On generalization of the LS-STAG immersed boundary method for Large Eddy Simulation and Detached Eddy Simulation

Valeria V. Puzikova
valeria.puzikova@gmail.com

Abstract

The general approach to the application of the LS-STAG method for the numerical solution of LES and DES equations is suggested. According to the concept of the LS-STAG method normal Reynolds or subgrid stress components are sampled on the base mesh (similar to pressure discretization) and shear ones are sampled in the upper right corners of the base mesh cells. Thus, for the shear Reynolds or subgrid stresses an additional mesh (xy-mesh) is introduced. In case of Reynolds Stress (RSM) LES and DES models, these meshes are used for transport equation solving for Reynolds or subgrid stresses. The result then is taken into account in the Helmholtz equation for the velocity. In case of Eddy Viscosity (EVM) LES or DES models eddy viscosity is sampled on the xy-mesh. In this research it is shown how to obtain the LS-STAG discretization of LES / DES equations and LES / DES turbulence models using the LS-STAG discretization developed for RANS equations and RANS-based turbulence models. To validate this approach the flow past circular airfoil at the Reynolds numbers Re $= 1000$ and Re $= 3900$ was simulated.

1 Introduction

The LS-STAG method [1] for viscous incompressible flows simulation combines the advantages of the MAC method, immersed boundary methods and level-set method. This method allows to solve on the Cartesian meshes problems when domain shape is irregular or it changes in the simulation process due to hydroelastic body motion. For these reasons, the LS-STAG method is very useful for solving such complicated problems of computational mechanics as coupled hydroelastic problems, biomechanic problems, problems of solid mechanics with deformable bodies.

However, the LS-STAG method, as all mesh methods, has a significant limitation when simulating flows with high Reynolds number: it requires extremely small space and time steps. It leads to significant increase in computational cost. The traditional method of solving this problem is RANS, LES, DES etc. turbulence models usage. Generalization of the LS-STAG method for LES and DES is presented in this research.

2 Governing equations

The problem is considered for 2D unsteady case when the flow around an airfoil assumed to be viscous and incompressible within the framework of LES and DES approaches. In contrast to direct numerical simulation (DNS) based on solution of Navier—Stokes equations and resolution of all turbulent movement scales, turbulence models usage involves a simulation of a turbulence scales contribution to the averaged motion (in case of RANS approach) or a simulation of scales that do not exceed the filter width Δ (in case of LES approach). In case of RANS approach one speaks of the Reynolds stress simulation and in case of LES approach one speaks of the subgrid stress simulation.

The Reynolds-averaged Navier—Stokes equations are solved in RANS approach, and the filtered Navier—Stokes equations are solved in LES approach instead of the Navier—Stokes equations. Usage of DES approash is means that RANS equations are solved in one part of the computational domain, and LES equations are solved in the other part. It is possible to write down the unified problem statement in dimensionless variables for all approaches, because the form of LES equations is similar to the form of RANS equations. So, the flow is described by the following LES / DES equations:

$$\nabla \cdot \mathbf{v} = 0, \quad \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\nabla p + \nu \Delta \mathbf{v} + \nabla \cdot \hat{\tau}^t. \tag{1}$$

Here $\mathbf{v}$ is the dimensionless Reynolds averaged of filtrated velocity, p is dimensionless Reynolds averaged of filtrated pressure, t is the dimensionless time, ν is the dimensionless viscosity coefficient, $\hat{\tau}^t$ is the Reynolds or subgrid stresses tensor. The boundary conditions are the following:

$$\mathbf{v}\big|_{\text{inlet}} = \mathbf{v}_\infty, \quad \frac{\partial \mathbf{v}}{\partial \mathbf{n}}\bigg|_{\text{outlet}} = 0, \quad \mathbf{v}\big|_{\text{airfoil}} = \mathbf{0}, \quad \frac{\partial p}{\partial \mathbf{n}}\bigg|_{\text{inlet \& outlet \& airfoil}} = 0. \tag{2}$$

The relationship between $\hat{\tau}^t$ and Reynolds averaged or filtrated flow variables is given by the turbulence model. In case of Reynolds Stress (RSM) RANS models, for example DRSM, ARSM, EARSM, the Reynolds stress transport equation is solved for simulating of $\hat{\tau}^t$. In case of Eddy Viscosity (EVM) RANS models the eddy viscosity ν^t (and the turbulent kinetic energy k in case of two-equation models) is simulated and Reynolds or subgrid stresses are evaluated using the Boussinesq eddy viscosity assumption [2]:

$$\tau^t_{xx} = 2\nu^t \frac{\partial u}{\partial x} + \frac{2}{3}k, \quad \tau^t_{yy} = 2\nu^t \frac{\partial v}{\partial y} + \frac{2}{3}k, \tag{3}$$

$$\tau^t_{xy} = \nu^t \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right). \tag{4}$$

Here τ^t_{xx} and τ^t_{yy} are the normal Reynolds or subgrid stresses, τ^t_{xy} are the shear Reynolds or subgrid stresses. The equations for ν^t and k, as well as initial and boundary conditions for them are is determined by the turbulence model.

