Model of a rotor oscillations having an initial deflection and mounted on anisotropic supports

Olga A. Volokhovskaia
olgaavol@yandex.ru

Abstract

We develop a mathematical model of the motion of the rotor with one disk installed on anisotropic supports and having an initial deflection of the shaft, which is manifested only in the operation of turbine unit. Features of model are that the equations of oscillations system as variables includes horizontal and vertical displacements of the axis connecting the centers of the end cross-sections of the shaft and that the resulted imbalance vector is the sum of the imbalance disk vectors and the vector of the initial disk deflection at the point of the disk attachment to the shaft. It is shown that when the shaft torsion is taken into consideration, the parametric members appear in equations of the system motion, leading to the possibility of parametric resonances in the system. An example of calculating the amplitudes of resonance oscillations of the high pressure rotor of the turbine K -300- 23.5 and assessed the contribution of the initial shaft deflection in the value of the rotor vibration amplitudes near the supports and in the center of the span is proposed.

1 Introduction

A normal vibration state of the turbine unit during operation is determined by many conditions. Among the basic conditions are detachment of the shafting from dangerous resonances and provision of its vibrostability during construction, quality balancing of the rotor at the manufacturing plant during operation, and dynamic balancing of shafting in its bearings on the power plant during repair.

However, the reasons for increased vibrations of the turbine unit rotors can sometimes be factors which appear only during operation rather than at during the traditional balancing procedure. The deflection of the rotor shaft is one of the most common factors. It can have different causes. Let us name the main ones.

Thermal instability of the rotor is a consequence of the heterogeneity of the rotor material with respect to values of the temperature extension coefficient. When the temperature of the thermally unstable rotor changes from room temperature to operating temperature, it acquires deflection. This deflection is constantly present during turbine unit operation and disappears only when it stops and cools to room temperature. The reversible thermal deflection in the middle of the span, if no more than 20 μm, is taken to be acceptable [1].

Nonuniform creep. During long operation of high-temperature rotors (average- and high-pressure rotors), rotor deflection is possible due to the heterogeneity of the material properties in the in the circumferential direction in high-temperature rotor areas.

The rotor deflection after the contact in the seals. When the rotor is passing the critical velocity, local heating and the appearance of local temperature shaft deformations occurs at the points of contact. After rotor cooling, the rotor residual deflection of the opposite sign occurs in the cause of the appearance of plastic deformation at the contact points.

Startup of the turbine with a deflected rotor. This evidence takes place when the rotor has a nonuniform temperature due to insufficient heating at low rotation speed before startup.

In the present paper, the author proposes a mathematical model for describing the motion of a single disk rotor with an initially bent shaft, installed on anisotropic bearings, which has certain features. They are the next ones: the system oscillation equations include the horizontal and vertical displacements of the axis connecting the centers of the end cross sections of the shaft as the variables, while the resulted imbalance vector is the sum of such vectors of the disk and the initial deflection in the attachment point of the disk on the shaft.

2 Physical model of the deflected rotor

Let us consider the simplest physical model of the rotor with an initially bent shaft. Let the disk of mass m be rigidly connected to the inertialess elastic initially bent round shaft in the geometrical center. The elasticity modulus of the shaft is E. The shaft is supported by elastic bearings with stiffness k_1 and k_2 in the horizontal and vertical directions.

The disk on the shaft is located symmetrically with respect to the bearings, the length of the half span is l. The initial curvature of the shaft (the arrow of the static deflection) δ and the disk eccentricity e are considered to be known. The rotor rotates with some angular velocity $\dot{\psi}$.

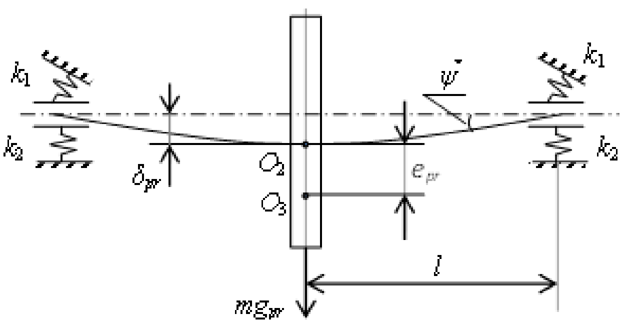

Figure 1: The model of single disk rotor with the deflected shaft.

