Mullins effect in the calculation of the stress-strain state of a car tire

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Abstract

In this paper, the stress-strain state calculation takes into account the Mullins effect. Experimental data are shown that demonstrate the substantial change in the properties of rubber after reloading. It is shown experimentally that the effect of the softening depends on the initial deformation of the elastomeric composite. Calculation of the stress-strain state of the tires was done both with and without the Mullins effect. This shows that there is a different softening of the material in different points of the tire, which indicates that it is necessary to take into account the Mullins effect when calculating stress-strain state, and do it considering the geometrical configuration of the tire and the strain at each point.

1 The experiment

To assess the Mullins effect, a cyclic tension test with increasing amplitude was performed. In the experiment, ring samples with an external diameter of 52 mm, an internal diameter of 44 mm, and a thickness of 4 mm were investigated. Owing to their shape, samples were easy to take out of grips and, as a result, the experiment procedure became much simpler. The extension-compression rate was chosen to be equal to 1% per minute. Under such slow loading, the stress-strain curve is close at most to the equilibrium curve, which is made possible because of the completion of polymer fiber sliding over the surface of inclusions. At each deformation level, the dwell time was taken to be equal to 10 minutes. To perform experiments, a universal test machine (Zwick Z-250) was used.

Figure 1 presents the plot of cyclic stretching of rubber compound with increasing amplitude, where the effect of softening is readily seen. The higher is deformation, the stronger is the softening effect. After reloading and 10 minute equalizing, the material partially restores, and the residual strain decreases compared to that occurred immediately after reloading. In the case of further stretching up to the previous strain, the curve is situated below the curve illustrating the previous stretching — material softening. When the value of the previous strain is exceeded, the behavior of the material is described by the stress-strain curve of the unprocessed sample, yet, after double elongation, it cannot be described by a single stress-strain curve. This is indicative of the onset of damage accumulation. In the experiment, it has been found that the mechanical properties of rubber compounds vary after the first stretching. The material softens, and the softening value depends on the previous deformation of elastomeric composites. Therefore, it seems incorrect to ignore the Mullins effect in calculations.

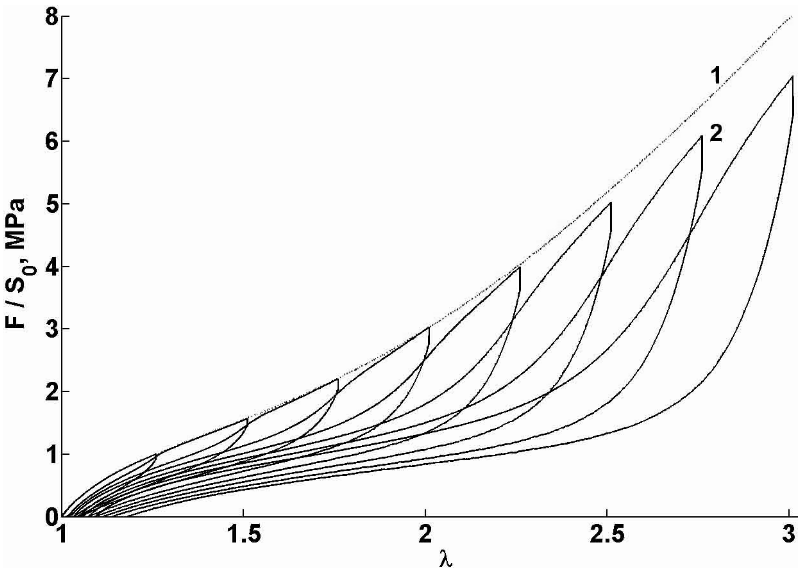

Figure 1: Cyclic stretching of rubber compound with increasing amplitude;
1 — stress-strain curve for the unprocessed sample, 2 — stress-strain curve under cyclic loading, $\mathbf{F}/\mathbf{S_0}$ — stress, $\lambda = 1 + \varepsilon/100$ — elongation ratio

2 Mathematical modeling

In the proposed numerical experiment, we simulate the Mullins effect in order to determine the stress-strain state in car tires. Taking into account the softening effect in rubber compound provides correct description of the stress-state state of a tire. What is more, in this case the composition of carbon black can be changed, which makes it possible to increase tire life, to improve tire grip, to find optimal tire designs and to increase the strength of the tire structure to improve fuel economy.

