ASPECTS OF FATIGUE CRACK GROWTH ASSESSMENT

Petinov S.V.[1,2]

[1]IPME RAS, [2]St.Petersburg Polytechnic University,
St.Petersburg, Russia.

Abstract

The rules for fatigue assessment of welded structures recommend carrying out the crack growth analysis by using the concept of equivalent cyclic loading. The concept provides the irregular loading history reducing to the equivalent cyclic loading based on application of the linear damage accumulation rule. In reducing the S-N criterion is applied; the recommendations imply linear elastic deformation of material whereas at the crack tip develops plasticity effects of which on the stress intensity depend on the loading conditions. Presented is the analysis of influence of material plasticity on fatigue crack growth in the stress concentration area when the equivalent cyclic stress concept and the direct damage summation procedure are applied. It is shown that considering of material plasticity insignificantly influences short structural cracks in stress concentrations of structural components.

Key words: Equivalent cyclic stress; Fatigue crack propagation; Stress concentration; Effect of plasticity on fatigue crack growth; Effective stress intensity factor; Damage accumulation technique

Crack growth: effects of material plasticity

The rules for fatigue residual life assessment of welded structures (IIW, EUROCODE 3, etc.) recommend evaluation of the fatigue crack growth by applying the linear elastic fracture mechanics methodology, LEFM, until the onset of a critical condition of structure, i.e. unstable fracture or development of a through crack in pipelines, etc.

The crack growth analysis within the LEFM is applicable, strictly, at the linear elastic behavior of material. In reality, even at moderate stress, the increase of stress ahead the crack tip is limited by the material plasticity. The area around the crack tip where plastic strain develops is defined as the crack tip plastic zone, the extent and the form of which ahead of the crack depend on the nominal stress, stress state, and material properties. Irwin had shown [6] that if the near the plastic zone stress field is characterized by singularity of the $1/\sqrt{r}$-type the stress intensity factor values can be defined and applied in analysis of crack extensions provided the increased compliance of the material ahead of the crack tip may be accounted for by introducing the *effective crack length* by the half-size of the plastic zone longer than the actual one.

In the case of monotonous tensile loading of an infinite plate with the through-thickness mode I crack length of *2a*, the stress intensity is defined as follows:

$$K_{\mathrm{I}} = \sigma\left(\pi a_e\right)^{1/2} = \sigma\left(\pi\left(a + r_p\right)\right)^{1/2} \qquad (1)$$

where a_e is the *effective crack length*, r_p is the conditional extension of the crack tip due to material plasticity. The latter is defined, e.g., in the case of plane stress as $r_p = K_{\mathrm{I}}^2 / 2\pi\sigma_y^2$, σ_y is the yield stress.

Considering definition of r_p equation (1) can be an approximation of stress intensity with an account for material plasticity:

$$K_I = \sigma(\pi a)^{1/2}(1 + w(\sigma/\sigma_y)^2))^{1/2} = \sigma(\pi a)^{1/2} M_p(\sigma, \sigma_y), \quad (2)$$

where $M_p(...)$ is termed the correction for the material plasticity, $w \approx 0.25$ is the empirical parameter established based on experiments and numerical simulation of the low-carbon steel elastic-plastic cyclic deformation at the crack tip [7].

At moderate nominal stresses in affected structural member, the stress distribution at the crack tip and surrounding plastic zone differs insignificantly from what may be expected as the elastic response of material. E.g., the nominal stress amplitude typically is essentially smaller than the yield stress, and in loading histories most frequently repeating and effective in the crack extensions stress amplitude may be assumed around and below $\sigma \approx 0.3\sigma_y$. Then the plasticity correction is $M_p(\sigma, \sigma_y) = (1 + w(\sigma/\sigma_y)^2))^{1/2} \approx 1.01$, and obviously, the role of material plasticity may be neglected.