The calculation scheme of the construction (a view in the projection on the plane passing through the bearing axis and the disk mass center) is shown in Figure 1: $e_{pr} = e\cos(\varphi - \psi_0)$, $\delta_{pr} = \delta\cos\psi_0 + \delta_{st}$, $mg_{pr} = mg\cos(\psi + \psi_0)$—are the eccentricity, total shaft flexure, and rotor weight projections on the specified plane given a fixed rotor; δ_{st} — is the static deflection of the shaft in this plane; the meaning of the angles φ and ψ_0 is evident from Figure 2.

Figure 2 conditionally shows the scheme of the rotating disk at some arbitrary time (a side projection). In this case, it is assumed that the bent shaft is in its initial state, i.e., not deformed by the forces P_1 and P_2 of the bearing reaction. With rigid bearings, the point O (Figure 1) is the projection on the disk plane of the fixed axis of the bearings. The point O_1 is the projection of the axis connecting the ends of the nondeformed bent shaft on the same plane. The point O_2 is the geometrical center of the disk (the attachment point of the disk on the shaft). The point O_3 is the mass center of the disk having the vector-eccentricity $\mathbf{e}$; the vector $\delta = \mathbf{O_1O_2}$ corresponds to the projection of the cantilever of the bent shaft on the disk plane and is equal to bending vector of the nondeformed shaft.

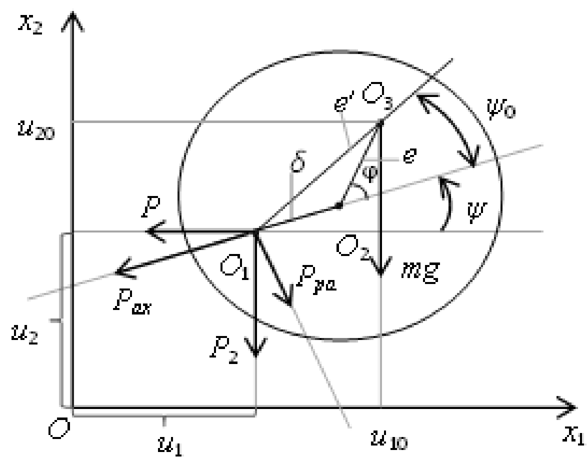

Figure 2: Projections of forces and displacements on the disk plane.

The reaction forces of bearings P_1 and P_2 should be such that the point O_1 after deformation of the shaft by these forces moves to the point . The bearing compliances will be taken into account by adding their values to the shaft compliances.

Let us decompose the reaction force of the bearings (the resultant force of P_1 and P_2) into an axial component P_{ax} acting along the axis O_2O_1 and a perpendicular component P_{pa}. Then, we obtain

$$P_{ax} = P_1 \cos\psi + P_2 \sin\psi; \quad P_{pa} = P_1 \sin\psi + P_2 \cos\psi. \tag{1}$$

Let us consider the deformation of the cantilever of length l under the action of these forces. The force P_{ax} causes only the cantilever bending, and the corresponding displacement u_{ax} along the axis O_1O_2 is expressed by

$$u_{ax} = -\frac{1}{2}P_{ax} \cdot \delta_{ben}; \qquad \delta_{ben} = \frac{l^3}{3EI}, \tag{2}$$

where δ_{ben} is the bending compliance of the cantilever beam of length l; EI is the stiffness of the shaft section at flexure. The force $P_{pa}/2$ causes both bending and torsion of the cantilever in the direction perpendicular to O_1O_2 axis (Figure 2):

$$u_{pa} = u_{ben} + u_{tors} = \frac{1}{2}P_{pa} \cdot \left(\delta_{ben} + \delta_{tors}\right). \tag{3}$$

