Calculation of the period of toroidal bubble pulsations during electrical discharge in an electrolyte in adiabatic approximation

Ruslan N. Medvedev   Vyacheslav S. Teslenko
ruslan@hydro.nsc.ru

Abstract

This paper presents a calculation of the period of oscillations of a toroidal bubble as a function of its maximum radius in self-sustained oscillations in an electrolyte. This dependence can be used to estimate the time of bubble growth on circular or annular current concentrators in an adiabatic approximation. A comparison with experimental results for circular diaphragms 0.1 mm in radius and annular concentrators 3 and 4.3 mm in radius shows that for sufficiently rapid release of energy on the current concentrators bubble oscillations are fairly accurately described by the dependence that was obtained.

1 Introduction

The mechanism of electrohydrodynamic self-oscillations on current concentrators in electrolyte have been described [1-8] for concentrators in the form of circular or annular holes in a dielectric film and in the form of circular or annular electrodes insulated laterally and placed in an electrolyte. In the case of diaphragm concentrators, a dielectric film is placed between the electrodes so that the current can flow only through the holes. In the case of metal current concentrators, the concentrator is a positive electrode with a small surface area, and the opposite electrode in the form of a plate of large area is placed at a large distance from the current concentrator.

It has been shown that on both annular and circular current concentrators bubbles form at the outer edge of the concentrator and grow in the form of a torus, which subsequently (in the case of a circular concentrator) is closed at the center and, thus, completely overlaps the current concentrator. Further growing, the bubble assumes a quasi-spherical shape, cools, and collapses, after which the process occurs again.

The maximum size of the bubble and its oscillation period depend on the applied voltage and electrolyte conductivity. A linear empirical relationship with a proportionality coefficient of about 0.8 s/m was found between the oscillation period and the radius of a circular hole a for $a = 0.05 - 0.5$ mm and voltages above 200 V [1].

This paper presents an approximate analytical calculation of the oscillation period of a toroidal bubble as a function of its maximum size using the relations described in [9], which will allow to calculate the parameters for designing self-sustained oscillators of a given frequency.

2 Theoretical calculations

Kedrinskii [9] calculated oscillations of a toroidal bubble and obtained an expression for the kinetic energy of the liquid for the growth of a toroidal bubble in an infinite volume of an ideal liquid in the approximation of a thin torus (the cross-sectional radius is much smaller than the outer radius $R << a$). The torus in cylindrical coordinates is shown in Fig. 1.

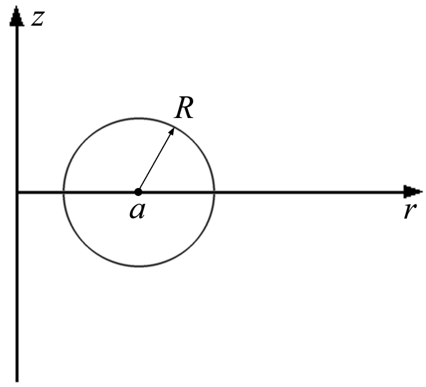

Figure 1: Cross section of a toroidal bubble in cylindrical coordinates.

In the approximation $R << a$ in cylindrical coordinates, the expression given in [9] for the kinetic energy of a moving liquid caused by an expanding bubble has the form:

$$K = 2\pi^2 \rho \dot{R}^2 R^2 a \ln \frac{8a}{R}, \tag{1}$$

where ρ is the density of the liquid and $\dot{R}$ is the velocity of the bubble boundary.

We introduce the dimensionless coordinates: $x = r/a$ and $y = z/a$ and normalize the liquid velocity $v(r, z)$ by the velocity of the bubble boundary: $\tilde{v} = \frac{v}{\dot{R}}$. Then, formula (1) can be written in dimensionless form

$$\tilde{K} = \frac{K}{\rho \dot{R}^2 a^3} = 2\pi^2 \tilde{R}^2 \ln \frac{8}{\tilde{R}}, \tag{2}$$

where $\tilde{R} = R/a$ is the dimensionless inner radius of the torus.

To find the limits of applicability of formula (1), we performed a numerical solution of the continuity equation for the expansion of a toroidal bubble in liquid using cylindrical coordinates. Assume the flow is nonvortical.

