On some problems of designs structural and topological optimization

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Abstract

Some designs structural and topological optimization problems, which mathematically can be formulated as bilinear control systems are considered. Specifically, by optimizing the structure of a glue layer, realizing friction contact between elastic finite rod vibrating under influence of boundary dynamical perturbations and fixed rigid base, we can reach to rod vibration damping without attraction of additional forces. Optimization of viscoelastic dampers distribution function under a finite elastic beam subjected to a moving load may leads to its vibration damping at a given moment. Topology optimization of an elastic foundation under rectangular plate bending under influence of moving load may provide to plate vibrations damping at given moment. The general technique based on Bubnov–Galerkin procedure is described and applied in all mentioned problems. Results of numerical calculations are also presented.

1 Introduction

Parallel to designs shape and form optimization, their structural and topological optimization is also of great interest [1, 2]. In most general statement design topology optimization requires minimization of a specific optimality criterion, describing material distribution in its spatial domain, which is not necessarily simply connected, under some differential and topological constraints. Design motion or static equilibrium equations may serve as differential constraints, then, domain equations may be considered as topological constraints. In general, such problems mathematically are formulated as constrained optimization problems with nonlinear objective functional.

Having significant practical importance design structural or topological optimization may leads not only to material minimal distribution in given domain, but also impact on processes, in which the design is involved. Nevertheless, such problems of optimization theory are important not only from applications point of view, but stand out also with investigation difficulty, because they are describing by bilinear systems of PDEs, i.e. PDEs with variable controllable coefficients. Bilinear systems are important subclass of nonlinear systems, having many applications in engineering, biology, economics and etc.

Control of bilinear systems have been investigated by many authors. For instance, control of bilinear systems of ordinary differential equations are considered in [3, 4], and of partial differential equations– in [5, 6, 7]. In monograph [5] many interesting problems are discussed, which are modeled by bilinear control systems.

In this paper some practical problems, concerning bilinear control systems are presented. A unified approach for those problems investigation based mainly on Fourier generalized transform is described. Only mathematical models and solution results of each problem are presented.

2 Solution technique

Bilinear control problems mathematically can be formulated in the following manner: choosing a function u from given set $\mathcal{U}$ of admissible controls minimize the criterion

$$\kappa[u] \to \min, \ u \in \mathcal{U},$$

under differential

$$\mathcal{D}_u[w] = \mathcal{P}(\boldsymbol{x}, t), \ t > 0, \tag{1}$$

and geometrical, i.e. topological constraints

$$\boldsymbol{x} \in \Omega.$$

Here $\Omega \subset \mathbb{R}^3$ is domain occupied by the design, $\mathcal{D}_u[\cdot]$ is a differential operator, defined in domain $\Omega \times \mathbb{R}^+$, acting on unknown function $w = w(\boldsymbol{x}, t)$ and containing control function as coefficient of that function, $\mathcal{P} : \mathbb{R}^3 \times \mathbb{R}^+ \to \mathbb{R}$ is a given right–hand side, satisfying certain conditions. Examples of operator $\mathcal{D}_u[\cdot]$ may be found, for instance, in [6, 7].

There should be given some conditions on boundary:

$$w(\boldsymbol{x}, t)\big|_{\boldsymbol{x} \in \partial\Omega} = w_\partial(t), \ t > 0, \tag{2}$$

$\partial\Omega$ as usual denotes the boundary of domain Ω. Some initial data are to be also given:

$$w(\boldsymbol{x}, 0) = w_0(\boldsymbol{x}), \ \left.\frac{w(\boldsymbol{x}, t)}{\partial t}\right|_{t=0} = \dot{w}_0(\boldsymbol{x}), \ \boldsymbol{x} \in \Omega. \tag{3}$$

Ensuring terminal data

$$w(\boldsymbol{x}, T) = w_T(\boldsymbol{x}), \ \left.\frac{w(\boldsymbol{x}, t)}{\partial t}\right|_{t=T} = \dot{w}_T(\boldsymbol{x}), \ \boldsymbol{x} \in \Omega, \tag{4}$$

which usually are taken to be zero, can be stated as main aim of control problem. In this case equation (1) and conditions (2) are considered in finite time interval $[0, T]$.