Let us determine the displacements under torsion u_{tors}, which are induced by $P_{pa}/2$. The displacement differential du_{tors} in the direction of P_{pa} under rotation of the current section z due to the torque $M_{tors} = P_{pa} \cdot y/2$ on the length dz and the displacement u_{tors} will be the following (Figure 3):

$$du_{tors} = \frac{M_{tors}dz}{GI_{tors}} \cdot y = -\frac{1}{2}P_{tors}\frac{y^2dz}{GI_{tors}}; \; u_{tors} = -\frac{1}{2}P_{pa}\int_0^l \frac{y^2dz}{GI_{tors}} = -\frac{1}{2}P_{pa}\delta_{tors}; \tag{4}$$

$$\left(\delta_{tors} = \frac{\delta^2 \, l}{2GI_{tors}}\right) .$$

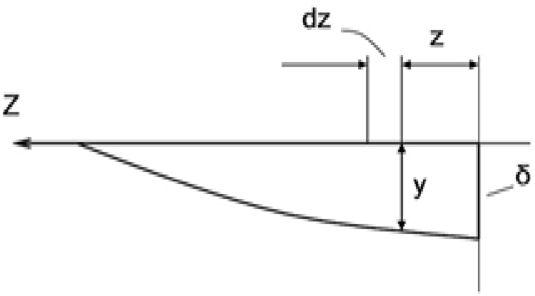

Figure 3: Determination of the shaft displacement at torsion.

During integration, it was taken here that $y = \delta\cos(\pi z/2l)$, GI_{tors} - is the stiffness of the shaft section under torsion.

Let us determine the projections of displacements u_1 and u_2 at axis Ox_1 and Ox_2:

$$u_1 = u_{ax}\cos\psi - u_{pa}\sin\psi; \quad u_2 = u_{ax}\sin\psi + u_{pa}\cos\psi.$$

By adding the bearing compliances in the direction of axes Ox_1 $(1/k_1)$ and Ox_2 $(1/k_2)$ with use of (1), (2) and (3) we find the displacements in the directions of these axes for the rotor installed on the anisotropic compliant bearings:

$$u_1 = -P_1\left[\delta_1 + \delta_{tors}\sin^2\psi\right] + P_2\delta_{tors}\sin\psi\cos\psi;$$

$$u_2 = -P_1\delta_{tors}\sin\psi\cos\psi - P_2\left[\delta_2 + \delta_{tors}\cos^2\psi\right], \tag{5}$$

$$\delta_1 = \delta_{bend} + 1/k_1\ , \quad \delta_2 = \delta_{bend} + 1/k_2. \tag{6}$$

Note that the correlation $\delta_{tors}/\delta_{bend} \sim (\delta/l)^2 << 1$ is of a sufficiently small value, so the values $\delta_{tors}/\delta_1 \sim 1$ and $\delta_{tors}/\delta_2 \sim 1$ are small according (6). Taking this into account, we solve system (5) for the reactions P_1 and P_2 conserving only the terms of the first order with respect to $\delta_{tors}/\delta_{bend}$. Considering the directions of P_1 and P_2 (Figure 1) against the displacements, we obtain

$$P_1/m = u_1\Omega_1^2 4\mu\chi_1\left[u_1\Omega_1^2\sin^2\psi u_2\Omega_2^2\sin\psi\cos\psi\right];$$

$$P_2/m = u_2\Omega_2^2 4\mu\chi_2\left[u_2\Omega_2^2\cos^2\psi u_1\Omega_1^2\sin\psi\cos\psi\right]; \tag{7}$$

$$\Omega_1{}^2 = 1/(\ m\delta_1\ ),\ \ \Omega_2{}^2 = 1/(\ m\delta_2\ ),\ \ \chi_1 = \delta_{bend}/(\ 2\delta_1);$$

$$\chi_2 = \delta_{bend}/(\ 2\delta_2\ ),\ \ \mu = \delta_{tors}/(\ 2\delta_{bend}).$$