A car tire undergoes large deformations, and therefore for its modeling the nonlinear theory of elasticity is used. In calculations, the Ogden model is applied [7].

The stress-state state of a tire is modeled using the commercial finite element ABAQUS, which describes the Mullins effect [8, 9]. To represent the softening phenomenon in terms of damage accumulation is the main hypothesis set forth by Mullins. A real physical mechanism of softening is different from that embedded in the ABAQUS software. It is based on viscoelasticity which suggests the recovery of mechanical properties [5, 6]. Damage accumulation in a tire rubber compound does not occur up to a maximum strain of 100%, and the operating strain range (when the wheel rolls along a flat road) does not exceed 50%. Despite the aforesaid, the ABAQUS describes fairly well the effect of softening in rubber compounds.

The problem under study deals with the stress-strain state of a car tire and can be physically formulated as follows. The cross-sectional view of the tire is shown in Fig. 2. The problem is a three-dimensional one.

The tire areas of extreme stress having multiple reinforcements in the form of wire and cord are changed to a simple material having the effective properties of an elastomeric composite. This has been done in order that the complex parts of the tire (cords, bulges) do not “mask” the observed effect during calculations.

We assume that a car tire comes into the contact with a road that is banked at an angle 30 degrees and makes two turns. The coefficient of friction between the tire and the road is 0.5. A force that presses the tire against the road is applied to the point A of

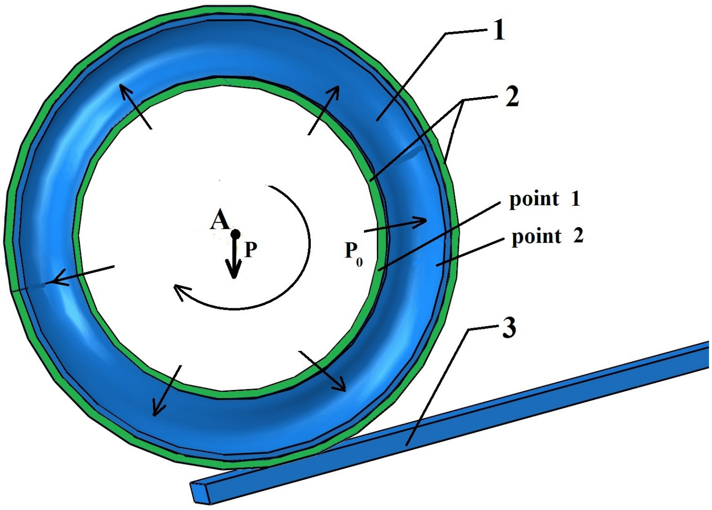

Figure 2: Computational scheme for the problem; 1 — car tire, 2 — cord, 3 — road; $\mathbf{P_0}$ — internal pressure in the car tire (for most cars the tire pressure is about 2.1 bar or 210 kPa); **P** — force exerted by a tire on the road surface

the tire. This force acts through the whole of calculation and remains unchanged. Time is assumed to be dimensionless: 1, 2 — first tire/road contact; 2, 3 — turn of the wheel without contact with a road, and 3, 4 — second turn of the wheel.

Maximum deformations in the car tire during the second turn of the wheel with and without consideration of the Mullins effect are shown in Fig. 3.