3 Transition from RANS-based turbulence models to subgrid (LES and DES) models

It is possible to distinguish the linear turbulence scale $l_{turb} = l_{turb}(\mathbf{r})$ in all turbulence models. With RANS approach is used this scale l_{turb} is equal to scale $l_{RANS} = l_{RANS}(\mathbf{r})$, which is determined by the turbulence model (table 1).

Table 1: Turbulence scale l_{RANS} for some turbulence models [2]

Turbulence model	l_{RANS}	Comments
SpallartAllmaras	d_w	d_w is the distance from the field point to the nearest wall
$k-\varepsilon$	$k^{3/2}\varepsilon^{-1}$	ε is the dissipation rate of the k
$k-\omega$, $k-\omega$ SST	$k^{1/2}(\beta^*\omega)^{-1}$	ω is the specific dissipation rate of the k, $\beta^* = 0.09$

In the case of the turbulence model usage within the LES framework the scale l_{turb} is equal to subgrid scale:

$$l_{LES} = C_{LES}\Delta. \tag{5}$$

Here $\Delta = \Delta(\mathbf{r})$ is the characteristic filter size at the point of computational domain with the radius vector $\mathbf{r}$, and C_{LES} is the empirical constant, which choice depends on the turbulence model and numerical method used to solve the problem (1), (2). Within the DES approach the linear turbulence scale l_{turb} is equal to hybrid linear scale

$$l_{DES} = \min\{l_{RANS}, C_{DES}\Delta\}. \tag{6}$$

Here C_{DES} is the empirical constant similar to C_{LES}, and the maximum of the mesh steps at the point of computational domain with the radius vector $\mathbf{r}$ is used as the characteristic filter size $\Delta = \Delta(\mathbf{r})$. Thus, DES operates as RANS in the domains where the mesh is too coarse and not suitable for resolving turbulent structures, i.e. at $C_{DES}\Delta > l_{RANS}$, and DES operates as subgrid model for LES in the domains where the grid is sufficiently fine [2].

4 Generalization of the LS-STAG method for LES and DES

The Cartesian mesh with cells $\Omega_{i,j} = (x_{i-1}, x_i) \times (y_{j-1}, y_j)$ is introduced in the rectangular computational domain. It is denoted that $\Gamma_{i,j}$ is the face of $\Omega_{i,j}$ and $\mathbf{x}^c_{i,j} = (x^c_i, y^c_j)$ is the center of this cell. Unknown components $u_{i,j}$ and $v_{i,j}$ of velocity vector $\mathbf{v}$ are computed in the middle of fluid parts of the cell faces. These points are the centers of control volumes $\Omega^u_{i,j} = (x^c_i, x^c_{i+1})\times(y_{j-1}, y_j)$ and $\Omega^v_{i,j} = (x_{i-1}, x_i)\times(y^c_j, y^c_{j+1})$ with faces $\Gamma^u_{i,j}$ and $\Gamma^v_{i,j}$ respectively (fig. 1).