3 The motion equations of the deflected rotor with anisotropic bearings (mathematical model)

The system of the deflected rotor motion equations consists of two equations of forward motion along the axis Ox_1 and Ox_2 taking the origin at the fixed point O of the space (Figure 1):

$$m\ddot{u}_{10} = -2h_1 m\dot{u}_1 - P_1, \quad m\ddot{u}_{20} = -2h_2 m\dot{u}_2 - P_2 - mg; \tag{8}$$

and the equation of change of the system kinetic moment with respect to the fixed axis Oz perpendicular to the disk plane

$$\dot{K}_O = \sum M_{ext}; \tag{9}$$

$$K_0 = I_{O3}\dot{\psi} + m(u_{10}\dot{u}_{20} - u_{20}\dot{u}_{10}); \quad \dot{K}_O = I_{O3}\ddot{\psi} + m(u_{10}\ddot{u}_{20} - u_{20}\ddot{u}_{10}); \tag{10}$$

$$L = (2h_1 m\dot{u}_1 + P_1)u_2 - (2h_2 m\dot{u}_2 + P_2)u_1 - mg\left[u_1 + e'\cos(\psi - \psi_{0)}\right] + M_{rot} - M_{br}, \tag{11}$$

where M_{ext} is the moment of external forces with respect to the axis Oz; M_{rot} and M_{br} are the rotatory and the braking moments (Figure 2); I_{O3} is the inertia moment of the disk with respect to the axis O_3z.

Substituting the expression for reactions of P_1 and P_2 and (7) into (8) and (9) and calculating the time derivative from the kinetic moment K_0, taking into account the correlations (Figure 2):

$$\begin{aligned} &u_{10} = u_1 + e'\cos(\psi + \psi_0), \quad u_{20} = u_2 + e'\sin(\psi + \psi_0);\\ &\ddot{u}_{10} = \dot{u}_1 + e'[\ddot{\psi} sin(\psi + \psi_0) + \psi^2 cos(\psi + \psi_0)];\\ &\ddot{u}_{20} = \dot{u}_1 + e'[\ddot{\psi} cos(\psi + \psi_0) + \dot{\psi}^2 sin(\psi + \psi_0)], \end{aligned} \tag{12}$$

and making the labor consuming transformations, we find the system of three equations of motion for the physical model in the following form:

$$\begin{aligned} &\ddot{u}_1 + 2h_1\dot{u}_1 + \Omega_1^2(1 - 2\mu\chi_1)u_1 + 2\mu\chi_1[\Omega_1^2 u_1\cos 2\psi + \Omega_2^2 u_2 \sin 2\psi] =\\ &= e'[\ddot{\psi}\sin(\psi + \psi_0) + \dot{\psi}^2\cos(\psi + \psi_0)];\\ &\ddot{u}_2 + 2h_2\dot{u}_2 + \Omega_2^2(1 - 2\mu\chi_2)u_2 + 2\mu\chi_2[\Omega_1^2 u_1\sin 2\psi + \Omega_2^2 u_2\cos 2\psi] =\\ &= e'[\ddot{\psi}\cos(\psi + \psi_0) + \psi^2\sin(\psi + \psi_0)] - g;\\ &\bar{I}_{O3}\ddot{\psi} + \Big\{2h_{1c}\dot{u}_1 - 2h_{2s}\dot{u}_2 + \Omega_{1c}^2[1 - \mu(3\chi_1 + \chi_2)]u_1 + \Omega_{2s}^2[1 - \mu(\chi_2 - \chi_1)]u_2\Big\}\sin\psi +\\ &+ \Big\{2h_{1s}\dot{u}_1 - 2h_{2c}\dot{u}_2 + \Omega_{1s}^2[1 + \mu(\chi_2 - \chi_1)]u_1 - \Omega_{2c}^2[1 - \mu(\chi_1 + \chi_2)]u_2\Big\}\cos\psi -\\ &-\mu(\chi_2 - \chi_1)[\Omega_{1c}^2 u_1 + \Omega_{2s}^2 u_2]\sin 3\psi + \mu(\chi_2 - \chi_1)[\Omega_{2c}^2 u_2 - \Omega_{1s}^2 u_1]\cos 3\psi = M_{rot} - M_{br}. \end{aligned} \tag{13}$$

