Geometrically nonlinear static theory of micropolar elastic thin shallow shells

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Abstract

Micropolar thin shallow shells are considered, the elastic deflections are comparable to their thickness and at the same time are small in relation to the basic size, at the same time as the small angles of relation of the normal to the middle surface before deformation, and their free rotations. Thus, in the deformation tensor and bending-torsion tensor takes into account not only linear but also the nonlinear terms in the gradients of displacements. The hypothesis method is developed and on this base static applied theory of micropolar elastic flexible shallow shells are constructed. Some practical problems are solved.

1 Introduction

In the monographs [1],[2] the geometrically nonlinear static and dynamic general theories of thin plates and shells are constructed on the basis of the classical elasticity theory.

In the works [3],[4] the applied static and dynamic general theories of micropolar thin plates with finite deflections are constructed on the basis of three-dimensional micropolar theory of elasticity.

In the given work as initial, the general variational principle of geometrically nonlinear three-dimensional static theory of micropolar elasticity shallow shells are developed (all basic equations and natural boundary conditions of the specified theory are followed from this principle). The kinematic and static hypotheses of the works [3],[4] are accepted in the basis of construction of the adequate general variational principle of geometrically nonlinear micropolar applied theory of elastic thin shallow shells. By verifying the resulting functional of applied theory of micropolar flexible shallow shells to it's all functional arguments balance equations, physical relations of elasticity, geometrical relations, and also natural boundary conditions are obtained.

2 The geometrically nonlinear model of three-dimensional micropolar elastic shallow shells with independent field of displacements and rotations

The shallow shell with the constant thickness $2h$ is considered as three-dimensional elastic micropolar isotropic body. The shell is attributed to the system of coordinate $x_1,\ x_2,\ z$. The coordinate surface $x_1,\ x_2$ is combined with the median surface of the shell. The axes Oz is directed along the normal of the median surface.

The variational functional of three-dimensional theory of micropolar elasticity with the finite displacements are resulted and looks like:

$$I=\int_{-h}^{h}\iint_S\left\langle W-\left\{\sigma_{11}\left[\gamma_{11}-\left(\frac{\partial V_1}{\partial x_1}+\frac{V_3}{R_1}+\frac{1}{2}\left(\frac{\partial V_3}{\partial x_1}\right)^2\right)\right]+\right.\right.$$

$$+\sigma_{22}\left[\gamma_{22}-\left(\frac{\partial V_2}{\partial x_2}+\frac{V_3}{R_2}+\frac{1}{2}\left(\frac{\partial V_3}{\partial x_2}\right)^2\right)\right]+\sigma_{33}\left[\gamma_{33}-\frac{\partial V_3}{\partial z}\right]+$$

$$+\sigma_{12}\left[\gamma_{12}-\left(\frac{\partial V_2}{\partial x_1}+\frac{1}{2}\frac{\partial V_3}{\partial x_1}\frac{\partial V_3}{\partial x_2}-\omega_3\right)\right]+\sigma_{21}\left[\gamma_{21}-\left(\frac{\partial V_1}{\partial x_2}+\frac{1}{2}\frac{\partial V_3}{\partial x_1}\frac{\partial V_3}{\partial x_2}+\omega_3\right)\right]+$$

$$+\sigma_{13}\left[\gamma_{13}-\left(\frac{\partial V_3}{\partial x_1}+\omega_2\right)\right]+\sigma_{31}\left[\gamma_{31}-\left(\frac{\partial V_1}{\partial z}-\omega_2\right)\right]+$$

$$+\sigma_{23}\left[\gamma_{23}-\left(\frac{\partial V_3}{\partial x_2}-\omega_1\right)\right]+\sigma_{32}\left[\gamma_{32}-\left(\frac{\partial V_2}{\partial z}+\omega_1\right)\right]+$$

$$+\mu_{11}\left[\chi_{11}-\frac{\partial \omega_1}{\partial x_1}\right]+\mu_{22}\left[\chi_{22}-\frac{\partial \omega_2}{\partial x_2}\right]+\mu_{33}\left[\chi_{33}-\frac{\partial \omega_3}{\partial z}\right]+$$

$$+\mu_{12}\left[\chi_{12}-\frac{\partial \omega_2}{\partial x_1}\right]+\mu_{21}\left[\chi_{21}-\frac{\partial \omega_1}{\partial x_2}\right]+\mu_{13}\left[\chi_{13}-\frac{\partial \omega_3}{\partial x_1}\right]+$$

