Damage and long-term strength criterion of elastic-viscous aging media

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Abstract

The problem of creep, damage and long-term strength of compressible elastic-viscous aging media is considered. The modified Maxwell equation expressed in the scale of effective time is used. The parameter of continuity is defined by a value of relative changes of density, which is an integral measure of the structure micro defects stored during long-term loading. It is assumed that the rate of brittle fracture depends on stress and the value of stored damage. Taking into account the noted assumptions the analytical relations for the parameter of continuity and the fracture criteria are derived.

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The problem of creep and long-term strength of polymer and composite materials with viscous and brittle mechanical characteristics is discussed. The majority polymers (polystyrene, polyacrylates, polyvinyl chloride and other) and composites, based on a polymer matrix, fractured when the value of residual deformation is small. During the long-term action of mechanical stresses and temperatures the interrelated processes of deformation and damage occur. Thus the damage parameter has a certain physical meaning and is associated with destructive processes, which are consisting of thermal and mechanical stages. In the case of brittle composite materials the damage is defined by degradation processes: loosing of continuity in contact zone fiber-matrix, fracture of fibers in defective volumes, the formation of cracks or voids in the matrix and other. These processes are accompanied by changes of the structure and properties as a result of chemical reactions.

In mechanics of scattered damage and brittle fracture the concept of continuity (Kachanov [1]) and damage (Rabotnov [2]) is considered. Following Kachanov, let's introduce the parameter of continuity ψ ($1 \geq \psi \geq 0$), which we define as a relative volume changes (loosening on the Novozhilov terminology [3]) or density $\psi = \rho/\rho_0$ (where ρ_0 is initial, ρ is current density) [4]. So this parameter is an integral measure of structure micro defects stored during long-term loading. In the world scientific literature there are numerous experimental investigations of the evolution of the parameter ψ during the creep of metals and composite materials [5-11]. In initial condition $t = 0$, $\rho = \rho_0$, $\psi = 1$ and in fracture moment $t = t_f$, $\rho = 0$, $\psi = 0$.

Since the real materials have random structure, so continuity parameter is a statistical characteristic, which can be defined by some kinetic equation. The form of a kinetic equation is determined according to the experimental results on long-term strength. In the common case these equations are based on two hypotheses formulated in [12, 13]. In according to the first hypothesis the rate of brittle fracture depends only on stress $\sigma(t)$

$$\frac{d\psi}{dt} = -f\left[\sigma(t)\right]. \tag{1}$$

In accordance with the second hypothesis and the conception of statistical physics the rate of brittle fracture depends on the stress and the value of stored damage

$$\frac{d\psi}{dt} = -f\left[\sigma(t), \psi\right]. \tag{2}$$

In equations (1)-(2), $\sigma(t)$ is stress, depending on time. In the creep case $\sigma(t) = \sigma_0 = \mathrm{const}$, $1 \geq \psi \geq 0$, $0 \leq t \leq t_f$ and from the solution of equations (1), (2) we can obtain the criteria of long-term strength

$$t_f = -1/f(\sigma_0), \tag{3}$$

$$t_f = -\int_t^0 \frac{d\psi}{f(\sigma_0, \psi)}. \tag{4}$$

When formulating the long-term strength criterion in the form of relations (1), (2) the condition of a constant stress during creep is accepted. In this regard it can be mentioned that the creep experiments are conducted when the applied value of load P is constant. Dropping from time to time the load we can achieve the condition of constant stress. However, the practical realization of this condition is not quite workable. The change of cross section of the specimen because of formation of pores and cracks and, accordingly, a correct estimate of the value of true stress is not quite possible.

Let's consider the creep problem of a tensile specimen made of elastic viscous aging material under the action of constant load P. As a rheological equation we will use the modified Maxwell equation, expressed in the scale of effective time [4]

$$\begin{aligned} &\tfrac{d\varepsilon}{d\omega} = \tfrac{1}{E}\tfrac{d\sigma}{d\omega} + \tfrac{\sigma}{\eta}, \\ &d\omega = f_1(\omega, \varepsilon, T, t)dt + f_2(\omega, \varepsilon, T, t)d\varepsilon, \end{aligned} \tag{5}$$

where ε is strain, T is temperature, t is time, E is the modulus of elasticity, η is the coefficient of viscosity.

Parameter ω is considered as a effective time, using which it is possible to describe the deformation aging processes and aging after quenching. According to equation (5) during instant active loadings this parameter corresponds to a deformation time ε. In a state of unloading and stabilization the parameter ω describes the kinetics of chemical processes of aging and reduces to a real time t. In the calculations according to formula (5) the parameter of effective time is defined by the following relation [14]

$$d\omega = ae^{kt}dt + bd\varepsilon, \tag{6}$$

where a, b, k are constants.

To determine the long-term strength in addition to equations (5)-(6) the relation for a parameter of continuity is considered in the form of a power law [15]

$$\frac{d\psi}{dt} = -A\sigma^n = -A\sigma_0^n \psi^n e^{n\varepsilon}, \tag{7}$$

where A, n are constants, σ is true, σ_0 is engineering stress.

In equation (7) the mass conservation law $\rho_0 l_0 F_0 = \rho l F$ is applied, from which it follows $\sigma = P/F = \sigma_0 F_0/F = \sigma_0 \psi e^{\varepsilon}$, $\sigma_0 = P/F_0$, $\varepsilon = \ln(l/l_0)$, l_0, F_0 are initial and l, F are current length and cross-section area of specimen.