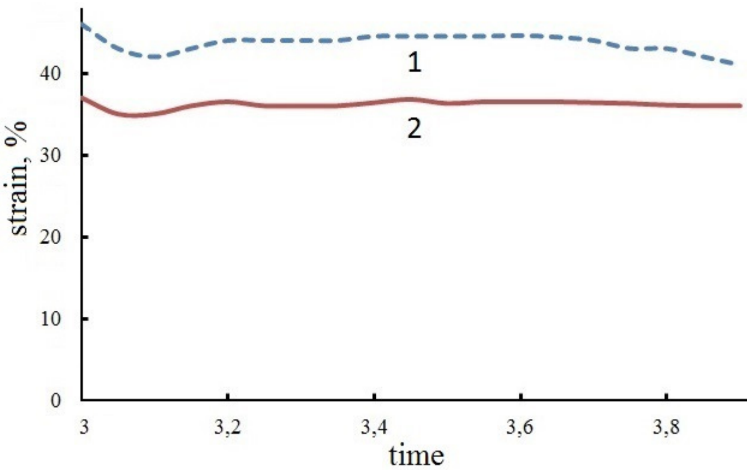

Figure 3: Strain curve for the contact surface of a tire during the second turn; 1 — the Mullins effect is taken into account, and 2 — the Mullins effect is neglected

Strains at different points of the car tire during the second turn of the wheel with and without consideration of the Mullins effect are shown in Fig. 4.

It follows from Figs. 3 and 4 that in the case when the Mullins effect is taken into consideration the tire deformation is larger and, consequently, the tire/road contact spot as well as the friction and wear will change.

The stress-strain state of the tire is examined at specified points (Fig. 2). The strain curves are shown in Figs. 5 and 6.

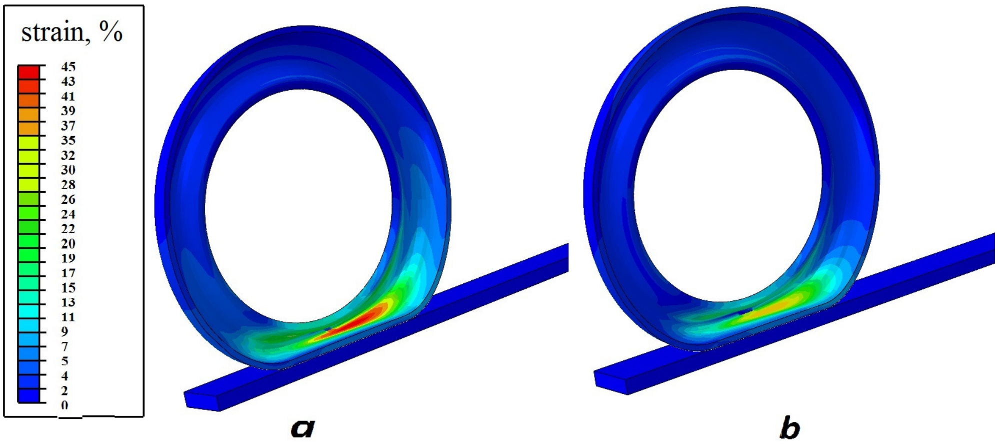

Figure 4: Tire deformations; *a* — the Mullins effect is taken into account, and *b* — the Mullins effect is neglected

During the first turn of the wheel, the Mullins effect is almost absent (dimensionless time 1, 2), but during the second turn (time 3, 4) the softening effect is quite considerable.