We introduced a dimensionless velocity potential φ: $\tilde{v} = -\nabla\varphi$. The velocity distribution was obtained by solving the Laplace equation in a dimensionless cylindrical system (the system is symmetrical with respect to the azimuthal angle):

$$\frac{1}{x}\frac{\partial}{\partial x}\left(x\frac{\partial \varphi}{\partial x}\right) + \frac{\partial^2 \varphi}{\partial y^2} = 0. \tag{3}$$

This equation was solved numerically by the finite element method using an irregular triangular mesh with sizes $x \in (0 : 100)$, $y \in (-100 : 100)$ with boundary conditions $\varphi|_{x,y\to\infty} \to 0$, $\left.\frac{\partial\varphi}{\partial x}\right|_{x=0} = 0$, $\left.\left(\partial\varphi/\partial x\right)^2 + \left(\partial\varphi/\partial y\right)^2\right|_{(x-1)^2+y^2=\tilde{R}^2} = 1$,

$\frac{\partial \varphi}{\partial y}\Big|_{y=0; x\leq 1-\tilde{R}, x\geq 1+\tilde{R}} = 0$, which correspond to the equality the potential to zero at infinity, equality of normal component of velocity to zero at the axis of symmetry and in the plane of the tore axis, absence of the phase penetration through the bubble's boundary. The inner radii of the torus were chosen in the range $\tilde{R} = 0.01 \div 0.99$.

After finding the numerical solution of equation (3), we calculated the gradient to find the dimensionless velocity. Next, it was integrated to find the kinetic energy $\tilde{K}_{num}$ by the formula:

$$\tilde{K}_{num} = \frac{\pi}{3} \sum_i (\tilde{v}_{i,1}^2 x_{i,1} + \tilde{v}_{i,2}^2 x_{i,2} + \tilde{v}_{i,3}^2 x_{i,3}) S_i, \quad (4)$$

where S_i is the area of the *i-th* triangular element of the mesh, and the summation over the nodes of each *i-th* triangular element of the mesh is written in parentheses.

The acceptability of the chosen size of the mesh was evaluated by calculating the kinetic energy for the volume $x \in (0:200)$ and $y \in (-200:200)$ for $\tilde{R} = 0.99$. The difference was 2%, so our choice is satisfactory.

Figure 2 shows a plot of the dimensionless kinetic energy $\tilde{K}_{num}$ versus the radius of the torus in comparison with expression (2). It is seen that the points of the numerical solutions agree with formula (2) up to $\tilde{R} = 1$ with accuracy no worse then 12%. Thus, it can be concluded that formula (2) adequately describes the kinetic energy of the liquid for the expansion of a toroidal bubble in the range $R \leq a$.

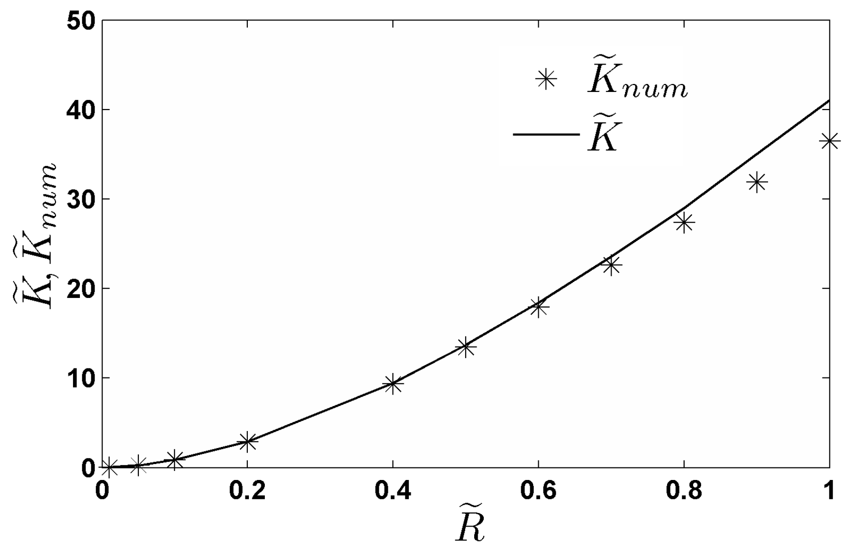

Figure 2: Kinetic energy versus radius of the torus in dimensionless variables.

