the whole volume of RVE;

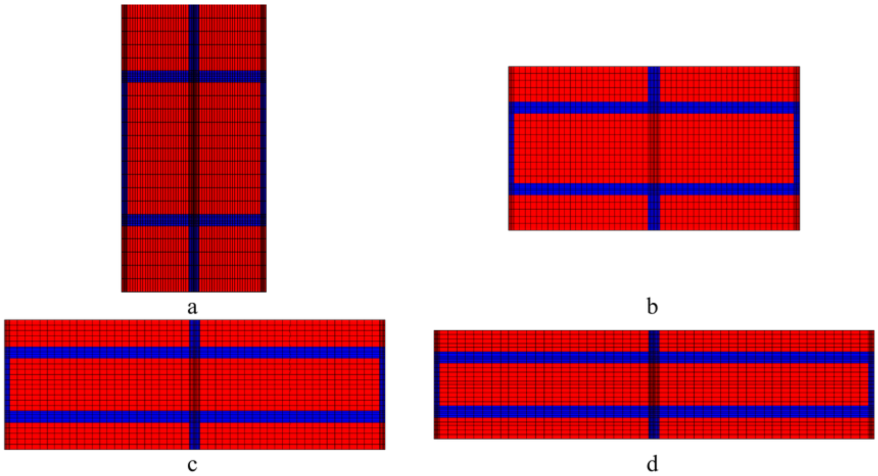

Figure 9: Finite element model for the various bricks aspect ratio: a) $l/h = 1$; b) $l/h = 4$; c) $l/h = 7$; d) $l/h = 10$.

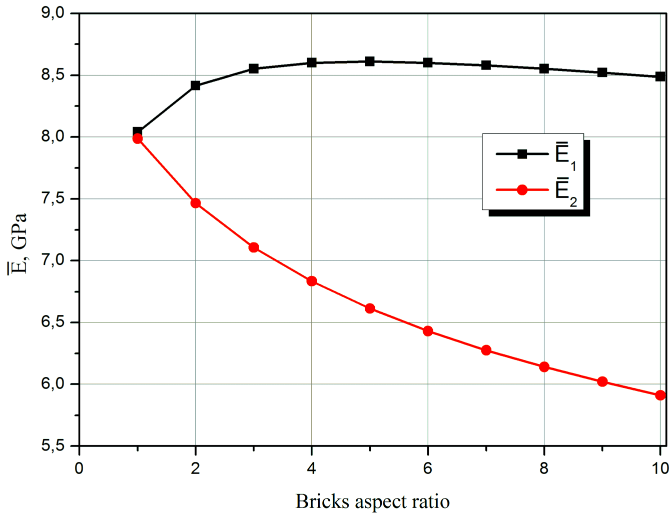

Figure 10: Dependence of the elastic moduli of masonry RVE on the bricks aspect ratio.

- increase on 20.9% for averaging over the whole volume of brick;
- decreases on 81.4% for averaging over the whole volume of mortar.

The same increase of the bricks aspect ratio under the compression in *vertical* direction leads to the varying of von Mises stress intensity:

- decreases on 34.9% for averaging over the whole volume of RVE;
- decreases on 40.7% for averaging over the whole volume of brick;
- increase on 7.6% for averaging over the whole volume of mortar.

The elastic modulus $E1$ does not change monotonically with increase of brick aspect ratio. $E1$ has a maximum value at $l/h = 5$. This aspect ratio corresponds to the typical brick aspect ratio used traditionally in the practice.

3.6 Comparison of the different homogenization approaches

The comparison of the obtained effective elastic moduli of the masonry with results of another numerical or analytical approaches [5, 6, 7, 9, 10] are summarized in Table 2. The results of current investigation, which are based on the FE homogenization and using of boundary conditions of periodicity are in a good agreement with other approaches based on FEM ([5, 9, 10]). The appreciable differences are observed with with elliptical cylinder model [7] and two-step method [6].

Table 2. Effective elastic properties of masonry.

Method of homogenization	$\bar{E}_1$, GPa	$\bar{E}_2$,GPa	$\overline{\nu}_{12}$,[-]
FEM, stack bond [10]	8.530	6.790	0.196
Periodic FEM model (current investigation)	8.609	6.708	0.200
FEM, running bond [10]	8.620	6.770	0.200
Periodic model, stack bond, [9]	8.568	6.850	0.191
Periodic model, running bond, [9]	8.574	6.809	0.197
Multilayer method Pande et al. [5]	8.525	6.906	0.208
Two-step method [6]	9.187	6.588	0.215
Elliptical cylinder model [7]	7.784	6.315	0.247

4 Conclusions

1. The effective elastic moduli of the masonry RVE have been obtained by means of the finite element homogenization under assumption of orthotropic resulting elastic properties.

2. The multivariant numerical experiments with varying parameters characterizing masonry have been performed. The effects of orientation of bricks on the stress state of masonry RVE has been analyzed.

3. The influence of brick aspect ratio and mortar thickness on the effective elastic moduli and stress state of masonry has been investigated.

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