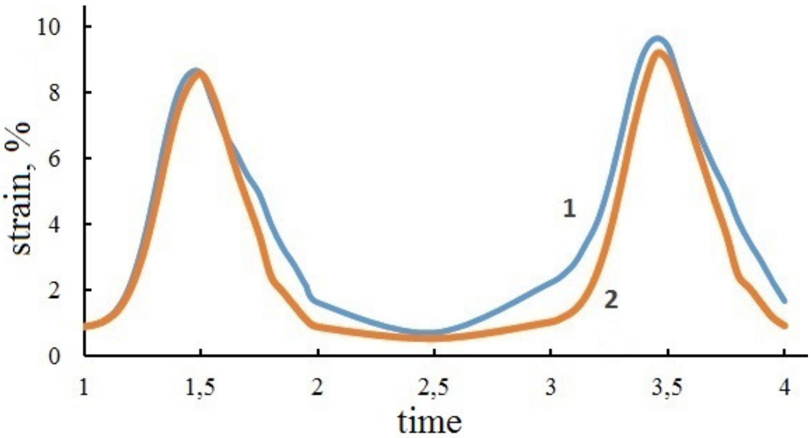

Figure 5: Time variation of deformation on the rim (point 1 fig. 2); 1 — the Mullins effect is taken into account, and 2 — the Mullins effect is neglected

As one can see (Fig. 6), the deformation ratio is highly different on various wheel surface areas. Deformation does not exceed 10% on the wheel rim, whereas near the contact surface (point 2 in Fig. 2) it is equal to 37%, and on the tire contact surface — 46%. Hence it can be concluded that deformation in the car tire is essentially non-uniform along the tire diameter, and therefore the Mullins effect will vary along the diameter. This fact should be taken into consideration when designing car tires.

Calculations in ABAQUS show that one needs to consider the Mullins effect when designing and developing car tires. In the future, the ABAQUS software package will be used to model, apart from damage accumulation, viscoelasticity as the most important softening mechanism.

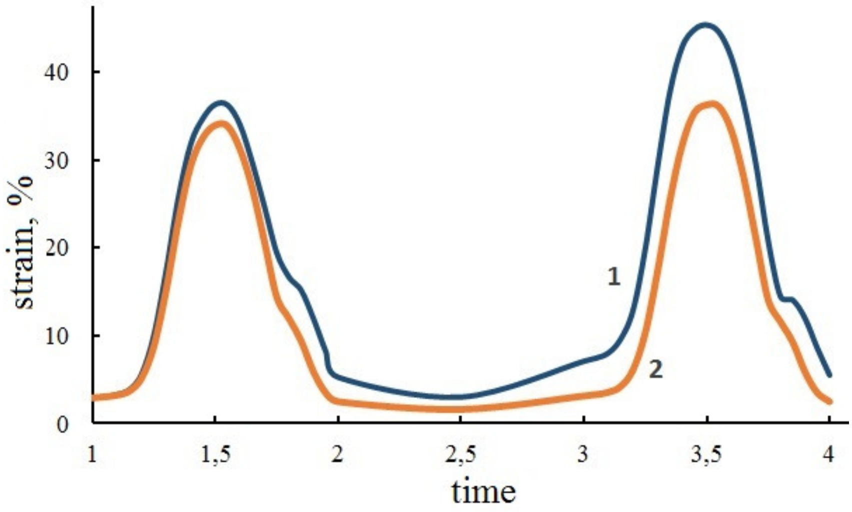

Figure 6: Time variation of deformation near the contact surface of the wheel during its rolling motion (point 2 fig. 2); 1 — the Mullins effect is taken into account, and 2 — the Mullins effect is neglected

3 Conclusion

The results of theoretical and experimental studies lead us to the following conclusions:
— In order to develop an accurate mathematical model for car tires one should take into consideration the Mullins effect which is most pronounced in elastomeric composites under cyclic loading conditions. The mechanical properties of rubber compounds change significantly after repeated deformation.
— Consideration of the Mullins effect substantially affects the design stress-state state of a tire.
— Different ratios of deformation and hence different degrees of softening can be observed at various tire surface points. For this reason, the stress-strain state of a tire should be calculated at each point of its diameter with consideration of diverse manifestations of the Mullins effect.
— The Mullins softening effect should be described not only in terms of damage accumulation, as in the ABAQUS software, but also in terms of viscoelasticity that is the most important softening mechanism.

Acknowledgements

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