

## MEAN STRESS EVOLUTION IN IRREGULAR CYCLIC LOADING OF ALUMINUM ALLOY

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**Abstract.** The mean stress evolution was studied in program loading under strain limits control including fragments of excessive compression and extension. Cyclic stress-strain curves were continuously recorded and analyzed. An effective cyclic curve is suggested for generalization of the cyclic properties of the material. Specific character of the mean stress development is observed in transitions from fully-reversed segments of irregular loading to segments with cyclic tensile or compressive loading: the induced tensile mean stress increases in repetitive fragments of tensile loading following the material cyclic hardening. The induced compressive mean stress in respective loading fragments occurs almost unaffected by the cyclic hardening mechanisms.

### General

The mean stress in service loading of structures may be caused by different factors. These are the self-weight of a structure, the aerodynamic loads due to the low-frequency part of the wind velocity pulsations spectra on the tall buildings and long-span bridges, loaded and unloaded conditions of the cargo ships, residual welding stress, etc. Apart from these, in realistic service loading of structures the mean stress varies in load excursions.

The effects of the mean stress on fatigue behavior of materials and structures were recognized since early studies of fatigue phenomenon. By the end of XIX Century Goodman [1] derived empirical formula defining the fatigue limit stress amplitude depending on the mean stress:

$$\sigma_R = \sigma_{-1} (1 - \sigma_m / \sigma_u), \quad (1)$$

where  $\sigma_{-1}$  is the fatigue limit stress at fully reversed cyclic loading,  $\sigma_m = (\sigma_{\max} + \sigma_{\min})/2$  is the mean stress in the load cycle,  $\sigma_u$  is the ultimate strength of the material,  $R = \sigma_{\min} / \sigma_{\max}$  is the load ratio.

Since that time the versions of (1) were suggested reflecting fatigue properties of particular materials. An important finding was the differentiation of the mean stress influence in the early fatigue crack initiation phase and in the crack growth stage [2, 3, etc.]. Kooistra and Lemcoe [4] found by recording cyclic strain in pressure vessels that the loading conditions at the stress concentration areas could be attributed to the cyclic straining, loading with the strain range control. Application of the developed by the time of the Strain-life criteria, e.g., Manson's criterion [5]:

$$\Delta\varepsilon = CN^{-\alpha} + BN^{-\beta}, \quad (2)$$

where  $\Delta\varepsilon$  is the strain range at the critical location in structure where fatigue crack initiation was expected,  $C$ ,  $B$ ,  $\alpha$  and  $\beta$  are the material constants, offered alternative assessment of the fatigue life of structural components. The problem of evaluation of the local strain range for practical analyses was solved by Neuber who suggested simple relationship [6]

$$\Delta\sigma \Delta\varepsilon = (K_t \Delta\sigma_n)^2 / E, \quad (3)$$

where  $\Delta\sigma$  is the local, elastic-plastic stress at the affected location,  $K_t$  is the theoretical stress concentration factor, to be calculated, e.g., by applying the finite-element technique on assumption of elastic material behavior, and  $\Delta\sigma_n$  is the nominal stress in the component.

To find the strain range from Eq.3 one would need in the experimentally obtained cyclic stress-strain diagram, cyclic curve. Lately, a direct experimentally-based approach which allowed evaluation of local cyclic strain for several grades of structural steel was developed in [7].

Keeping with this concept, and to consider the effects of mean stress in the crack initiation phase, Smith, Watson and Topper [8] assumed an equivalent strain amplitude at a stress concentration with an emphasis on the tensile part of the stress cycle, the  $P_{SWT}$  parameter:

$$P_{SWT} = ((\sigma_a + \sigma_m) \varepsilon_a E)^{1/2}. \quad (4)$$

Zacher and Seeger [9] indicated that this parameter should be modified to distinguish tensile and compressive mean stresses:

$$P_{mod} = ((\sigma_a + k\sigma_m) \varepsilon_a E)^{1/2}, \quad (5)$$

where  $k$  is the correction for the mean stress sign effect on the hysteresis loop.