The energy conservation law for the adiabatic expansion of a toroidal bubble in a liquid is written as:

$$E = K + A + W = 2\pi^2 \rho \dot{R}^2 R^2 a \ln \frac{8a}{R} + p_a V + \frac{pV}{\gamma - 1}, \quad (5)$$

where A is the work against the external pressure, W is the internal energy of the bubble, p_a is the pressure (atmospheric) in the liquid, p is the gas pressure in the bubble, $V = 2\pi^2 a R^2$ is the volume of the bubble, and γ is the adiabatic exponent of the gas in the bubble.

In an adiabatic expansion of the bubble, the pressure in it is given by

$$p(R) = p_0 \left(\frac{R_0}{R}\right)^{2\gamma}, \tag{6}$$

where p_0 and R_0 are the initial pressure and bubble radius.

At the initial time, the bubble has a minimum radius, K=0 and A=0, and all the energy is concentrated in the internal energy of the bubble. At the time of the maximum size, K=0. Equating the energy at the beginning of the expansion to the energy at the time of the maximum radius and using the expression for the bubble pressure (6), we obtain $p_0 R_0^2 = (\gamma-1) p_a R_{\max}^2 + p_0 R_0^2 \left(\frac{R_0}{R_{\max}}\right)^{2\gamma-2}$. It is seen that the second term on the right side is negligible compared with the left side, i.e., at the end of the bubble growth, its internal energy is negligible compared with the work done, so that we can write $E \approx 2\pi^2 a R_{\max}^2 p_a$. Substituting this expression into (5) and expressing p_0 as $p_0 = (\gamma-1) p_a \left(\frac{R_{\max}}{R_0}\right)^2$, we obtain:

$$\rho \dot{R}^2 R^2 \ln\left(\frac{8a}{R}\right) = p_a (R_{\max}^2 - R^2) - p_a R_{\max}^2 \left(\frac{R_0}{R}\right)^{2\gamma-2}. \tag{7}$$

The second term on the right side of (7) is significant only in the initial stage of growth when $R \sim R_0$ and it is therefore can be ignored. Next, expressing the velocity of the bubble boundary from (7), we obtain the relation:

$$\dot{R}^2 = \frac{p_a (R_{\max}^2 - R^2)}{\rho R^2 \ln(8a/R)}, \tag{8}$$

which helps to obtain the dependence of the time of expansion of the bubble on its radius, i.e., the solution of the inverse problem:

$$t = a\sqrt{\frac{\rho}{p_a}} \int_0^{\tilde{R}} \sqrt{\frac{\ln(8/\xi)}{\tilde{R}_{\max}^2 - \xi^2}}\, \xi d\xi. \tag{9}$$

From equation (9), we can obtain the expression for the oscillation period of a toroidal bubble by setting the upper limit of integration equal to $\tilde{R}_{\max}$ and doubling the right side:

$$\tau = 2a\sqrt{\frac{\rho}{p_a}} \int_0^{\tilde{R}_{\max}} \sqrt{\frac{\ln(8/\xi)}{\tilde{R}_{\max}^2 - \xi^2}}\, \xi d\xi. \tag{10}$$

The integral in (10) cannot be expressed in terms of elementary functions, numerical integration shows that it can be described by the approximate relation $1.65\tilde{R}_{\max}$ with accuracy 7%. Thus, we obtain an approximate relation between the oscillation period of a toroidal bubble and its maximum radius:

$$\tau \approx 3.3 R_{\max} \sqrt{\rho/p_a}. \tag{11}$$

A comparison of (11) with the well-known Rayleigh formula for a spherical bubble [10] shows that the oscillation period of a toroidal bubble is almost twice that of a spherical bubble.