Now, let us use Bubnov–Galerkin procedure [8] to solve the control problem under consideration. Suppose, that we have already found a complete system of basic (approximate) functions $\{\varphi_k(\boldsymbol{x}, t)\}_{k=0}^n$ for boundary value problem (1)–(2), first one of which satisfies non–homogeneous boundary conditions (2), and the rest part– corresponding homogeneous boundary conditions. Then the residue, obtaining as a result of substituting approximate solution

$$w_n(\boldsymbol{x}, t) = \varphi_0(\boldsymbol{x}, t) + \sum_{k=1}^{n} \alpha_k \varphi_k(\boldsymbol{x}, t), \ \boldsymbol{x} \in \Omega, \ t \in (0, T), \tag{5}$$

where α_k are unknown constants, into equation (1) will be

$$\mathcal{R}_n(\boldsymbol{x}, t) = \mathcal{D}_u[w_n] - \mathcal{P}(\boldsymbol{x}, t), \ \boldsymbol{x} \in \Omega, \ t > 0. \tag{6}$$

According to Bubnov–Galerkin procedure, unknown coefficients α_k should be determined from orthogonality conditions of approximate functions $\{\varphi_k(\boldsymbol{x}, t)\}_{k=0}^n$ to residue (6) [8]:

$$\int_0^T \int_\Omega \mathcal{R}_n(\boldsymbol{x}, t)\varphi_k(\boldsymbol{x}, t) d\boldsymbol{x} dt = 0, \ k = \overline{0; n}. \tag{7}$$

System (7) provides linear algebraic equations with respect to unknown coefficients α_k. After determination of those coefficients and substitution in approximate solution (5), by satisfying terminal data (4) after some algebraic transformations and simplifications with respect to control function we will obtain

$$\int_0^T \int_\Omega u \cdot \mathcal{K}_n(\boldsymbol{x}, t) d\boldsymbol{x} dt = \mathcal{M}_k, \ k = \overline{0; n}, \tag{8}$$

where kernels $\mathcal{K}_n(\boldsymbol{x}, t)$ and constants $\mathcal{M}_k$ depend only on parameters of system (1),(2).

If for some $n_0 \in \mathbb{N}$ the residue (6) is identically zero: $\mathcal{R}_{n_0}(\boldsymbol{x}, t) \equiv 0$, then corresponding function $w_{n_0}(\boldsymbol{x}, t)$ (5) is the exact solution of problem (1),(2). Otherwise, increasing the number n of terms in (5) the exact solution can be approximated by function (5) with required accuracy.

From system (8) the desired control function can be determined in several manners, for example, by method, proposed in [9], treating system (8) as moments problem [9, 10]. Convenience of this approach is that it is possible not only to find the explicit form of controls, but also to establish the conditions for its existence [9]–[13].

Problems, where control function does not explicitly depend on some variables are also important. In such cases we suggest to proceed in as follows. Let us suppose, that control function u does not explicitly depend on parameter t. Then, introducing an operator $\mathcal{A}_T[\cdot]$ defined on whole real axis and acting as

$$\mathcal{A}_T[f] \equiv f_1(\cdot, t) = \begin{cases} f(\cdot, t), \ t \in [0, T]; \\ 0, \ t \notin [0, T], \end{cases}$$

we can extend system (1), (2) for all real t and also include in it initial and terminal data (3), (4). Indeed, it is easy to see, that the explicit form of that operator can be constructed by means of characteristic function (indicator) of segment $[0, T]$:

$$\mathcal{A}_T[f] = \chi_{[0,T]}(t) f(\cdot, t).$$

In view of representation of characteristic function $\chi_{[0,T]}(t)$ by means of Heaviside unit–step function, defined as [14]

$$\theta(t) = \begin{cases} 1, \ t > 0; \\ 0, \ t < 0, \end{cases}$$

as follows [7, 11, 12, 13]