Analytical solutions of interrelated equations (5), (6), (7) are possible in the case of some reasonable assumptions. Let's solve the equation (5) without account of damage processes. Taking into account (6) and initial conditions $t = 0$, $\varepsilon = 0$, $\sigma = \sigma_0 = const$, the solution of equation (5), is written in the form

$$\varepsilon = \frac{\sigma}{E}\left[1 + \frac{a(e^{kt} - 1)}{k\tau\left(1 - \frac{\sigma b}{E\tau}\right)}\right], \tag{8}$$

where $\tau = \eta/E$ is relaxation time.

Introducing the relation (8) into equation (7) and solving it with initial condition $t = 0$, $\psi = 1$, we will obtain

$$\psi = \left\{1 + \frac{A\sigma_0^{n+1} an(1-n)}{E^2\tau\left(1 - \frac{\sigma_0 b}{E\tau}\right)}\left[e^{\frac{n\sigma_0}{E}} - e^{kt} e^{\frac{n\sigma_0}{E}\left[1 + \frac{a}{E\tau k\left(1 - \frac{\sigma_0 b}{E\tau}\right)}\left(e^{kt} - 1\right)\right]}\right]\right\}^{\frac{1}{1-n}}. \tag{9}$$

Curve of continuity parameter ψ according to formula (9) is shown on Fig. 1.

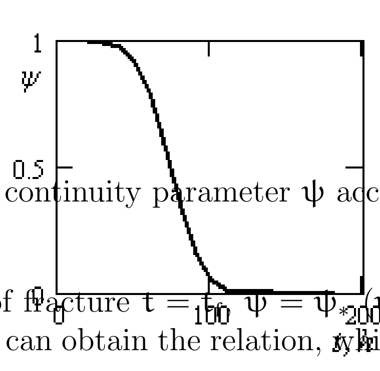

Figure 1: Curve of continuity parameter ψ according to formula (9).

Taking the condition of fracture $t = t_f$, $\psi = \psi_*$ (ψ_* is a value of continuity at fracture time) from (9) we can obtain the relation, which is reduced to the following

$$kt_f + A_1 e^{kt_f} + B = 0, \tag{10}$$

where $A_1 = \frac{n\sigma_0 a}{E^2\tau k\left(1-\frac{\sigma_0 b}{E\tau}\right)}$, $B = \frac{n\sigma_0}{E} - \frac{n\sigma_0 a}{E^2\tau k\left(1-\frac{\sigma_0 b}{E\tau}\right)} - \ln\left(e^{\frac{n\sigma_0}{E}} - \frac{(\psi_*^{1-n}-1)E^2\tau\left(1-\frac{\sigma_0 b}{E\tau}\right)}{(1-n)A\sigma_0^{n+1}an}\right)$.

The solution of equation (10) is

$$t_f = -\frac{W(A_1 e^{-B}) - B}{k}, \quad (11)$$

where W is the Lambert function defined as

$$W_0(x) = \sum_{n=1}^{\infty} \frac{(-n)^{n-1}}{n!} x^n.$$

Taking into this decomposition only the first two terms of series, we obtain the solution of equation (10) in the form

$$t_f = -\frac{A_1 e^{-B} - (A_1 e^{-B})^2 - B}{k}. \quad (12)$$

In calculations according to formulas (9) and (12) the following values of coefficients were used: $E = 2000$ MPa, $a = 0,8$ [h]$^{-1}$, $b = 3$, $\tau = 35$ h, $n = 2$, $k = 0,1$ [h]$^{-1}$, $A = 0,1$ [MPa]$^{-2}$, $\sigma_0 = 80$ MPa, $\psi_* = 0,1$. These coefficients were chosen to obtain a qualitative description of damage parameter and long-term strength curves.

On Fig. 2 the theoretical curve of long-term strength using criterion (12) is shown.

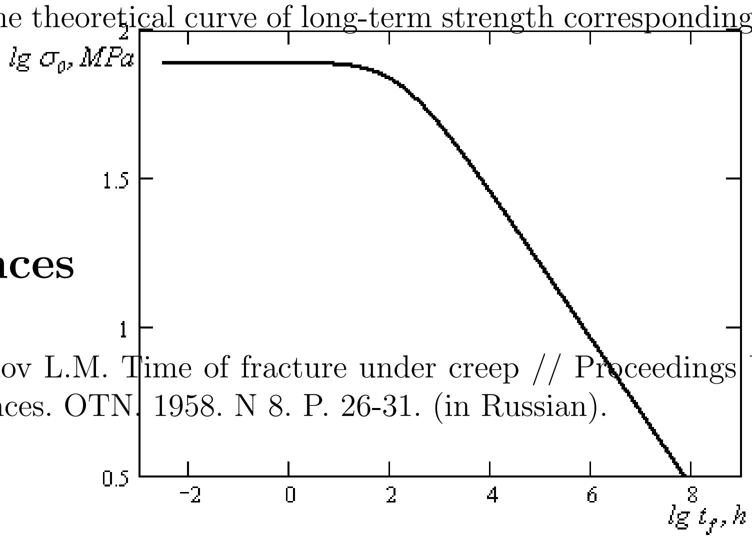

Figure 2: The theoretical curve of long-term strength corresponding to the criterion (12).

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