3 Experiments

The experiments were performed in a 1-5% NaCl solution in water (mass concentration) with diaphragms and metal current concentrators of the following types:

1) concentrator in the form of a circular hole of radius a= 0.1 mm in a Teflon dielectric film of the thickness of 20 μm;

2) concentrator in the form of an annular hole of radius a= 4.3 mm and 0.35 mm wide in a polyester film of the thickness of 0.1 mm;

3) metal annular electrode (of positive polarity) of radius a = 3 mm and 0.3 mm wide.

The range of voltages applied to the electrodes from the discharge capacitor was 100 – 800 V. Bubble oscillations were recorded by the high-speed video camera MotionXtra GE-LE. In some cases, an inductance coil L = 7.7 – 18 mH was connected to the discharge circuit to increase the energy release in bubbles [6].

The maximum bubble radius and oscillation period were determined from video records. The delay from the bubble collapse to the initiation of growth of the next bubble, required for the overheating of the electrolyte by the current, was not taken into account in the data processing.

Figure 3 compares the experimental dependence of the dimensionless period of bubble oscillations $\tilde{\tau} = \tau * \frac{1}{a}\sqrt{\frac{p_a}{\rho}}$ on their maximum radius $\tilde{R}_{\max}$ with the theoretical formula:

$$\tilde{\tau} \approx 3.3\tilde{R}_{\max}. \quad (12)$$

It is seen that the results of experiments with the inductance coil L connected to the discharge circuit differ from those without that one. For circular diaphragms, the maximum radius with the connection of the coil for the same period exceeds the radius without the coil.

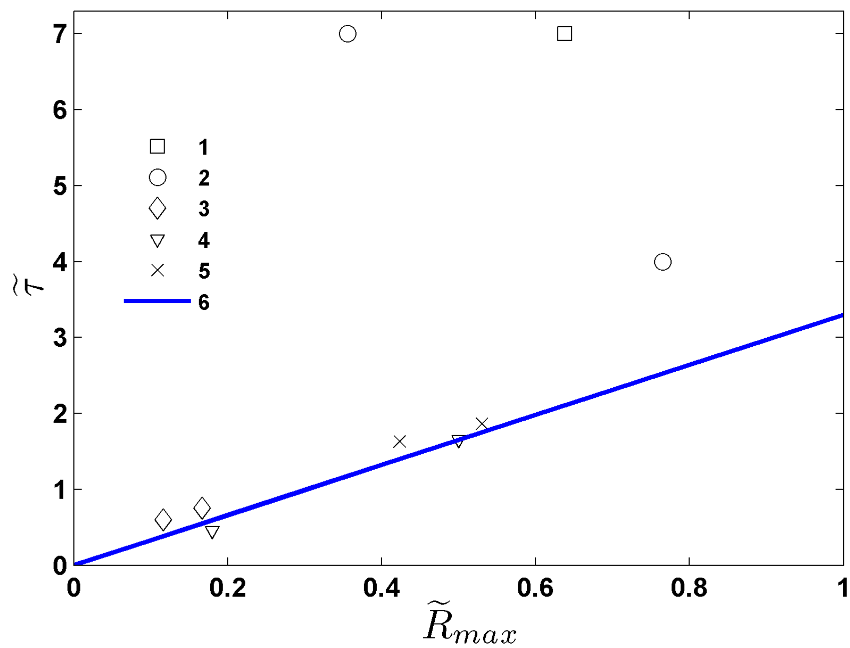

Figure 3: Bubble oscillation period versus maximum radius: 1) – setup (1), L=7,7mH, 2) – setup (1), L=0, 3) – setup (3), L=0, 4) – setup (3), L=7,7mH, 5) – setup (2), L=7,7÷18mH, 6) – formula (12).

4 Analysis of results

It is evident from the plot in Fig. 3 that for annular concentrators, the bubble oscillation periods are in good agreement with the approximate formula (12), whereas for circular holes, this is not so. Some agreement with theory is observed only for relative radii obviously larger than unity. This is due to the fact that the adiabatic model of bubble oscillations is not applicable in this case because the bubble growth on a circular concentrator is accompanied by heating of the electrolyte by the current in the central part of the hole, which causes evaporation into the bubble during its growth. In this case, two mechanisms operate: adiabatic (due to the initial energy of the bubble) and thermal (due to evaporation from the walls) [11]. The thermal mechanism dominates for slow energy release, and the adiabatic mechanism for rapid energy release, as is seen on the plot. We note that for slow energy release, the bubble shape differs significantly from a toroidal shape; in this case, the hole is gradually filled with coalescing bubbles from the edge to the center.