In (13), besides the designations used above, the following were additionally adopted:

$$\begin{aligned} &\bar{I}_{O3} = I_{O3}/(me'); \quad \bar{M}_{rot} = M_{rot}/(me'); \quad \bar{M}_{br} = M_{br}/(me');\\ &2h_{1c} = 2h_1\cos\psi_0; \quad 2h_{1s} = 2h_1\sin\psi_0; \quad 2h_{2c} = 2h_2\cos\psi_0; \quad 2h_{2s} = 2h_2\sin\psi_0;\\ &\Omega_{1c}^2 = \Omega_1^2\cos\psi_0; \quad \Omega_{1s}^2 = \Omega_1^2\sin\psi_0; \quad \Omega_{2c}^2 = \Omega_2^2\cos\psi_0; \quad \Omega_{2s}^2 = \Omega_2^2\sin\psi_0. \end{aligned} \tag{14}$$

Let the angle φ between the disk eccentricity vector $\mathbf{e}$ and the bending arrow of the bent shaft δ be known (Volokhovskaia 2). Then, the trigonometrically functions of the angle ψ_0 and the value $\mathbf{e}'$ are determined by the following formulas:

$$\cos\psi_0 = \frac{\delta + e\,\cos\varphi}{e'}; \quad \sin\psi_0 = \frac{e\sin\varphi}{e'}; \quad e' = \sqrt{\delta^2 + e^2 + 2\delta e\,\cos\varphi}. \tag{15}$$

The initial conditions of the problem depend on the conditions of the turbounit startup. To obtain the solution of (13) corresponding to the startup of the cold rotor with a formed initial deflection, we can write the following:

$$t \;=\; 0: \; u_1 = u_1 = u_2 = \dot{u}_2 = 0; \quad \psi = \dot{\psi} = \; 0 \tag{16}$$

In order to solve system (13) at startup, with a thermally unstable rotor, we need to know the relationship between the modulus of the bending arrow δ and the time $[0, t_{work}]$ corresponding to the time required for heating the rotor up to the working temperature. In the first approximation, we can use only the piece-wise-linear relationship of the form

$$\delta * (t) = at, \; a = \delta/t_{work} \;\text{ at }\; 0 < t < t_{work}; \; \delta * (t) = \delta \text{ at } t > t_{work}.$$

In this case, the initial conditions of the problem take the form:

$$t = 0: \quad u_1 = \dot{u}_1 = u_2 = \dot{u}_2 = 0; \quad \psi = \dot{\psi} = 0; \quad \delta^* = 0. \tag{17}$$

The system of differential equations of motion (13) with initial conditions (16) and (17) corresponds to the Cauchy problem, which can be solved both numerically and analytically using the small parameter method.

4 Analysis of the motion equations

If the torsion of the bent shaft is not taken into account, i.e., $\mu = \delta_{tors}/(2\delta_{bend}) = 0$, which is available since $\mu << 1$, then, the deflected rotor will behave as the unbalanced one, which eccentricity $\mathbf{e}'$ is made of the vector of proper imbalance $\mathbf{e}$ with respect to the point O_2 where the disk is fixed on the shaft and the vector of initial deflection of the shaft δ in this point $e' = |\mathbf{e} + \delta| > e$ (Figure 2). Thus, the preliminary bending of the shaft depending on the modulus values and the mutual location of $\mathbf{e}$ and δ can lead to an increase and decrease in the eccentricity $\mathbf{e}'$ (Figure 4), as well as in the acting centrifugal inertial forces and levels of rotor vibrations caused by these forces. In the figure Figure 4 the picture (a) corresponds to $e' = |\mathbf{e} + \delta| > e$, and the picture (b)—to $e' = |\mathbf{e} + \delta| < e$.