Early crack extensions in stress concentration

However, in case of a crack initiated at a stress concentration zone, the material plasticity may affect the crack propagation. Initially the macroscopic crack grows under the influence of local stress elevation, and material plasticity at the crack tip is enhanced by the mechanical conditions. Approximate solution of the problem may be suggested as follows. In the initial phase of fatigue failure when the crack grows in the stress concentration area and does not significantly influence the component compliance, the stress intensity factor approximate expression can be given in the form:

$$K_I\left(a, \sigma_n, \sigma_y\right) = M_C\left(a,..\right) M_{SC}\left(K_t, a\right) M_P\left(a, \sigma_y\right) \sigma_n \left(\pi a\right)^{1/2}, \quad (3)$$

where $M_C(a,...)$ is the correction for the compliance of the cracked component, which may be taken $M_C(a,...) \approx 1$, $M_{SC}(K_t, a,...)$ is the correction for the local stress elevation, K_t is the theoretical stress concentration factor, and $M_p(...)$ is the plasticity correction.

With the above arguments, the latter can be suggested in the following form which allows for the local stress increase:

$$M_p\left(a, S_n, \sigma_y\right) = \left(1 + w\left(M_{sc}(K_t, a) S_n / \sigma_y\right)^2\right)^{1/2}, \quad (4)$$

where S_n is the nominal, characteristic stress range, in the component under the scope, σ_y is the yield stress. The use of the yield stress in this correction is assumed provisionally, provided the parameter w is defined considering the cyclic plasticity of the material. Corrections M_{SC}, M_P to be obtained from systematic analyses of the stress-strain field in the vicinity of the crack tip when the crack propagates from the notch root in a structural detail.

To illustrate the effects of material plasticity in the crack propagation analyses at irregular loading a conditional structural detail is selected, axially cyclically loaded 100-mm wide plate made of low-carbon steel ($\sigma_y = 240$ MPa) with central elliptic hole 47 mm wide. The hole is used as the initiator of fatigue process and the crack extensions were modeled also by the finite-element procedure. The pre-crack at the notch root and the near crack tip stress fields were analyzed. Theoretical stress concentration factor found by the finite-element analysis (FEA) is $K_t = 2.85$. The material monotonous and cyclic elastic-plastic behavior was modeled by the FEM using the flow theory of plasticity; to obtain the generalized cyclic stress-strain diagram, the hour-glass specimens were tested under the strain-range control conditions. It was found that the correction for the stress concentration in the initial phase of the crack propagation might be approximated in the following form [8]:

$$M_{SC}\left(a,K_t\right)=1+\left(K_t-1\right)\Big/\left(1+a/12\right)^5, \tag{5}$$

where a is the crack length measured from the hole.

The rules for fatigue assessment of welded structures [1], [3], [5], etc. recommend performing the crack growth analyses based on the concept of equivalent cyclic loading. Accordingly the concept the random loading should be reduced to the equivalent cyclic loading based on the principle of equal fatigue damage. Following this principle, the continuous stress probability distribution recommended in the rules should be reduced to the sequence of blocks composed of fragments of cyclic loading. These fragments are specified by the number of load cycles (or by relative numbers with regard to intended fatigue life) and respective stress ranges (or amplitudes). The damage summation is recommended to carry out in the common form where the environment loading history is presented as a step-wise histogram:

$$D=\sum_i n_i/N_i=C^{-1}\sum_i n_i\cdot(S_i)^m=(N^*/C)\sum_i p_i\cdot(S_i)^m, \tag{6}$$

where i is the number of equivalent[1] cyclic stress components in the stress block, n_i is the number of equivalent stress cycles in stress block components, N^* is the number of stress «cycles» which the structure should to withstand through the service life, $p_i=n_i/N^*$ is the fraction of the stress cycles in the life-long loading history attributed to equivalent cyclic stress range S_i, N_i is the number of cycles to failure at constant stress range S_i, C and m are parameters of a fatigue failure criterion, appropriate design S-N curve:

$$N(S)=C/S^m. \tag{7}$$

The design S-N curve typically is recommended as a «two-slope» criterion. However, evaluation of the equivalent stress range S_i may be provided by applying the «mono-slope» criterion as shown in the below. The damage summation scheme (6) also can be used in the integral form, when the loading history is given as a continuous probability distribution of stress range, $p(S)$ (probability density):

$$D=\sum_i n_i/N_i=N^*\int_{S_{\min}}^{S_{\max}}(p(S)/N(S))dS \tag{8}$$

Now the efficiency of the equivalent cyclic loading in the case of crack initiation and growth in the stress concentration will be analyzed. The equivalent cyclic stress range is defined according Elber's finding [4]:

$$S_{eq}=(\sum_i p_i(S_i)S_i^m)^{1/m}, \tag{9}$$

where $p_i(S_i)$ is the probability of the stress range to fall into the sub-range of histogram with the characteristic stress S_i, m is the empirical parameter, $m=3$ for structural steels.

[1] The term «equivalent» is applied here since the irregular loading is substituted by a cyclic loading within "i"-th stress sub-range

The loading history of the considered structural component with the crack initiating elliptic hole is assumed a random zero-to-tension succession (composed of a series of stationary processes) characterized by exponential probability distribution (exceedance),

$$Q = \exp(-S / a_s), \tag{10}$$

in which a_s is the scale parameter of the distribution. This parameter value is assumed $a_s = 10$ MPa. The corresponding maximum stress range is equal $S_{\max} = a_s(-\ln Q) = 10 \cdot 18.42 = 184.2$ MPa (characterized by the exceedance, $Q = 10^{-8}$).

The parameters of Paris equation, $da / dl = C\Delta K^m$, for the low-carbon steel under the scope are: $C = 3.02 \cdot 10^{11}, m = 2.67,$ [8]. The number of loading cycles necessary for the crack to symmetrically grow up to 10 mm should be evaluated.

The equivalent stress range (9) is used to calculate the equivalent stress intensity factors, ΔK_{eq}. Fatigue life of a structural component, the number of load cycles necessary to grow the crack up to assumed size, is found by integrating the Paris equation.

The implementation of the procedure presumes transformation of the long-term stress probability distribution into the equivalent histogram, block-type irregular loading scheme. The rules for fatigue design do not explicitly indicate the procedure of evaluation of characteristic (equivalent) stress range for every of the block-loading component. It is noted, e.g. [3], that the number of the steps in a loading block should be not less than 20. In the example, the step components of the block-type equivalent of the long-term stress distribution may be defined using the approach explained in [9], [10].

The equivalent stress intensities are calculated by using equation (3):

$$\Delta K_{eq}\left(a, S_{n,eq}, \sigma_y\right) = M_{SC}\left(K_t, a\right) M_P\left(a, \sigma_y\right) S_{n,eq}\left(\pi a\right)^{1/2}, \tag{11}$$

where $S_{n,eq}$ is the equivalent by the fatigue damage nominal stress in a sub-range of the block-loading scheme, and with the above defined corrections.

The two versions of the analysis may be realized. The first may be based on evaluation of the equivalent cyclic stress and respective equivalent stress intensity range; the latter calculated using the equation (3), so that the plasticity correction would be applied to the equivalent stress intensity only.

The whole range of stresses is subdivided provisionally into the following steps, classes: 0-26, 26-52, 52-78, 78-104, 104-130, 130-156 and 156-182 MPa. The lower class may be assumed non-effective, if the threshold stress intensity considered; however, firstly, the threshold SIF is neglected.

The equivalent number of load cycles n_i for every class in the histogram can be found by using (3) on assumption that expected fatigue life is equal to $N^* = 10^8$ cycles. Further, for every class, step, of histogram the «probability of occurrence» of the stress range within the step limits is found as, $p_i = n_i / N^*$.