The left-hand part of Eqns 4, 5 as follows from Eq. 3 has to be the local stress amplitude (on assumption of elastic material behavior) or the product  $K_t \sigma_n$ . In Eq.5 the parameter  $k$ , however, can not be a constant in irregular loading histories.

These and the other proposals are addressed to the loading conditions where the mean stress remains unchanged in the course of the cyclic loading of a structure. Same suggestion is applied in the current rules for fatigue assessment and design of welded structures [10-15], etc.

However, the development of microplastic and plastic strains in load excursions at the stress concentration areas provides conditions for the mean stress part relaxation and respective local stress redistribution to maintain the equilibrium [16]. The same mechanisms of plasticity at cyclic loading work in the presence of residual welding stresses [17, 19]. And more than that, the mean stress varies depending on the changes in the loading conditions of the structure and randomly varies in the service loading successions; considering effects of the mean stress on the current values of the cyclic curve parameters deemed important in the damage counting procedures, e.g., Rainflow editing [18]. The technique of experimental evaluation of the service loading induced strain is described elsewhere, e.g., in [18, 20].

Therefore it is reasonable to study the cyclic elastic-plastic properties and mean stress behavior under irregular cyclic loading in conditions of the stress concentration modeled by the cyclic straining. Of special interest are the cyclic curves and the mean stress evolution in the cyclically non-isotropic material, aluminium alloy.

### Material and testing program

Material is the cyclically hardening weldable aluminium alloy AlMg61 (Mg 5.4%, Si 0.5%, Fe 0.2%), static proportionality stress  $\sigma_{0.2} = 180$  MPa, ultimate strength  $\sigma_u = 340$  MPa. The hour-glass specimens with the gage part diameter 6 mm were machined from extruded plate material. The tests were carried out under the strain range control on electro-mechanical testing machine at the frequency of 7 cycles per min. In the course of the tests the axial load-transverse strain diagrams were continuously recorded and converted into the “axial stress-axial strain” diagrams using the incremental theory of plasticity relationship [21]:

$$\varepsilon_x = (1 - 2\nu)(\sigma_x / E) - 2\varepsilon_y, \quad (6)$$

where  $\varepsilon_x$  and  $\varepsilon_y$  are the axial and transverse strain, respectively,  $\sigma_x$  is the axial stress and  $\nu$  is the Poisson's ratio at the elastic material behavior. Eq. 6 is applicable in the range of axial strain up to 0.05 [7, 19].

The loading programs included, firstly, the material testing at the cyclic loading (strain range controlled); the main program of loading successions consisted of the "basic" cyclic symmetrical loading components and periodical cyclic overload tensile or / and compressive fragments, also were applied the block-type programs with varied strain range asymmetry. Fragments of several loading programs are shown in Fig.1.

The experiments and the analysis were focused on:

- evaluation of the strain hardening of the material and suggestion of the cyclic curve for application in assessment of fatigue damage of structural components,
- defining the influence of the loading succession on the cyclic strain hardening of material,
- description of the mean stress evolution in loading successions with variable parameters
- and suggesting of the material cyclic properties characterization at the variable loading in the stress concentration conditions.