$$\left.\left.+\mu_{31}\left[\chi_{31}-\frac{\partial \omega_1}{\partial z}\right]+\mu_{23}\left[\chi_{23}-\frac{\partial \omega_3}{\partial x_2}\right]+\mu_{32}\left[\chi_{32}-\frac{\partial \omega_2}{\partial z}\right]\right\}\right\rangle dx_1dx_2dz-$$

$$-\iint_{S^+}\left[q_1^+V_1+q_2^+V_2+q_3^+V_3+m_1^+\omega_1+m_2^+\omega_2+m_3^+\omega_3\right]_{z=h}dx_1dx_2+$$

$$+\iint_{S^-}\left[q_1^-V_1+q_2^-V_2+q_3^-V_3+m_1^-\omega_1+m_2^-\omega_2+m_3^-\omega_3\right]_{z=-h}dx_1dx_2+$$

$$+\int_{-h}^{h}dz\int_{l_1'}\left(\sigma_{21}^0V_1+\sigma_{22}^0V_2+\sigma_{23}^0V_3+\mu_{21}^0\omega_1+\mu_{22}^0\omega_2+\mu_{23}^0\omega_3\right)dx_1+$$

$$+\int_{-h}^{h}dz\int_{l_1''}\left[\sigma_{21}\left(V_1-V_1^0\right)+\sigma_{22}\left(V_2-V_2^0\right)+\sigma_{23}\left(V_3-V_3^0\right)+\right.$$

$$\left.+\mu_{21}\left(\omega_1-\omega_1^0\right)+\mu_{22}\left(\omega_2-\omega_2^0\right)+\mu_{23}\left(\omega_3-\omega_3^0\right)\right]dx_1+$$

$$+\int_{-h}^{h} dz \int_{l_2'} \left(\sigma_{21}^0 V_1 + \sigma_{22}^0 V_2 + \sigma_{23}^0 V_3 + \mu_{21}^0 \omega_1 + \mu_{22}^0 \omega_2 + \mu_{23}^0 \omega_3\right) dx_2 +$$

$$+\int_{-h}^{h} dz \int_{l_2''} \Big[\sigma_{21}\left(V_1 - V_1^0\right) + \sigma_{22}\left(V_2 - V_2^0\right) + \sigma_{23}\left(V_3 - V_3^0\right) +$$

$$+\mu_{21}\left(\omega_1 - \omega_1^0\right) + \mu_{22}\left(\omega_2 - \omega_2^0\right) + \mu_{23}\left(\omega_3 - \omega_3^0\right)\Big] dx_2 \quad (1)$$

Here surface integrals are extended on the face surfaces $S^+, S^-(z = \pm h)$ and on the lateral surface of the shell, where on one part the external strains and moments are set, and on the other part displacements and rotations are set; quantities with the top indexes zero are the set external force stresses and couple stresses on the certain part l_1 of the contour of the median surface of the shell and displacements and rotations on the other part l_2 of the same contour, and also $l_1 = l_1' \bigcup l_1''$, $l_2 = l_2' \bigcup l_2''$. W is the density of potential energy of deformations:

$$W = \frac{1}{2}\Big(\sigma_{11}\gamma_{11} + \sigma_{22}\gamma_{22} + \sigma_{12}\gamma_{12} + \sigma_{21}\gamma_{21} + \sigma_{13}\gamma_{13} + \sigma_{23}\gamma_{23} + \sigma_{32}\gamma_{32} +$$

$$+\mu_{11}\chi_{11} + \mu_{22}\chi_{22} + \mu_{33}\chi_{33} + \mu_{12}\chi_{12} + \mu_{21}\chi_{21} + \mu_{13}\chi_{13} + \mu_{31}\chi_{31} + \mu_{23}\chi_{23} + \mu_{32}\chi_{32}\Big) \quad (2)$$

Here V_i, V_3 are components of the displacement vector; ω_i, ω_3 are components of the independent rotation vector; σ_{ii}, σ_{ij}, σ_{i3}, σ_{3i}, σ_{33} are components of the force stresses tensor; μ_{ii}, μ_{ij}, μ_{i3}, μ_{3i}, μ_{33} are components of the couple stresses tensor; γ_{ii}, γ_{ij}, γ_{i3}, γ_{3i} are components of the deformations tensor; χ_{ii}, χ_{ij}, χ_{i3}, χ_{3i} are components of the bends-torsions tensor.