$$\chi_{[0,T]}(t) = \theta(t) - \theta(t - T), \ t \in \mathbb{R},$$

we have

$$\mathcal{A}_T[f] = [\theta(t) - \theta(t - T)] f(\cdot, t).$$

Applying operator $\mathcal{A}_T[\cdot]$ to system (1), (2) we will have

$$\mathcal{D}_u[w_1] = \mathcal{P}_1(\boldsymbol{x}, t) + \Pi(\boldsymbol{x}, t), \ \boldsymbol{x} \in \Omega, \ t \in \mathbb{R}, \tag{9}$$

$$w_1(\boldsymbol{x}, t)\big|_{\boldsymbol{x} \in \partial\Omega} = w_{1\partial}(t), \ t \in \mathbb{R}, \tag{10}$$

at that initial and terminal data are included in term $\Pi(\boldsymbol{x}, t)$, which contains also derivatives of Heaviside function, i.e. Dirac delta functions and its derivatives.

It is easy to observe, that transformed function $w_1(\boldsymbol{x},t)$ is compactly supported in Ω, where it coincides with the main function $w(\boldsymbol{x},t)$.

Applying Fourier real generalized integral transform [14], defined as

$$\mathcal{F}_t[f(\cdot,t)] \equiv \overline{f}(\cdot,\sigma) = \int_{-\infty}^{\infty} f(\cdot,t)e^{i\sigma t}dt,$$

where $\sigma \in \mathbb{R}$ is the transform parameter, and $\mathcal{F}_t[\cdot]$ is the Fourier operator, to system (9), (10) we reduce the order of differential operator $\mathcal{D}_u[\cdot]$ and eliminate the variable, on which control function does not explicitly depend:

$$\mathcal{L}_u[\overline{w}_1] = \overline{\mathcal{P}}_1(\boldsymbol{x},\sigma) + \overline{\Pi}(\boldsymbol{x},\sigma),\ \boldsymbol{x} \in \Omega,\ \sigma \in \mathbb{R}, \tag{11}$$

$$\overline{w}_1(\boldsymbol{x},\sigma)\big|_{\boldsymbol{x}\in\partial\Omega} = \overline{w}_{1\partial}(\sigma),\ \sigma \in \mathbb{R}, \tag{12}$$

where $\mathcal{L}_u[\cdot] = \mathcal{F}_t[\mathcal{D}_u[\cdot]]$.

Writing approximate solution $\overline{w}_{1n}(\boldsymbol{x},\sigma)$ of system (10), (11) according to Bubnov–Galerkin procedure (5)–(7), and taking into account, that corresponding function $w_{1n}(\boldsymbol{x},t)$ is compactly supported in Ω, we can proceed, for instance, as it is done in [15]. Namely, well known Wiener–Paley–Schwartz theorem states [14, 15], that the extension of $\overline{w}_1(\boldsymbol{x},\sigma)$ function in whole complex plane $\mathbb{C}$ is an analytic entire function of variable $z = \sigma + i\varsigma$, satisfying inequality

$$|z^\nu \overline{w}_1(\boldsymbol{x},z)| \le C_\nu e^{\vartheta|\varsigma|}, z \in \mathbb{C},$$

for all $\boldsymbol{x} \in \Omega$, $\nu = 0,1,2,...$ and corresponding real constants C_ν. ϑ is a positive real parameter, depending only on parameter T.

Extending $\overline{w}_{1n}(\boldsymbol{x},\sigma)$ function for all $z = \sigma + i\varsigma$, in order to fulfill conditions of Wiener–Paley–Schwartz theorem we must provide non–existence of poles like singularities for $\overline{w}_{1n}(\boldsymbol{x},z)$ function. It can be done, for example by equating to zero the denominator of that extension namely in such points of complex plane, where its numerator turns to zero. Those two conditions will provide necessary constrains, from which optimal controls can be found. For example, if we use Bubnov–Galerkin procedure to solve the problem under consideration, then determining required constants α_k from system (7) of linear algebraic equation as

$$\alpha_k = \frac{\Delta_k}{\Delta},\ k = \overline{1;n},$$

where Δ and Δ_k are main and auxiliary determinants of system (7), and then substituting them in approximate solution form $\overline{w}_{1n}(\boldsymbol{x},z)$, extended for all $z = \sigma + i\varsigma$, we must equate to zero functions $\Delta_k(z)$, where functions $\Delta(z) = 0$. It can be proved [15], that if one of auxiliary determinants $\Delta_k(z)$ for a fixed k is equal to zero in zeros of functions $\Delta(z) = 0$, then the rest determinants $\Delta_k(z)$ are equal to zero simultaneously, therefore we may use only one of them.