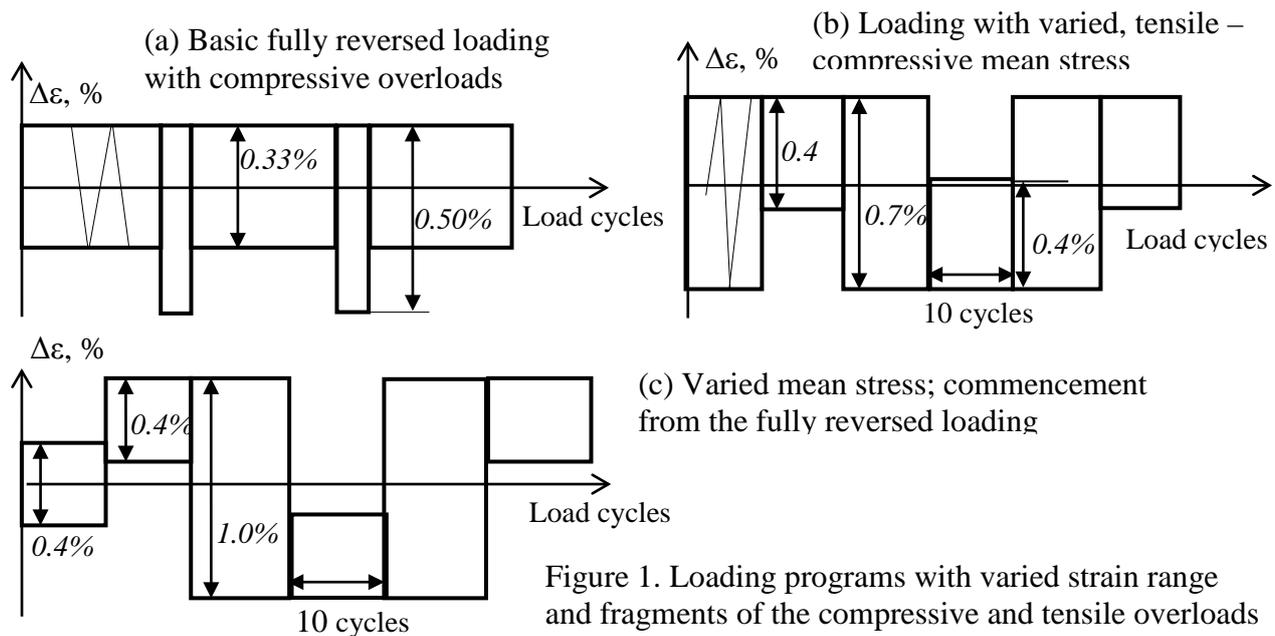


Figure 1. Loading programs with varied strain range and fragments of the compressive and tensile overloads

## Results of experiments

*Cyclic strain range-controlled test results.* Tests were carried out at the fully reversed cyclic loading under the strain range control. The transverse strain was varied in the range  $\Delta\varepsilon_y = 0.0033 - 0.010$ . The lower boundary of the range was established by the principle of intelligibility of the plastic strain component, and the upper boundary was selected as meeting the requirement of the specimen stability at the compressive part of load cycle.

Analysis of the records provided evaluation of the cyclic diagrams of the material and characterizing of the cyclic strain hardening. The cyclic stress-strain curves were approximated applying the modified Ramberg-Osgood formula:

$$\Delta\varepsilon = \Delta\varepsilon_e + \Delta\varepsilon_p = \Delta\sigma / E + K(\Delta\sigma - \Delta\sigma_c)^n \quad (7)$$

where  $K$  is the cyclic compliance,  $\Delta\sigma_c$  is the cyclic proportionality stress range and  $n$  is the cyclic strain hardening exponent.

It was found that the cyclic strain hardening exponent  $n$  does not depend on the general cyclic hardening of material and occurs, approximately,  $n = 2.40$ . The cyclic compliance occurred varying in a wide range following the general trend of cyclic hardening,  $K = (0.52 \dots 2.61) \times 10^{-8}$ . Cyclic proportionality stress range,  $\Delta\sigma_c$ , follows the same trend.

Analysis of the cyclic hardening of the alloy revealed the general regularity. In the dimensionless form, when the stress ranges were related to the “basic” value, defined by the  $d_b = 0.35$ <sup>1</sup>, where  $d = n/N$ , is the ratio of the past load cycles and the load cycles at specimen failure<sup>2</sup>, the cyclic hardening was presented in the unified form, as shown in Fig.2. Respectively, the stress range, the cyclic hardening considered, can be approximated in the following form:

$$\Delta\sigma(d) = \Delta\sigma(d_b) \exp a(d - d_b), \quad (8)$$

where  $a$  is an empirical constant,  $a \approx 0.28$ ,  $d$  is the above relative number of past load cycles related to the stress range  $\Delta\sigma(d)$ . Consequently, the cyclic stress-strain curve can be expressed as:

$$\Delta\varepsilon = \Delta\sigma(d) / E + K(\Delta\sigma(d) - \Delta\sigma_c(d))^n \quad (9)$$

This equation reflects the fact that cyclic hardening at the strain-range controlled loading realizes as the increase of the material resistance, and at the same time, as the decrease of the plastic cyclic compliance, the hysteresis loop width,  $\Delta\varepsilon_p$ .