It is naturally, the functional (1) to call the full functional of three-dimensional micropolar theory of elasticity of shallow shells at finite deflections. On its basis the variational equation ($\delta I = 0$) can be obtained. All general equations and natural boundary conditions of the micropolar elasticity problem at finite displacements are obtained from this equation.

3 The geometrically nonlinear theory of micropolar elastic thin shallow shells with big deflections

Hypothesis of works [3], [4] are accepted in the base of the offered theory of micropolar elastic geometrically nonlinear thin shallow shells for the purpose of reduction the geometrically nonlinear three-dimensional theory of micropolar elasticity to the corresponding tow-dimensional theory. Also, instead of the components of stresses and couple stresses tensors there are entered integrated characteristics, that are statically equivalent to them: strains (T_{ii}, S_{ij}, N_{i3}, N_{3i}), moments (M_{ii}, M_{ij}, L_{ii}, L_{ij}, L_{i3}, L_{33}) and hypermoments (Λ_{i3}):

$$T_{ii} = \int_{-h}^{h} \sigma_{ii} dz,\ S_{ij} = \int_{-h}^{h} \sigma_{ij} dz,\ N_{i3} = \int_{-h}^{h} \sigma_{i3} dz (i \leftrightarrow 3),\ M_{ii} = \int_{-h}^{h} \sigma_{ii} z dz$$

$$M_{ij}=\int_{-h}^{h}\sigma_{ij}zdz,\ L_{mn}=\int_{-h}^{h}\mu_{mn}dz(m,n=1,2,3),\ \Lambda_{i3}=\int_{-h}^{h}\mu_{i3}zdz \qquad (3)$$

The formula of the averaged functional I_0 of micropolar thin shallow shells are obtained from the formula (1) of the three-dimensional theory according to the accepted hypothesis:

$$I_0=\iint_S\Bigg\langle W_0-\Bigg\{T_{11}\left[\Gamma_{11}-\left(\frac{\partial u_1}{\partial x_1}+\frac{w}{R_1}+\frac{1}{2}\left(\frac{\partial w}{\partial x_1}\right)^2\right)\right]+M_{11}\left[K_{11}-\frac{\partial\psi_1}{\partial x_1}\right]+$$

$$+T_{22}\left[\Gamma_{22}-\left(\frac{\partial u_2}{\partial x_2}+\frac{w}{R_2}+\frac{1}{2}\left(\frac{\partial w}{\partial x_2}\right)^2\right)\right]+M_{22}\left[K_{22}-\frac{\partial\psi_2}{\partial x_2}\right]+$$

$$+S_{12}\left[\Gamma_{12}-\left(\frac{\partial u_2}{\partial x_1}+\frac{1}{2}\frac{\partial w}{\partial x_1}\frac{\partial w}{\partial x_2}-\Omega_3\right)\right]+M_{12}\left[K_{12}-\left(\frac{\partial\psi_2}{\partial x_1}-\iota\right)\right]+$$

$$+S_{21}\left[\Gamma_{21}-\left(\frac{\partial u_1}{\partial x_2}+\frac{1}{2}\frac{\partial w}{\partial x_1}\frac{\partial w}{\partial x_2}+\Omega_3\right)\right]+M_{21}\left[K_{21}-\left(\frac{\partial\psi_1}{\partial x_2}+\iota\right)\right]+$$

$$+N_{13}\left[\Gamma_{13}-\left(\frac{\partial w}{\partial x_1}+\Omega_2\right)\right]+N_{31}\left[\Gamma_{31}-(\psi_1-\Omega_2)\right]+$$

$$+N_{23}\left[\Gamma_{23}-\left(\frac{\partial w}{\partial x_2}-\Omega_1\right)\right]+N_{32}\left[\Gamma_{32}-(\psi_2+\Omega_1)\right]+$$

$$+L_{11}\left[\kappa_{11}-\frac{\partial\Omega_1}{\partial x_1}\right]+L_{22}\left[\kappa_{22}-\frac{\partial\Omega_2}{\partial x_2}\right]+L_{33}\left[\kappa_{33}-\iota\right]+$$

$$+L_{12}\left[\kappa_{12}-\frac{\partial\Omega_2}{\partial x_1}\right]+L_{21}\left[\kappa_{21}-\frac{\partial\Omega_1}{\partial x_2}\right]+L_{13}\left[\kappa_{13}-\frac{\partial\Omega_3}{\partial x_1}\right]+$$