The damage and equivalent stress range values at every step of the block may be conditionally estimated by using equations (7) and (8) where the parameters of S-N curve are applied: $C = 3.51 \cdot 10^{11}, \quad m = 3.$ [2]

$$d_i = (N^* / Ca_s) \int_{S_{\min}}^{S_{\max}} S^m \exp(-(S / a_S)) dS \tag{12}$$

[2] Earlier it was shown that partial damage and equivalent stress values are practically independent of the S-N parameters [10]

Respectively, the equivalent stress ranges are found for every of the steps corresponding to the values of damage:

$$d_i = n_i / N_i = n_i S_i^m / C; \quad S_i^{eq} = (d_i C / n_i)^{1/m}.$$

The mentioned characteristics, i.e., n_i , p_i , d_i and equivalent stress values, $S_{eq,i}$ for every sub-range of the stress histogram were calculated and finally, the "total" equivalent stress range (6) calculated considering the power parameter of the Paris' equation, $m = 2.67$, is:

$$S_{eq} = \left(\sum_i p_i S_i^m \right)^{1/m} = (p_0 S_{0eq}^m + p_1 S_{1eq}^m + p_2 S_{2eq}^m + p_3 S_{3eq}^m + p_4 S_{4eq}^m + p_5 S_{5eq}^m + p_6 S_{6eq}^m)^{1/m} = 17.18 \text{ MPa}.$$

Further, fatigue life of the component, equivalent cyclic loading applied, can be found by integrating Paris' equation:

$$N(S_{n,eq}) = C^{-1} \int_{a_0}^{a_f} (\Delta K_{eq}(S_{n,eq}, a))^{-m} da, \tag{13}$$

where C and m are the Paris' equation parameters, a_0, a_f are the crack extension assumed limits: $a_0 = 0.2,\ a_f = 5.0,$ mm.

Analysis shows that when plasticity was not considered the number of load cycles for the crack to grow from 0.2 to 5.0 mm (both sides of the notch) was $N(S_{n,eq}) = 1.66 \cdot 10^6$ cycles.

Taking into account plasticity correction (4) resulted in $N(S_{n,eq}) = 1.648 \cdot 10^6$ cycles, e.g. the influence of material plasticity is negligible when the irregular loading is reduced to the equivalent cyclic loading.

The second version may be focused on the direct application of the linear damage summation rule (6):

$$D = \sum_i n_i / N_i = \sum_i n_i(S_{i,eq}) / N_i(S_{i,eq}), \tag{14}$$

where the total damage components, $n_i / N_i = n_i(S_{i,eq}) / N_i(S_{i,eq})$, should be calculated considering the damages due to the equivalent cyclic loading in the sub-ranges of the loading histogram,

$$N_i(S_{i,eq}) = C^{-1} \int_{a_0}^{a_f} (\Delta K_{i,eq}(S_{i,eq}, a))^{-m} da. \tag{15}$$

Further, the number of equivalent stress cycles in every of the i steps is found as

$$n_i(S_{i,eq}) = N^* \int_{S_{i,\min}}^{S_{i,\max}} p(S) dS \tag{16}$$

The probability density stress distribution function is found from (10):

$$p(S) = -dQ / dS = a_S^{-1} \exp(-S / a_S),$$

and $N^* = 10^8$, is the total number of the load (stress) excursions through the expected life of the component. Equivalent stress values for calculation of $N_i(S_{i,eq}),\ n_i(S_{i,eq})$ are taken from Table 1.

To make comparisons, when the damage summation was applied, the total expected number of loading cycles, N^*, was related to estimated damage index values. In case when the equivalent cyclic stress procedure was used, the fatigue life values were estimated considering the stress intensity threshold and neglecting it. The threshold value of the stress intensity factor range for normal structural steel was taken according [2]: $\Delta K_{th} = 5\ Ì\ Ï\ à\sqrt{ì}$. Respectively, the average threshold stress range, lower-most non-effective, was assumed $S_{th} = 26$ MPa. The results of the both procedures application are presented in Table 2.