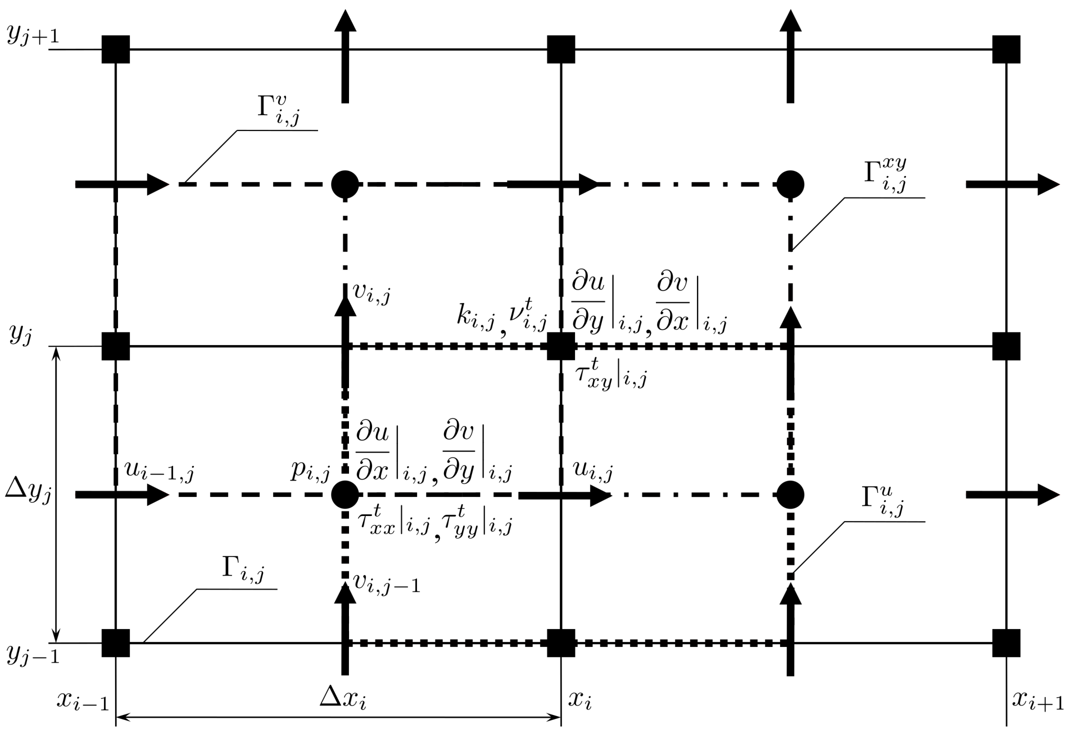

Figure 1: Staggered arrangement of the variables on the LS-STAG mesh

Cells which the immersed boundary intersects are the so-called 'cut-cells' [1]. These cells contain the solid part together with the liquid one. The level-set function φ [3] is used for immersed boundary Γ^{ib} description. The boundary Γ^{ib} is represented by a line segment on the cut-cell $\Omega_{i,j}$. Locations of this segment endpoints are defined by a linear interpolation of the variable $\varphi_{i,j} = \varphi(x_i, y_j)$. The cell-face fraction ratios $\vartheta^u_{i,j}$ and $\vartheta^v_{i,j}$ are introduced. They take values in interval $[0, 1]$ and represent the fluid parts of the east and north faces of $\Gamma_{i,j}$ respectively.

To preserve the five-point structure of the MAC method stencil we need to make distinction between the discretization of the normal and shear stresses (fig. 1). It is proposed to sample the normal and shear Reynolds or subgrid stresses similarly. It is conveniently to sample the eddy viscosity ν^t and turbulence kinetic energy k at the same points as the shear stresses. Thus, in case of the LS-STAG method usage within LES and DES approaches the fourth mesh with cells $\Omega^{xy}_{i,j} = (x^c_i, x^c_{i+1}) \times (y^c_j, y^c_{j+1})$ is needed. The faces of these cells are $\Gamma^{xy}_{i,j}$ (fig. 1) and their areas are $M^{xy}_{i,j}$. If $i = \overline{1, N}$, $j = \overline{1, M}$, xy-mesh contains $E_{xy} = (N-1) \cdot (M-1)$ cells.

It is posible to assign a weight $\alpha_{i,j}$ to each cell $\Omega_{i,j}$ of the base mesh:

$$\alpha_{i,j} = \begin{cases} 0, & \text{if } \Omega_{i,j} \text{ is the solid cell,} \\ 1/3, & \text{if } \Omega_{i,j} \text{ is the triangular cell,} \\ 1/4, & \text{otherwise.} \end{cases}$$

Then $M^{xy}_{i,j}$ can be expressed through the area of base mesh cells:

$$M^{xy}_{i,j} = \alpha_{i,j-1} V_{i,j-1} + \alpha_{i-1,j} V_{i-1,j} + \alpha_{i,j} V_{i,j} + \alpha_{i,j+1} V_{i,j+1}.$$

Here $V_{i,j}$ is the area of the $\Omega_{i,j}$.

Since ν^t and shear Reynolds or subgrid stresses (4) are sampled at the same points, it follows that

$$\tau^t_{xy}|_{i,j} = \nu^t_{i,j}\Big(\frac{\partial u}{\partial y}\Big|_{i,j} + \frac{\partial v}{\partial x}\Big|_{i,j}\Big),$$

whereas averaged values of turbulent viscosity $\overline{\nu^t}_{i,j}$ and the turbulent kinetic energy $\overline{k}_{i,j}$ should be used for the computation of the normal Reynolds or subgrid stresses (3):

$$\tau^t_{xx}|_{i,j} = 2\overline{\nu^t}_{i,j}\frac{\partial u}{\partial x}\Big|_{i,j} + \frac{2}{3}\overline{k}_{i,j},\ \ \tau^t_{yy}|_{i,j} = 2\overline{\nu^t}_{i,j}\frac{\partial v}{\partial y}\Big|_{i,j} + \frac{2}{3}\overline{k}_{i,j},$$

$$\overline{\nu^t}_{i,j} = \alpha_{i,j}(\nu^t_{i,j} + \nu^t_{i,j-1} + \nu^t_{i-1,j} + \nu^t_{i-1,j-1}),\ \ \overline{k}_{i,j} = \alpha_{i,j}(k_{i,j} + k_{i,j-1} + k_{i-1,j} + k_{i-1,j-1}).$$