$$+L_{23}\left[\kappa_{23}-\frac{\partial\Omega_3}{\partial x_2}\right]+\Lambda_{13}\left[l_{13}-\frac{\partial\iota}{\partial x_1}\right]+\Lambda_{13}\left[l_{13}-\frac{\partial\iota}{\partial x_1}\right]\Bigg\}\Bigg\rangle dx_1dx_2-$$

$$-\iint_{S^+}\Big[q_1^+u_1+q_1^+h\psi_1+q_2^+u_2+q_2^+h\psi_2++q_3^+w+$$

$$+m_1^+\Omega_1+m_2^+\Omega_2+m_3^+\Omega_3+m_3^+h\iota\Big]dx_1dx_2+$$

$$+\iint_{S^-}\Big[q_1^-u_1-q_1^-h\psi_1+q_2^-u_2-q_2^-h\psi_2++q_3^+w+$$

$$+m_1^-\Omega_1+m_2^-\Omega_2+m_3^-\Omega_3-m_3^-h\iota\Big]dx_1dx_2+$$

$$+\int_{l_1'}\left(S_{21}^0u_1+T_{22}^0u_2+M_{21}^0\psi_1+M_{22}^0\psi_2+N_{23}^0w+L_{21}^0\Omega_1+L_{22}^0\Omega_2+L_{23}^0\Omega_3+\Lambda_{23}^0\iota\right)dx_1+$$

$$+\int_{l_1''}\Big[S_{21}\left(u_1-u_1^0\right)+T_{22}\left(u_2-u_2^0\right)+M_{21}\left(\psi_1-\psi_1^0\right)+M_{22}\left(\psi_2-\psi_2^0\right)+N_{23}\left(w-w^0\right)+$$

$$+L_{21}\left(\Omega_1-\omega_1^0\right)+L_{22}\left(\Omega_2-\Omega_2^0\right)+L_{23}\left(\Omega_3-\Omega_3^0\right)+\Lambda_{23}\left(\iota-\iota^0\right)\Big]dx_1+$$

$$+\int_{l_2'}\left(T_{11}^0 u_1+S_{12}^0 u_2+M_{11}^0\psi_1+M_{12}^0\psi_2+N_{13}^0 w+L_{11}^0\Omega_1+L_{12}^0\Omega_2+L_{13}^0\Omega_3+\Lambda_{13}^0\iota\right)dx_2+$$

$$+\int_{l_2''}\Big[T_{11}\left(u_1-u_1^0\right)+S_{12}\left(u_2-u_2^0\right)+M_{11}\left(\psi_1-\psi_1^0\right)+M_{12}\left(\psi_2-\psi_2^0\right)+N_{13}\left(w-w^0\right)+$$

$$+L_{11}\left(\Omega_1-\omega_1^0\right)+L_{12}\left(\Omega_2-\Omega_2^0\right)+L_{13}\left(\Omega_3-\Omega_3^0\right)+\Lambda_{13}\left(\iota-\iota^0\right)\Big]dx_2+ \qquad (4)$$

W_0 is the average density of potential energy of deformation of micropolar shallow shell:

$$W_0=\frac{1}{2}\Big(T_{11}\Gamma_{11}+T_{22}\Gamma_{22}+S_{12}\Gamma_{12}+S_{21}\Gamma_{21}+M_{11}K_{11}+M_{22}K_{22}+M_{12}K_{12}+$$

$$M_{21}K_{21}+N_{13}\Gamma_{13}+N_{31}\Gamma_{31}+N_{23}\Gamma_{23}+N_{32}\Gamma_{32}+L_{11}\kappa_{11}+L_{22}\kappa_{22}+$$

$$+L_{33}\kappa_{33}+L_{12}\kappa_{12}+L_{21}\kappa_{21}+L_{13}\kappa_{13}+L_{23}\kappa_{23}+\Lambda_{13}l_{13}+\Lambda_{23}\Big) \qquad (5)$$