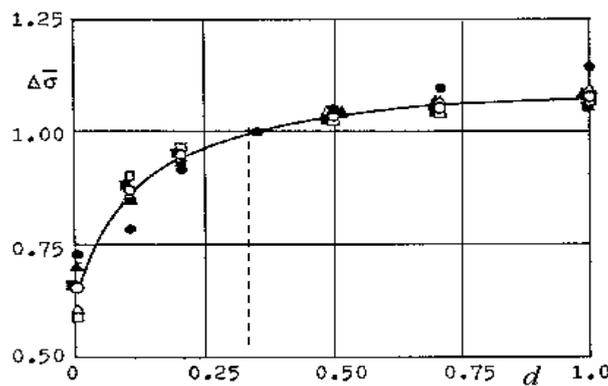


Figure 2. Generalized cyclic hardening of the material; the base stress range is defined at the damage index value  $d = 0.35$

Fig.3 illustrates evolution of the cyclic plasticity of the alloy. The notable difference of the cyclic plastic strain which revealed the tests may be explained by the inhomogeneity of the extruded material.

<sup>1</sup> Manson suggested it as 0.50 [5]

<sup>2</sup> The specimen failure was defined by the initiation of distortion of the tensile part of the hysteresis loop

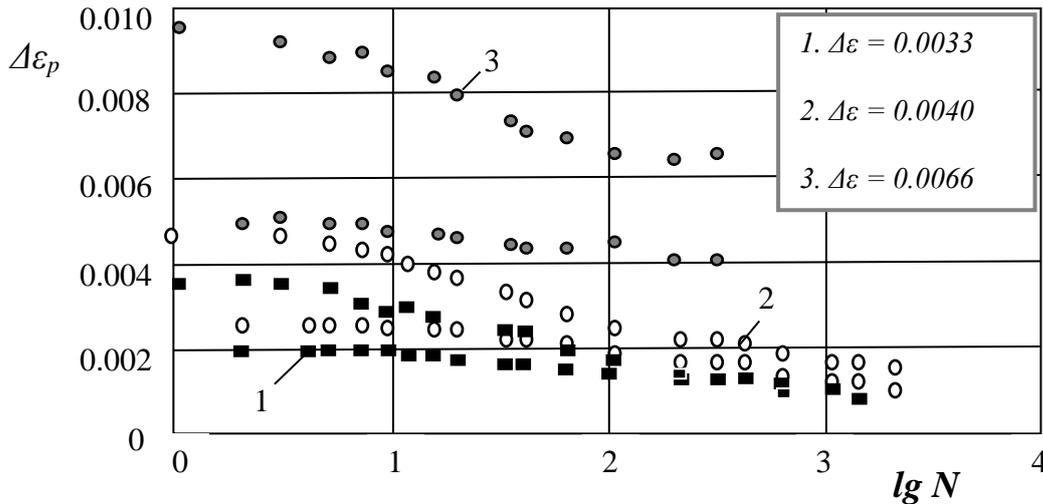


Figure 3. Plastic strain range (hysteresis loop width) evolution at the cyclic strain-range-controlled loading of the alloy

In program loading successions the mean stress develops under the influence of the «loading history» itself, changes of the load limits in the loading fragments, and of the cyclic hardening. Specific of the cyclic hardening is the observed hardening anisotropy.

Fig.4 shows kinetics of the mean stress, alternatively, tensile and compressive, following the loading program, Fig.1,b. As seen, in transition from the symmetrical loading fragments (strain range  $\Delta\varepsilon = 0.01$ ) to fragments of the tensile cyclic loading ( $\Delta\varepsilon = 0.004$ ) the mean stress monotonously increases reflecting the general cyclic hardening of the alloy.