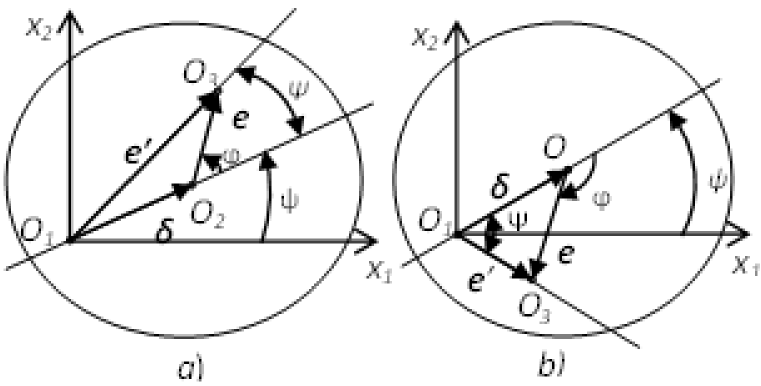

Figure 4: Determination of the resulted eccentricity vector

Upon taking the torsion deformation at the deflected rotor oscillations $\mu = \delta_{tors}/(2\delta_{bend}) \neq 0$. into account, the system of the equations of motion becomes parametric, and the parameter is the rotation angle of the rotor ψ (Figure 2). The given parametric form causes oscillations of twice the frequency concerning the working frequency. It was found that the system of equations of motion turns out to be similar to that of the rotor on the shaft having different principal moments of inertia of cross section [2]. The coefficients of parametric terms are proportional to the square of relationship between the initial deflection arrow and a half shaft span length δ^2/l^2. They are relatively small values. However, the parametric properties of the system can play an important role in resonance velocities (having double the value of the working one, namely, 100 Hz). Moreover, the equations of motion with respect to forward displacements and the angle of rotor rotation cease to be independent. Therefore, the first two equations of the system will involve the parametric terms containing the harmonics with unit and triple frequencies, namely, through the second derivative of the rotation angle $\ddot{\psi}$ included in the right-hand part of the equations. This fact can lead to the appearance of resonance velocities, which exceed the values of the working velocity by three times. However, it should be mentioned that the parametric terms containing the harmonics exist only when the rotor is placed on the anisotropic bearings, i.e., at $\chi_1 \neq \chi_2$.

The proposed physical model of the deflected rotor has three degrees of freedom, while its motion is described by three connected differential equations. Such type of corresponding mathematical model of the system means that the initially deflected rotor cannot rotate with constant velocity. This is explained by the presence of the terms containing the forward displacements in the rotor rotation angle equation. The correlation between the system equations significantly complicates its analytical and numerical analyses.

Let us evaluate the relative value of a change in the rotor angular velocity per revolution using the third equation from (13). To simplify the algorithm, we will consider that the rotor bearings are isotropic ($\chi_1 \neq \chi_2$) and damping in the system can be neglected ($h1 \approx h2 \approx 0$). Taking the above assumptions into account, we write the equation of the kinetic moment changing in the following form:

$$\frac{1}{m} I_{O3}\ddot{\psi} + L = \frac{1}{m}(M_{rot} - M_{br});$$

$$L \sim \Omega^2 e'[u_1 \sin(\psi + \psi_0) - u_2 \cos(\psi\check{}\psi_0)]\check{}2\mu\Omega^2 e'[u_1 \sin\psi - u_2 \cos\psi]\cos\psi_0. \quad (18)$$

The condition $\dot{\psi} = \omega = const$ becomes possible if the correlation $M_{rot} = M_{br} + L$ is satisfied. Assume that the rotatory moment is absolutely balanced by the loading moment $M_{rot} = M_{br}$. Supporting that $u_1 \sim u_2 \sim a$, we find the evaluation for L and $\ddot{\psi}$ by considering (18):

$$L \sim \Omega^2 e'[u_1 \sin(\psi + \psi_0) - u_2 \cos(\psi - \psi_0)] - 2\mu\,\Omega^2 e'[u_1 \sin\psi - u_2 \cos\psi]\cos\psi_0;$$

$$\ddot{\psi} \sim \Omega^2 e' am/I_{O3} \sim \Omega^2 e' a/r^2. \quad (19)$$

Taking $\Omega/\omega = n$ (for the rotors of turbounits $n \approx 0.5 - 2.5$, i.e. $n \sim 1$), from (19) and using the above assumption of the equality of the rotatory and loading moments, we obtain the expression for a relative change in the rotational velocity per revolution:

$$\Delta\omega/\omega \sim n^2(e'a/r^2)\omega\Delta t \sim (e'a/r^2)2\pi. \quad (20)$$

According to data [1], the eccentricities for steam turbines do not exceed the largest value, which is several thousandth parts of the radius of inertia r for the disk. The value of admissible reversible bending in the span center δ is 20 μm [1], i.e., the value $e' = |\mathbf{e} + \delta|$ is commensurable with the value of e (Figure 4). Therefore, if we take $e' \sim 10^-3r$, $a \sim 10e' \sim 10^-2r$ according to [1], assessment (20) gives $\Delta\omega/\omega \sim 10^{-4}$, i.e., during one turnover the angular velocity of the rotor has changes in value of the order of 0.01%. Consequently, we can neglect the third equation of system (13) given the above assumptions with sufficient precision, and the rotational velocity can be taken to be constant: $\dot{\psi} = \omega = const$.