It is conveniently to sample the linear turbulence scale l_{turb} and the characteristic filter size Δ for LES and DES at the same points as the ν^t and k. We recall that the maximum mesh step at the given point of the computational domain is used as a filter size Δ for DES approach. Since we deal with xy-mesh, the characteristic filter size is defined as a following:

$$\Delta_{i,j} = \Delta^{\max}_{i,j} = \max\{\Delta y^{xy}_{i-1,j}, \Delta y^{xy}_{i,j}, \Delta y^{xy}_{i+1,j}, \Delta x^{xy}_{i,j-1}, \Delta x^{xy}_{i,j}, \Delta x^{xy}_{i,j+1}\},$$

$$\Delta y^{xy}_{i,j} = \frac{1}{2}(\vartheta^u_{i,j}\Delta y_j + \vartheta^u_{i,j+1}\Delta y_{j+1}),\ \ \Delta x^{xy}_{i,j} = \frac{1}{2}(\vartheta^v_{i,j}\Delta x_i + \vartheta^v_{i+1,j}\Delta x_{i+1}).$$

The following filter can also be used on the LS-STAG mesh within LES approach:

$$\Delta_{i,j} = \Delta^{vol}_{i,j} = \sqrt{M^{xy}_{i,j}}.$$

Thus, the LS-STAG discretization of LES / DES equations and turbulence LES / DES models can be easily obtained from the LS-STAG discretization of RANS equations and RANS-based turbulence models developed in [4] by using formulae (5), (6). The development of the LS-STAG discretization for the Spalart—Allmaras (S-A) turbulence model [5] is described in [4] as an example.

5 Numerical experiments

The flow past circular airfoil was simulated using the developed modification of the LS-STAG method at the Reynolds numbers Re $= 1000$ (on non-uniform meshes 120×148 with $\Delta t = 5 \cdot 10^{-3}$ and 240×296 with $\Delta t = 10^{-3}$) and Re $= 3900$ (on non-uniform meshes 120×148 with $\Delta t = 10^{-3}$ and 240×296 with $\Delta t = 5 \cdot 10^{-4}$); $C_{LES} = 0.20$, $C_{DES} = 0.65$. The time averaged drag coefficient C_{xa} and the Strouhal number Sh were computed. The coefficient C_{xa} is obtained by averaging over a large period of time the unsteady load $C_{xa}(t) = \dfrac{2F_{xa}(t)}{\rho V^2_\infty}$. Computational results are shown in table 2. These results are in good agreement with experimental data and results of numerical simulations.

Table 2: Comparison of C_{xa} and Sh with established results from the literature

Turbulence model	Number of cells	Re = 1000		Re = 3900	
		C_{xa}	Sh	C_{xa}	Sh
Experiment [6]		0.98	0.21	0.93	0.22
Experiment [7]		1.12	—	1.01	—
LES [8]	1 103 520	—	—	1.08	—
SV LES [9]	30 720	—	—	1.01	0.22
FV LES [9]	855 040	—	—	1.07	0.24
$k-\varepsilon$ [10]	46 304	0.995	0.15	1.00	0.15
Real $k-\varepsilon$ [10]	46 304	—	0.17	—	0.20
SST $k-\omega$ [10]	46 304	—	0.23	—	0.25
$k-\varepsilon$ [11], ANSYS	388 550	1.17	—	0.74	—
SST $k-\omega$ [11], ANSYS	388 550	0.99	—	0.62	—
LES [11], ANSYS	388 550	1.15	0.21	1.07	—
S-A LES (Δ^{vol}), present study	17 760	1.13	0.26	0.82	0.26
S-A LES (Δ^{vol}), present study	71 040	1.04	0.24	1.09	0.25
S-A LES (Δ^{max}), present study	17 760	1.13	0.26	0.82	0.17
S-A LES (Δ^{max}), present study	71 040	1.03	0.24	1.08	0.25
S-A DES, present study	17 760	1.13	0.26	0.81	0.23
S-A DES, present study	71 040	1.00	0.23	1.01	0.23

6 Conclusion

The key points of the LS-STAG method generalization for LES and DES are described. For the shear Reynolds or subgrid stresses and for the eddy viscosity an additional mesh (xy-mesh) is introduced. It is shown how to obtain the LS-STAG discretization of LES / DES equations and LES / DES turbulence models using the LS-STAG discretization developed for RANS equations and RANS-based turbulence models. To validate this approach the flows past a circular airfoil at the Reynolds numbers Re $= 1000$ and Re $= 3900$ were simulated. Computational results are in good agreement with established results from the literature.

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Valeria V. Puzikova, Russia, 105005 Moscow, 2nd Baumanskaya, 5