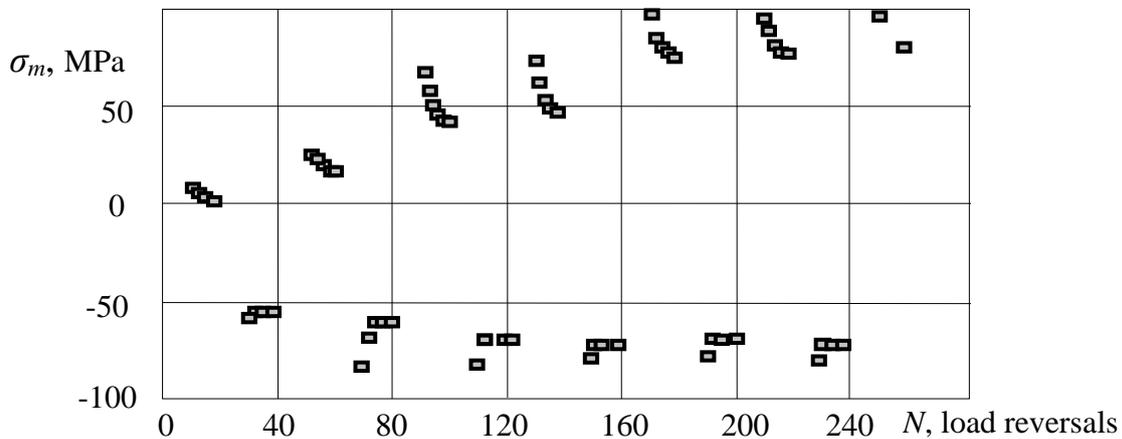


Figure 4. Mean stress evolution in the program loading. The loading scheme is shown in Fig.1,b

In transitions from fully-reversed cyclic loading to fragments with the mean compressive component (again,  $\Delta\varepsilon = 0.004$ ) the mean compressive stress also increases but this process rapidly stabilizes, and minute increases are seen immediately at transitions. In case of the program loading with completely tensile and completely compressive components inserted into intensive cyclic loading carried out accordingly scheme Fig.1,c, in fragments of the tensile cyclic strain the mean stress increases reflecting, again, the cyclic hardening with minute relaxation, same as shown in Fig.4. In transitions from the fully-reversed loading with  $\Delta\varepsilon = 0.01$  to the compressive cycling with

the strain range  $\Delta\varepsilon = 0.004$  the compressive mean stress occurs practically independent of the succession and is controlled almost solely by the current loading conditions, and the “minute” relaxation of the mean stress is insignificant.

It should be noted, that in stress concentration areas of actual structural components the localized stress redistribution may affect the quantitative character of the mean stress formation and experimental studies of specifics of the mean stress, of its influence on the fatigue resistance of material at critical locations in the crack initiation phase are needed. The mechanisms of the mean stress development and of its role in elastic-plastic deformation in irregular loading successions may be more complicated than described by Eqns 4, 5.

## Summary

Experimental study of the cyclic stress-strain behavior of cyclically hardening aluminium alloy revealed common character of hardening in cyclic loading at the strain limits control in the range of measurable plastic strains. The effective cyclic curve obtained at the dimensionless number of cycles,  $d = n/N = 0.35$  ( $n$  is the past number of load cycles, and  $N$  is the number of load cycles prior to material failure), is suggested for generalization of the cyclic properties of the material.

Specific character of the mean stress development is observed in transitions from fully-reversed segments of irregular loading to segments with cyclic tensile or compressive loading: the induced tensile mean stress increases in repetitive fragments of tensile loading following the material cyclic hardening. The induced compressive mean stress in respective loading fragments occurs almost unaffected by the cyclic hardening mechanisms.

Further experimental studies of the material behavior in stress concentration areas at irregular loading are thought necessary to complete understanding and description of the cyclic elastic-plastic properties and fatigue failure resistance of the cyclically hardening materials.

The same is important in case of the cyclically stable structural steels, fatigue characterization of which is far but complete yet, in particular, within the strain-criterion based damage accumulation approach [22] development of which is seen offering assessment of structural fatigue from the very crack initiation phase up to the onset of critical condition.

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