Function $w_{1n}(\boldsymbol{x},t)$, defined as Fourier inverse transform of function $\overline{w}_{1n}(\boldsymbol{x},\sigma)$ satisfying Wiener–Paley–Schwartz theorem conditions, is compactly supported in $[-\vartheta,\vartheta]$, which can be reduced to $[0,T]$ by linear change of variable t:

$$t = \frac{t_* + \vartheta}{2\vartheta}T.$$

In this case also we may reduce the solution to some moments problem. Besides, the most notable fact is that in this case unlike [15] we are able to reduce controls determination to *finite* system of moments problem.

3 Distribution optimization

Longitudinal vibration damping problem is considered for a finite elastic rod by optimizing adhesive (glue) layer distribution between some part of the rod and a fixed rigid base. The rod is subjected to boundary dynamical perturbations, while the other end of rod is elastically clamped (Fig. 1). Glue layer distribution intensity in contact area should be minimized.

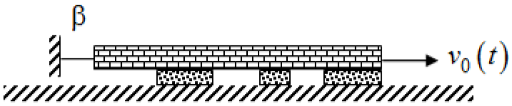

Figure 1: Illustration of the rod.

Assuming, that adhesive layer is deformed in pure shear conditions, differential equations of rod vibrations takes the form [7]

$$\mathcal{D}_u[w] \equiv \frac{\partial^2 w(x,t)}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 w(x,t)}{\partial t^2} - \alpha^2 u(x) w(x,t) = 0,\ (x,t) \in (-l,l) \times (0,T),$$

under boundary conditions

$$\left[E\frac{\partial w(x,t)}{\partial x} - \gamma w(x,t)\right]\Bigg|_{x=-l} = 0,\ E\frac{\partial w(x,t)}{\partial x}\Bigg|_{x=l} = [\theta(t) - \theta(t-\tau)]v(t),$$
$$t \in (0,T).$$

Here E is rod Young modulus, γ is clamping factor, α^2 represents the ratio of glue layer and rod elastic properties and geometric characteristics. Dimensionless function $u(x)$ denotes control factor and describes adhesive layer distribution in contact area.

It is assumed, that boundary perturbations vanishes at some moment $\tau > 0$, and our aim is the providing terminal data at given moment $T > \tau$

$$w(x,T) = \frac{\partial w(x,t)}{\partial t}\Bigg|_{t=T} = 0,\ x \in (-l,l),$$

for certain initial ones.

Set $\mathcal{U}$ contains essentially bounded measurable non–negative functions, compactly supported in $[-a,l] : \mathcal{U} = \{0 \le u \in L^\infty[-a,l] : u(x) \equiv 0, x \notin [-a,l]\}$.

Applying the part of the technique described above, which is not include Bubnov–Galerkin procedure, it is proved, that

Theorem 1 ([7]). *Resolving control function $u^o(x)$ optimal in the sense of criterion [9]*

$$\kappa[u] = \max_{x\in[-a,l]} u(x),\ u \in \mathcal{U},$$

is defined as

$$u^o(x) = \sum_{j=0}^{N}[\theta(x - x^o_{2j}) - \theta(x - x^o_{2j+1})],\ x \in [-a,l], \tag{13}$$

and determined by specifying switching points $-a < x^o_{2j} < x^o_{2j+1} < 1$ ($x^o_0 = -a$, and $x^o_{2N+1} = l$). The switching points are calculated from system of restrictions of equality type:

$$\Gamma[x^o_{2j}, x^o_{2j+1}, z_k] = M_k,\ k = 1,2,3,...,$$

where, in general, complex numbers z_k are determined from characteristic transcendent equation reads as:

$$\Lambda_\beta(z)\sin[\lambda_-(z)l] + \cos[\lambda_-(z)l] = 0,\ z \in \mathbb{C}.$$

Here

$$\lambda_+(\sigma) = \frac{|\sigma|}{c},\ \lambda_-(\sigma) = \left[\frac{\sigma^2}{c^2} - \alpha^2\right]^{\frac{1}{2}}.$$

The rest notations are given in [7].