Let us notice, that in the specified theory the displacements, independent rotations, components of deformation and bend-torsion tensors are expressed by formulas:

$$\gamma_{ii}=\Gamma_{ii}\left(x_1,x_2\right)+zK_{ii}\left(x_1,x_2\right),\ \gamma_{ij}=\Gamma_{ij}\left(x_1,x_2\right)+zK_{ij}$$

$$\gamma_{i3}=\Gamma_{i3}\left(x_1,x_2\right),\ \gamma_{3i}=\Gamma_{3i}\left(x_1,x_2\right)$$

$$\chi_{ii}=\kappa_{ii}\left(x_1,x_2\right),\ \chi_{33}=\iota\left(x_1,x_2\right),\ \chi_{ij}=\kappa_{ij}\left(x_1,x_2\right),\ \chi_{i3}=\kappa_{i3}\left(x_1,x_2\right)+zl_{i3}\left(x_1,x_2\right)$$

$$V_i=u_i\left(x_1,x_2\right)+z\psi_i\left(x_1,x_2\right),\ V_3=w\left(x_1,x_2\right)$$

$$\omega_i=\Omega_i\left(x_1,x_2\right),\ \omega_3=\Omega_3\left(x_1,x_2\right)+z\iota\left(x_1,x_2\right) \qquad (6)$$

$u_1,\ u_2,\ w$ are displacements of the points of the shallow shell's median surface along the axes $x_1,\ x_2,\ z$; ψ_1,ψ_2 are full angels of the rotation; $\Omega_1,\ \Omega_2,\ \Omega_3$ are free rotations of the initially normal element of the shallow shell's median surface round the lines $x_1,\ x_2,\ z$; ι is intensity of the full rotation along the z; Γ_{ii} are elongation deformations along the $x_1,\ x_2$; $\Gamma_{ij},\ \Gamma_{i3},\ \Gamma_{3i}$ are shears in the corresponding planes; K_{ii} are flexures of the shallow shell's median surface caused by the stresses; K_{ij} are torsions of the shallow shell's median surface caused by the stresses; $\kappa_{ii},\ \kappa_{33}$ are flexures of the shallow shell's median surface caused by the couple stresses; κ_{ij} are torsions of the shallow shell's median surface caused by the couple stresses; l_{i3} are hyper shears of the shallow shell's median surface caused by the couple stresses.

By the verifying I_0 to it's all functional arguments, the general equations and the natural boundary conditions of the micropolar elastic geometrically nonlinear thin shallow shells with the independent fields of displacements and rotations are obtained from the variational equation $\delta I_0=0$. These equations and boundary conditions are follows as:

Balance equations

$$\frac{\partial T_{ii}}{\partial x_i}+\frac{\partial S_{ji}}{\partial x_j}=-\left(p_i^+-p_i^-\right),\ \frac{\partial M_{ii}}{\partial x_i}+\frac{\partial M_{ji}}{\partial x_j}-N_{3i}=-h\left(p_i^++p_i^-\right)$$

$$\frac{\partial N_{13}}{\partial x_1}+\frac{\partial N_{23}}{\partial x_2}+\frac{\partial}{\partial x_1}\left[T_{11}\frac{\partial w}{\partial x_1}+\frac{1}{2}\left(S_{12}+S_{21}\right)\frac{\partial w}{\partial x_2}\right]+$$

$$+\frac{\partial}{\partial x_2}\left[\frac{1}{2}(S_{12}+S_{21})\frac{\partial w}{\partial x_1}+T_{22}\frac{\partial w}{\partial x_2}\right]-\frac{T_{11}}{R_1}-\frac{T_{22}}{R_2}=-(p_i^+-p_i^-)$$

$$\frac{\partial L_{ii}}{\partial x_i}+\frac{\partial L_{ji}}{\partial x_j}+(-1)^j(N_{j3}-N_{3j})=-(m_i^+-m_i^-)$$

$$L_{33}-\frac{\partial \Lambda_{13}}{\partial x_1}-\frac{\partial \Lambda_{13}}{\partial x_1}-(M_{12}-M_{21})=h(m_3^++m_3^-)$$

$$\frac{\partial L_{13}}{\partial x_1}+\frac{\partial L_{23}}{\partial x_2}+(S_{12}-S_{21})-\frac{L_{11}}{R_1}-\frac{L_{22}}{R_2}=-(m_3^+-m_3^-) \quad (7)$$

Elasticity relations

$$T_{ii}=\frac{2Eh}{1-\nu^2}[\Gamma_{ii}+\nu\Gamma_{jj}],\quad M_{ii}=\frac{2Eh^3}{3(1-\nu^2)}[K_{ii}+\nu K_{jj}]$$

$$S_{ij}=2h[(\mu+\alpha)\Gamma_{ij}+(\mu-\alpha)\Gamma_{ji}],\quad M_{ij}=\frac{2h^3}{3}[(\mu+\alpha)K_{ij}+(\mu-\alpha)K_{ji}]$$