After reaching the hole radius and closure at the center, a toroidal bubble rapidly takes the shape of a sphere and then grows and collapses as a spherical one. Therefore, the slope of the curve of the period versus maximum size decreases, and in the limit $\tilde{R}_{\max} \to \infty$, it tends to 1.83 [10].

It should be borne in mind that the self-oscillation period is longer than the time of bubble oscillations because there is a delay in the formation of the bubble relative to the start of current flow.

5 Conclusions

It is shown that relation (1) describes the kinetic energy of an expanding toroidal bubble with an accuracy of 12% up to the closure of the bubble boundary at the center ($R = a$).

Using formula (1), an approximate relation between the oscillation period of a toroidal bubble and its maximum size (11) was obtained which is consistent with experimental data on self-sustained oscillations on annular current concentrators.

For self-sustained oscillations on circular concentrators, the adiabatic approximation can be used only for sufficiently rapid release of energy (high voltage across the electrodes). The dominant mechanism in this case is generally the thermal one.

Most of the papers in reference list can be found at http://swsl.newmail.ru.

Acknowledgements

This work was supported by the Russian Federation for Basic Research (Grant No. 13-08-00838).

References

[1] V. S. Teslenko, A. P. Drozhzhin and A. M. Kartashov. Autooscillations generated during a diaphragm discharge in an electrolyte. TPL, 2001, v. 27, no. 10, pp. 883-885.

[2] V. S. Teslenko, A. P. Drozhzhin and G. N. Sankin. Autocyclic Circular Breakdowns with Induced Bubble Collapse in Electrolyte. TPL, 2006, Vol. 32, No. 2, pp. 149-152.

[3] V. S. Teslenko, R. N. Medvedev, A. P. Drozhzhin and G. N. Sankin. Mechanism of self-oscillations on diaphragm current concentrators // Proceedings of the 8th International

Conference Modern Problems of Electrophysics and Electrohydrodynamics of Liquid, St. Petersburg, 2006, pp. 139-142.

[4] V. S. Teslenko, R. N. Medvedev, A. P. Drozhzhin. Electrohydrodynamic self-oscillation effect on multiple current concentrators // Procs. of the IIIrd International Scientific Congress GEO-Siberia-2007, 25-27.04.2007, SGGA, Novosibirsk, 2007, vol. 4, part 2, pp. 178-183.

[5] V. S. Teslenko, R. N. Medvedev, A. P. Drozhzhin, Self-Synchronization of Electrohydrodynamic Autooscillations during Multichannel Discharges in Electrolyte // TPL, 2007, Vol. 33, No. 10, pp. 833-836.

[6] R. Medvedev, V. Teslenko, A. Drozhzhin. Electrohydrodynamic self-synchronization of self-oscillations on two diaphragm current concentrators in electrolyte // Physics Letters A, 2008, vol. 373, pp. 102-106.

[7] V. S. Teslenko, R. N. Medvedev, A. V. Zaikovskii. Self-oscillations on annular current concentrators in an electrolyte // Dynamics of Continuous Media, 2010, issue 126, pp. 146-155.

[8] V. S. Teslenko, R. N. Medvedev. Electrohydrodynamic self-oscillations on linear and circular current concentrators in electrolyte // TPL, 2011, Vol. 37, No. 5. pp. 472-475.

[9] V. K. Kedrinskii. Hydrodynamics of explosion: experiment and models. Izd-vo SO RAN, Novosibirsk, 2000.

[10] K. A. Naugol'nykh and N. A. Roi. Electrical discharges in water. Nauka, Moscow, 1971.

[11] V. P. Skripov, Metastable liquid, Nauka, Moscow, 1972.

Ruslan N. Medvedev, Lavrentyeva ave. 15, Novosibirsk, Russia
Vyacheslav S. Teslenko, Lavrentyeva ave. 15, Novosibirsk, Russia