Control function $u^o(x)$ (15) corresponds to piecewise realized contact between rod and rigid base. $\Pi(x,t)$ function (9) contains initial functions and in this case is equal to:

$$\Pi(x,t) = -\frac{1}{c^2}[w_0(x)\delta'(t) + \dot{w}_0(x)\delta(t)],$$

where $\delta(t)$ is Dirac delta function.

Numerical calculations are done and switching points are determined for some values of system parameters α, a, τ. It is observed, that with increasing parameter α, switching points get closer to each other, which corresponds to almost discrete contact between rod and rigid base, which should be expected.

Bending vibration damping problem is considered for a simply supported, finite, elastic beam, subjected to moving load with constant speed and intensity by optimizing viscoelastic dampers distribution law under the beam (Fig. 2). The density of dampers distribution under the beam should be minimized.

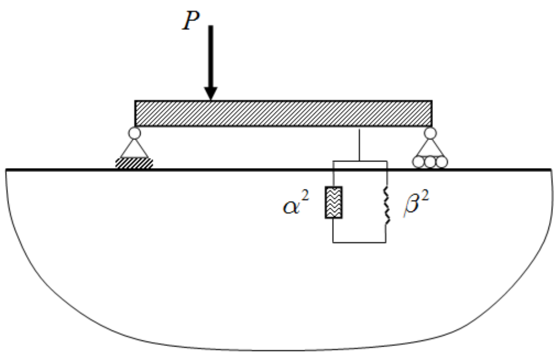

Figure 2: Illustration of the beam with viscoelastic damper.

On the basis of Kelvin–Voigt model of viscoelastic body [16]–[19] beam bending vibrations can be described by partial differential equation with variable controllable coefficient (all quantities and variables are dimensionless)

$$\mathcal{D}_u[w] \equiv \frac{\partial^4 w(x,t)}{\partial x^4} + u(x)\left[\alpha^2\frac{\partial w(x,t)}{\partial t} + \beta^2 w(x,t)\right] + \gamma^2\frac{\partial^2 w(x,t)}{\partial t^2} = F(x,t),$$
$$x \in (-1,1),\ t \in (0,T),$$

under boundary conditions, corresponding to simply supported ends of the beam

$$w(-1,t) = \left.\frac{\partial^2 w(x,t)}{\partial x^2}\right|_{x=-1} = 0,\ w(1,t) = \left.\frac{\partial^2 w(x,t)}{\partial x^2}\right|_{x=1} = 0,\ t \in (0,T).$$

Above α^2 is viscous, and β^2– elastic factors of dampers. Right hand side of equation (16) describes influence of moving load and have the form

$$F(x,t) = P\delta(x+1-vt)\cdot[\theta(t)-\theta(t-\tau)],\ x\in(-1,1),\ t\in(0,T).$$

It is supposed, that the moving load isolated from the beam in a given moment $\tau > 0$. Our aim is the providing terminal data at given moment $T > \tau$

$$w(x,T) = \left.\frac{\partial w(x,t)}{\partial t}\right|_{t=T} = 0,\ x\in(-1,1).$$

Set $\mathcal{U}$ contains measurable non–negative functions, compactly supported in $[-1,1]$: $\mathcal{U} = \{0 \le u \in L^1[-1,1] : u(x)\equiv 0, x\notin[-1,1]\}$, which is everywhere dense in $L^1[-1,1]$.

Taking system $\{\sin(\pi kx)\}_{k=1}^{n}$ as basis system of approximate functions and applying the whole technique described in section 2, we have