- Equivalent stress procedure (13) applied, plasticity considered: $N(S_{eq}) = 1.649 \cdot 10^6$ cycles, and when plasticity was neglected: $N(S_{eq}) = 1.661 \cdot 10^6$ load cycles;
- When the threshold SIF and plasticity considered, $N(S_{eq}) = 2.553 \cdot 10^6$ cycles, and if the plasticity not accounted for, $N(S_{eq}) = 2.567 \cdot 10^6$ cycles;
- The "direct" damage summation (14) application, plasticity considered, results in: $N(S_{eq}) = 1.606 \cdot 10^6$, and without plasticity $N(S_{eq}) = 1.648 \cdot 10^6$ load cycles.

From these, it may be concluded that the equivalent stress procedure and the direct damage summation result in close figures. Together with the above data, this allows also to conclude that material plasticity insignificantly affects fatigue life of structural detail even when the crack extensions are enhanced by the stress concentration. Taking into account the non-damaging stress range due to the threshold stress intensity *in this example* substantially influences estimated fatigue life of the component; however, again, fatigue life occurs unaffected by the material cyclic plasticity.

One more conclusion may be derived from the above analyses: the number of steps, classes in the block-type presentation of the long-term stress distribution may be provisional, not necessarily recommended 20, when the characteristic stress of every sub-range is obtained based on the principle of fatigue damage equivalence.

Conclusions

Reduction of the continuous long-term stress distribution by the explicit transformation to the step-vise block-type format and further to the equivalent cyclic loading based on the principle of fatigue damage equivalence provides a straight-forward assessment of the fatigue crack propagation.

Material cyclic plasticity plays insignificant role in estimation of the crack propagation in structural components even when the crack extensions are affected by the local stress elevation at the stress concentration areas. This conclusion, however, may not be general, since it is related to implementation of the linear damage mechanics techniques.

References

[1] BS EN 1993-1-9:2005 EUROCODE 3: Design of Steel Structures – Part 1-9: Fatigue. P.36. ISBN 0 580 46079 7

[2] Burnside, O.H., et al. (1984) Long-term Corrosion Fatigue of Welded Marine Steels. SSC-326, Ship Structure Committee. P.420

[3] DNV Classification Notes No.30.7 (2010) Fatigue Assessment of Ship Structures. Det Norske Veritas, Hovik, Norway. P.108

[4] Elber, W. (1976) Equivalent Constant-Amplitude Concept for Crack Growth under

Spectrum Loading. ASTM STP 595, P.236-250

[5] Fricke, W. (2008) Guideline for the Fatigue Assessment by Notch Stress Analysis for Welded Structure // Int. Institute of Welding, IIW-Doc. XIII-2240r-08/XV-1289r-08. Cambridge, Abington. P.35

[6] Irwin, G.R. (1961) In Proc. 7th Sagamore Ordnance Mater. Res. Conf., Vol.4, Syracuse University Press, Syracuse, N.Y. P.63-78

[7] Letova, T.I., Petinov, S.V. (1992) On the Initial Phase of Fatigue Failure of Structural Components // «Problems of Mechanical Engineering and Machine Reliability» No.5, pp.19-24 (О начальной фазе усталостного разрушения элементов конструкции. «Проблемы машиностроения и надежности машин»)

[8] Petinov, S.V. (2003) Fatigue Analysis of Ship Structures. Backbone Publishing Co., Fair Lawn, NJ, USA, P.262

[9] Petinov, S.V., Guchinsky, R.V. (2013) Fatigue Design of Expansion Joint in Ship Superstructure // Proc., The XLI International Summer School - Conference «Advanced Problems in Mechanics», SPb-Repino, SPb, IPME RAS, P.219-213

[10] Petinov, S.V., Guchinsky R.V. (2013) Fatigue Assessment of Ship Superstructure at Expansion Joint // International Journal of Maritime Engineering, RINA, Vol.155, Part A4, P.A201-A209