5 Example

As noted above, the influence of the rotor shaft torsion is very small. Therefore, the main index of the rotor vibration activity is the value of its amplitude on the critical velocities at forced oscillations with the eccentricity increased due to the initial shaft deflection. Let us calculate the resonance oscillations on the lowest frequencies of the deflected high-pressure rotor of $K - 300 - 23.5$ turbine during run-out. The rotor has the mass distributed along the span and bears on two identical bearings with elliptic bore liners. We take parameter from [3]: mass M=9600kg, length $2l$=5.5 m, stiffness of shaft section $EI = 5.15 \times 10^8$ N$\cdot m^2$, the coefficients of stiffness and damping of the oil layer of the bearings in the principal axes of the corresponding tensors $c_{11} = 0.11 \times 10^9$ N/m; $c_{22} = 1.16 \times 10^9$ N/m; $c_{12} = c_{21} = 0.33 \times 10^9 N/m$; $b_{11} = 0.45 \times 10^6kg\cdot s^1$; $b_{12} = b_{21} = 0.60 \times 10^6$ kg$\cdot s^1$; $b_{22} = 4.7 \times 10^6$ kg$\cdot s^1$; and the working angular velocity $\omega = 314$ rad/s.

Let us neglect the third equation of system (13) and the value of rotor gravity in comparison with the inertial forces of rotation, as well as take $\mu = 0$ (no shaft torsion). Then, the first two equations of system (13) become disconnected.

Moreover, we consider that $e = 0$, i.e., $e' = \delta$ (the rotor is completely unbalanced at room temperature). With the heating test, the rotor bending is equal to the acceptable value δ_0=20μm.

The formula for resonance amplitudes for the unbalanced rotor was obtained in [3]:

$$A_j = \varphi_{j(k)}(\Omega_{(k)}/H_{(kk)}) \cdot \nu_k, \tag{21}$$

Where A_j is the amplitude at the point j ; $\varphi_{j(k)}$ is the value of k-th normalized principal form of oscillations at the point j (hereinafter, no summation over indices); H_{kk} is the reduced damping coefficient by the k-th oscillation form of the rotor:

$$H_{kk} = B_{11}\varphi_{1k}^2 + B_{22}\varphi_{2k}^2 + ... + B_{(\mathrm{nn})}\varphi_{(\mathrm{n})k}^2. \tag{22}$$

Here, $B_{11}, B_{22}, \ldots, B_{(nn)}$ are the damping coefficients from the first to n bearings; $\varphi_{1k}, \varphi_{2k}\varphi_{nk}$ are the amplitude values of the principal form of number k on the bearing necks. For the deflected rotor with distributed parameters, the reduced imbalance is

$$\nu_k = \int_0^l \delta(z)\varphi_k(z)dm; \qquad (dm = \rho F dz) \tag{23}$$

where ρ is the material density; F is the rotor cross-section area; and $\varphi_k(z)$ is the k-th oscillation form. Assume that the “residual” bending of the rotor is

$$\delta(z) = \delta_0 \sin(\pi z/l). \tag{24}$$

By specifying the principal form of oscillations according to [3] in the form

$$\varphi_k = a_k + b_k \cdot \sin(\pi z/l); \qquad (k = 1, 2)\,. \tag{25}$$

With values of rotor parameters taken above, we find:

$$a_1 = 0,5970/\sqrt{M};\ b_1 = 0,6055/\sqrt{M};\ a_2 = 0,1079/\sqrt{M};\ b_2 = 1,2752/\sqrt{M}. \tag{26}$$