$$N_{i3}=2h[(\mu+\alpha)\Gamma_{i3}+(\mu-\alpha)\Gamma_{3i}],\quad N_{3i}=2h[(\mu+\alpha)\Gamma_{3i}+(\mu-\alpha)\Gamma_{i3}]$$

$$L_{ii}=2h[(\beta+2\gamma)\kappa_{ii}+\beta(\kappa_{jj}+\iota)],\quad L_{33}=2h[(\beta+2\gamma)\iota+\beta(\kappa_{11}+\kappa_{22})]$$

$$L_{ij}=2h[(\gamma+\epsilon)\kappa_{ij}+(\gamma-\epsilon)\kappa_{ji}],\quad L_{i3}=2h\frac{4\gamma\epsilon}{\gamma+\epsilon}\kappa_{i3},\quad \Lambda_{i3}=\frac{2h^3}{3}\frac{4\gamma\epsilon}{\gamma+\epsilon}l_{i3} \quad (8)$$

Geometrically relations

$$\Gamma_{ii}=\frac{\partial u_i}{\partial x_i}+\frac{w}{R_i}+\frac{1}{2}\left(\frac{\partial w}{\partial x_i}\right)^2,\quad \Gamma_{ij}=\frac{\partial u_j}{\partial x_i}-(-1)^j\Omega_3+\frac{1}{2}\left(\frac{\partial w}{\partial x_1}\right)\left(\frac{\partial w}{\partial x_2}\right)$$

$$\Gamma_{i3}=\frac{\partial w}{\partial x_i}+(-1)^j\Omega_j,\quad \Gamma_{3i}=\psi_i-(-1)^j\Omega_j,\quad K_{ii}=\frac{\partial \psi_i}{\partial x_i}-(-1)^j\iota$$

$$\kappa_{ii}=\frac{\partial \Omega_i}{\partial x_i},\quad \kappa 33=\iota,\quad \kappa_{ij}=\frac{\partial \Omega_j}{\partial x_i},\quad \kappa_{i3}=\frac{\partial \Omega_3}{\partial x_i},\quad l_{i3}=\frac{\partial \iota}{\partial x_i} \quad (9)$$

Boundary conditions (on $x_i=const$)

$$T_{11}=T_{11}^0,\quad S_{12}=S_{12}^0,\quad M_{11}=M_{11}^0,\quad M_{12}=M_{12}^0$$

$$T_{11}\frac{\partial w}{\partial x_1}+\frac{1}{2}(S_{12}+S_{21})\frac{\partial w}{\partial x_2}+N_{13}=N_{13}^0,\quad L_{11}=L_{11}^0,\quad L_{12}=L_{12}^0,\quad \Lambda_{13}=\Lambda_{13}^0 \quad (10)$$

The obtained system of the equations (7)-(9) and boundary conditions (10) are the mathematical static model of the geometrically nonlinear micropolar elasticity thin shallow shells with independent fields of displacements and rotations with the full account of shear deformations.

Let us notice, that from constructed model the geometrically linear model of micropolar shallow shells are obtained, if the nonlinear members to ignore, and the geometrically nonlinear Timoshenko type classical model are also obtained, if to put $\alpha=0$.

It is necessary to have in view, that the mathematical dynamic model of geometrically nonlinear micropolar elastic thin shallow shells with independent fields of displacements and rotations are obtained, if the corresponding inertial members $2\rho h\frac{\partial^2 u_i}{\partial t^2}$, $2\rho h\frac{\partial^2 w}{\partial t^2}$, $\frac{2\rho h^3}{3}\frac{\partial^2 \psi_i}{\partial t^2}$, $2Jh\frac{\partial^2 \Omega_i}{\partial t^2}$, $2Jh\frac{\partial^2 \Omega_3}{\partial t^2}$, $\frac{2Jh^3}{3}\frac{\partial^2 \iota}{\partial t^2}$ to add.

Let us notice, that the formulated above variational problem corresponds to the most general variational principle of the micropolar elastic thin shallow shells. Therefore, from the last result the Lagrange and Kastiliano type principles of micropolar elastic thin shallow shells will follow as special cases. The direct methods of their approach decision can be made to each of obtained variational equations (in particular Ritz and Galerkin methods). Using these methods, the boundary problem of the theory of flexible micropolar elastic thin shallow shells can be reduce to the decision of the nonlinear algebraic equations system.

As special case, the variational principle of geometrically nonlinear classical theory of elastic thin shallow shells [3],[4] will follow from the variational principle of geometrically nonlinear micropolar theory of elastic thin shallow shells.

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