Theorem 2. *Resolving control function $u^o(x)$ optimal in the sense of criterion [9, 11, 12, 13]*

$$\kappa[u] = \int_{-1}^{1} u(x)dx,\ u\in\mathcal{U},$$

is defined as

$$u^o(x) = \sum_{j=1}^{N}\delta(x-x_j^o),\ x\in[-1,1], \tag{14}$$

and determined by specifying switching points $-1 < x_j^o < x_{j+1}^o < 1$. The switching points are calculated from system of restrictions of equality type:

$$\Delta_k(x_j^o, z_\iota) = 0,\ \iota=\overline{1;2n}, \tag{15}$$

where, in general, complex numbers z_ι are determined from characteristic transcendent equation

$$\Delta(z) = 0.$$

Here Δ and Δ_k are the main and auxiliary determinants of linear system

$$\sum_{k=1}^{n}\alpha_k\Lambda_{km} = \Omega_m,\ m=\overline{1;n},$$

where

$$\Lambda_{km} = \left[(\pi k)^4 - \gamma^2\sigma^2\right]\delta_k^m + [\beta^2 - i\sigma\alpha^2]J_{km}[u],$$

$$J_{km}[u] = \int_{-1}^{1} u(x)\sin(\pi kx)\sin(\pi mx)dx,\ \Omega_m = \int_{-1}^{1}\overline{\Pi}(x,\sigma)\sin(\pi mx)dx,$$

δ_k^m is the Kronecker delta.

Control function $u^o(x)$ (14) corresponds to discrete (piecewise) distribution of dampers under the beam. $\Pi(x,t)$ function (9) contains initial function and in this case is equal to:

$$\Pi(x,t) = F(x,t) + \alpha^2 u(x)w_0(x)\delta(t) + \gamma^2[w_0(x)\delta'(t) + \dot{w}_0(x)\delta(t)].$$

Calculation of switching points from system of nonlinear restrictions, obtained by decomposition of real and imaginary parts of equation (15) can be interpreted as problem of non–linear programming and attacked by efficient numerical methods [20]. Numerical calculations are done and switching points are determined for some values of system parameters $\alpha,\beta,\gamma,\tau,P,v$.

4 Topology optimization

An elastic, isotropic, solid, homogeneous rectangular plate of thickness $2h$ is considered, the middle plane of which occupies domain

$$\Phi = \{(x, y) : x \in [-l, l], y \in [-d, d]\} \in \mathbb{R}^2,\ \min(l, d) \gg h.$$

Let the plate is lying on one parametric linear–elastic foundation distributed in domain Φ by a controllable law, simply supported by edges $x = \pm l$ and $y = \pm d$, as well as subjected to a moving load with intensity P distributed on the upper surface of plate by nonzero law $r = r(y)$. Then, on the basis of Kirchhoff hypothesis differential equation of plate middle plane bending vibrations takes the form

$$\mathcal{D}_u[w] \equiv D\Delta\Delta w(x, y, t) + \alpha^2 u(x, y)w(x, y, t) + 2\rho h \frac{\partial^2 w(x, y, t)}{\partial t^2} = F(x, y, t),$$
$$(x, y) \in \Phi,\ t \in (0, T),$$

under boundary conditions corresponding to plate simply supporting will be

$$w(\pm l, y, t) = \left.\frac{\partial^2 w(x, y, t)}{\partial x^2}\right|_{x=\pm l} = 0,\ w(x, \pm d, t) = \left.\frac{\partial^2 w(x, y, t)}{\partial y^2}\right|_{y=\pm d} = 0.$$

where $w(x, y, t)$ is the vertical component of plane displacement, D is the plate flexural rigidity, ν is the plate Poisson ratio, $u(x, y)$ is the intensity of foundation distribution in domain Φ, α^2 is the subgrade coefficient, $F(x, y, t)$ characterizes the influence of moving load on plate:

$$F(x, y, t) = P[\theta(t) - \theta(t - \tau)]\delta(x + l - vt)r(y),$$

$\tau < T$ is the moment of moving load isolation from plate surface, v is the constant velocity of load motion.

Our aim is $u^o(x, y)$ control function determination, having minimal intensity among admissible controls $u \in \mathcal{U}$, ensuring terminal data at given moment T

$$w(x, y, T) = 0,\ \left.\frac{\partial w(x, y, t)}{\partial t}\right|_{t=T} = 0,\ (x, y) \in \Phi,$$

if some initial ones are given.

Set $\mathcal{U}$ contains essentially bounded measurable non–negative functions, compactly supported in $\Phi : \mathcal{U} = \{0 \le u \in L^\infty(\Phi) : u(x, y) \equiv 0, (x, y) \notin \Phi\}$.