Substituting (26) and (25) into (23) and taking (24) into account, we find after itegration that

$$\nu_1 = \sqrt{M} \cdot \delta_0 \cdot 0,6828;\ \nu_2 = \sqrt{M} \cdot \delta_0 \cdot 0,7063;\ \Omega_1 = 118,0\ s^{-1};\ \Omega_2 = 168,38\ s^{-1};$$

$$H_{11} = 33,4133 s^{-1};$$

$$H_{22} = 11,4041 s^{-1}. \tag{27}$$

Using (27) and (21) we find the values of resonance amplitudes in the horizontal ($k = 1$) and vertical ($k = 2$) directions at the most demonstrative points on the rotor axis in terms of vibration levels.

The amplitudes on the bearing necks are $A_1 = 1.4315\delta_0$, $A_2 = 1.1254\delta_0$. Taking the existing constraint on the bending values in the heating test $\delta_{tol} = 20\mu m$ and assuming that $\delta_0 = \delta_{tol}$, we obtain the acceptable values of the amplitudes: $A_1 = 28.76\mu\, m$, $A_2 = 22.51\mu\, m$. Consequently, the constraint on the bending in the heating test $\delta_{tol} = 20\mu\, m$ means that the rotor of the high-pressure cylinder of the turbine $K - 300 - 23.5$ will have a double amplitude of vibration (the value according to the vibration norms) near the bearings $2A \cong 45 - 60\mu\, m$.This value knowingly satisfies the domestic vibration norms for new units, which are $2A \leq 80\mu\, m$ for the nominal rotation frequency. Note, that the parameters Ω_1, H_{11}, Ω_2, H_{22}, calculated at the nominal frequency $\omega = 314\ rad/s$, change insignificantly at the rotation frequency drop up to 168.38 rad/s and 118.0 rad/s during turbounit run-out. It is so, because the critical frequencies at $\omega = \Omega_1$ and $text\omega = \Omega_2$ are little different from the eigen frequencies of the system, while the damping coefficients H_{11} and H_{22} on the critical frequencies are little different from the calculated ones on the working frequency, in terms of their value.

When the rotor is running out due to the angular acceleration (slowdown) the real amplitudes during passing the resonances will be lower. On the other hand, the values obtained for amplitudes are related to the ideal balanced rotor. In practice, they will be added to the oscillation amplitudes caused by the rotor imbalance (Figure 4).

6 Conclusions

The system of equations of motion of the deflected rotor coincides with the corresponding system for initially rectilinear rotor, if the following replacements are implemented:

— the displacements u_1 and u_2, which are the shaft deflections at the attachment point of the geometrical disk center, are replaced by the displacements of some reduced point of the disk, which is placed on the intersection between the straight line connecting the centers of shaft end sections and the disk plane;

— the imbalance vector is replaced by the sum of the disk imbalance vector with the vector of disk geometrical center displacement with respect to the reduction point.

When the shaft torsion is taken into account , the system of motion equations is significantly complicated: the parametric terms are appear in it and determine the presence of the driving forces and the possible occurrence of parametric resonance of the first, second and third multiplicity.

The above singlemass model is easy to apply to the general case of oscillation of the multimass rotor, which will allow one to calculate the resonance oscillation of the real rotor with the given shaft deflection.

The tolerance used for the shaft bending in the span center in the test is justified.

Acknowledgements

The Author sincerely thanks A. G. Kostyuk for discussions and useful comments.

References

[1] Shubenok, L. A., Ed.*Prochnost' elementov parovykh turbin* (Strength of Steam Turbines Elements), Moscow: Mashgiz, 1962. (in Russian)

[2] F. M. Dimentberg. *Izgibnye kolebaniya vrashchayushchikhsya valov (Bending Vibration of Rotating Shifts)*, Moscow: Izd. Akad. Nauk SSSR, 1959. (in Russian)

[3] A. G. Kostyuk *Dinamika i prochnost' turbomashin* (Dynamics and Strength for Turbomachines), Moscow: Izd. Mosk. Energet. Inst., 2000. (in Russian)

Olga A. Volokhovskaia, Shosse Enthusiastov 98, build. 4 , app. 423, 111531, Moscow, Russian Federation