Introducing dimensionless variables and quantities, applying the technique described above, it is proved, that applying the whole technique described in section 2 for system of approximate functions $\{\sin(\pi kx)\sin(\pi ky)\}_{k=1}^n$, we have

Theorem 3. *Resolving control function $u^o(x, y)$ optimal in the sense of criterion [7, 9]*

$$\kappa[u] = \max_{(x,y)\in\Phi} u(x, y),\ u \in \mathcal{U},$$

is defined as

$$u^o(x, y) = \sum_{j=1}^{N} [\theta(x - x_j^o) - \theta(x - x_{j+1}^o)] \cdot [\theta(y - y_j^o) - \theta(y - y_{j+1}^o)], \tag{16}$$

and determined by specifying switching points $\{x_j^o, y_j^o\}_{j=1}^N \subset (-1,1)\times(-1,1)$. *The switching points are calculated from system of restrictions of equality type:*

$$\Delta_k(x_j^o, y_j^o, z_\iota) = 0,\ \iota = \overline{1;2n}, \tag{17}$$

where, in general, complex numbers z_ι *are determined from characteristic transcendent equation*

$$\Delta(z) = 0.$$

Here Δ and Δ_k are main and auxiliary determinants of system

$$\sum_{k=1}^{n} \alpha_k \Lambda_{km} = \Omega_m,\ m = \overline{1;n},$$

where

$$\Lambda_{km} = [\mathcal{D}\Gamma_k - \sigma^2]\delta_k^m + \alpha^2 J_{km}[u],\ \Gamma_k = \pi^4 k^4 [1+\gamma^2]^2,$$

$$\Omega_m = \int_{-1}^{1}\int_{-1}^{1} \overline{W}(x,y,\sigma)\sin(\pi m x)\sin(\pi m y)dxdy,$$

$$J_{km}[u] = \int_{-1}^{1}\int_{-1}^{1} u(x,y)\sin(\pi k x)\sin(\pi m x)\sin(\pi k y)\sin(\pi m y)dxdy.$$

Control function $u^o(x,y)$ (16) corresponds to discrete (piecewise) distribution of dampers under the beam. $\Pi(x,t)$ function (9) contains initial functions and in this case is equal to:

$$\Pi(x,t) = F(x,y,t) + w_{01}(x,y)\delta'(t) + \dot{w}_{01}(x,y)\delta(t).$$

Calculation of switching points from system of nonlinear restrictions, obtained by decomposition of real and imaginary parts of equation (17) can be interpreted as problem of non–linear programming and attacked by efficient numerical methods [20]. Numerical calculations are done and switching points are determined for some values of system parameters.

Optimal topology of elastic foundation is plotted against as moving load, as well as plate elastic and geometric characteristics (Fig. 3–4).

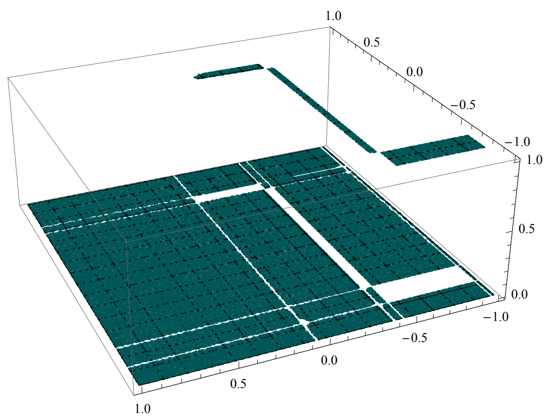

Figure 3: Illustration of elastic foundation optimal topology.

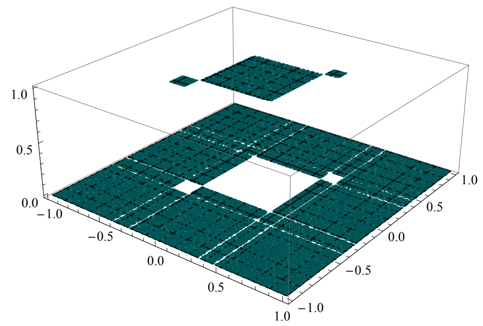

Figure 4: Illustration of elastic foundation optimal topology.

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