Numerical simulation of the effect of softening in materials on changes in the stress-strain state of elastomeric articles

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Abstract

The paper explores the necessity of taking into account the effect of softening encountered in rubbers (Mullins effect) during physical and numerical modeling of industrial rubber articles. For more precise computations, the Ogden-Roxburgh model is modified by changing constants to functions found from the analysis of experimental data. The modified model implemented in commercial package ANSYS allows us to perform a comparative analysis of changes in the stress-strain state of a car tyre with and without taking into account the effect of softening in rubbers.

The effect of softening encountered in rubbers was first observed by Bouasse and Carriere (1903) for unfilled polymers [1]. Despite this fact, the influence of this effect on the stress-strain state of real products is frequently ignored when modeling and designing rubber articles. It is commonly recognized that incorporation of superfine filler particles into rubbers leads to a remarkable improvement in the strength and fatigue properties of the material. However, this also gives rise to an increased softening [2, 3]. In the present paper, we discuss the necessity of taking into account the effect of softening encountered in rubbers during physical and numerical modeling of industrial rubber articles and consider a suitable way to modify the Ogden-Roxburgh model for use in numerical simulations properties.

Experimental data concerning the mechanical properties of filled rubbers were obtained from cyclic tension tests with increasing amplitude (the material is assumed to be elastic). Experiments showed a significant change in the mechanical properties of rubber already in the region of moderate elongations (up to 1.5) (Fig. 1) [4].

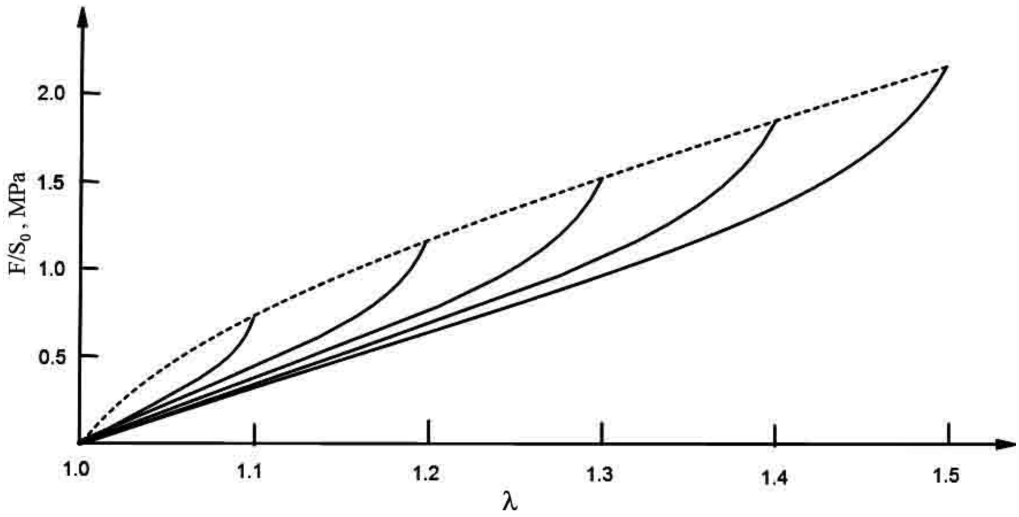


Figure 1: Experimental data obtained for rubber softening under uniaxial loading conditions. The dashed curve indicates the first cycle of loading, and the solid curves show unloading and subsequent loading of the softened material when the value of maximum elongation does not exceed that of elongation achieved during the previous deformation history of the material; F/S_0 — reduced stress; λ — elongation

We have computed Cauchy stress tensor of the Elastic energy potential:

$$\sigma_i = \frac{1}{\lambda_1\lambda_2\lambda_3}\sum_i \frac{\partial U}{\partial \lambda_i}\lambda_i,$$

In our investigation, we have used the Ogden-Roxburgh model describing rubber softening [5]. The strain energy density is split into deviatoric and volumetric parts and can be expressed as

$$U = \eta U_{dev} + U_{vol} + \phi(\eta),$$

where $\phi(\eta)$ is the function used to assess the amount of lost energy; η is the damage parameter. The function $\phi(\eta)$ has the form:

$$\frac{\partial \phi(\eta)}{\partial \eta} = \left(m + \beta U_{dev}^{m}\right) erf^{-1}\left(r(1-\eta)\right) - U_{dev}^{m}.$$

The damage of the material $\eta \in (0, 1]$ (at η=1, the material is assumed to be undeformed) is calculated by the formula [3]:

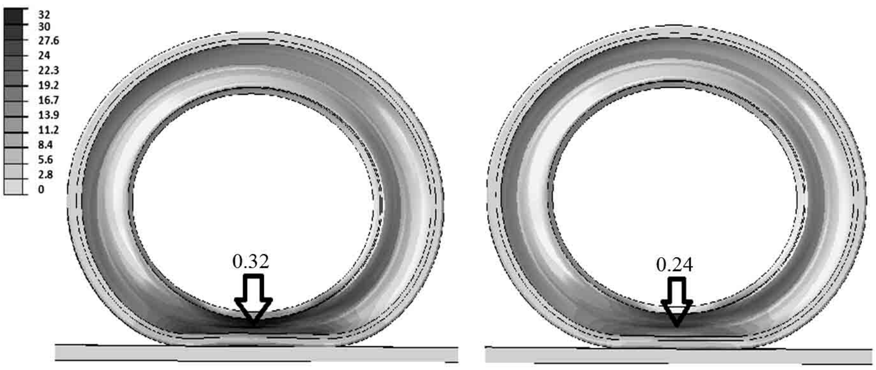
$$\eta = \begin{cases} 1, & \left[U_{dev} > U_{dev}^{m}\right] \\ 1 - \frac{1}{r} erf\left(\frac{U_{dev}^{m} - U_{dev}}{m + \beta U_{dev}^{m}}\right), & \left[U_{dev} < U_{dev}^{m}\right] \end{cases},$$

where $U_{dev}^m = max(U_{dev})$, $erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-\omega^2} d\omega$.

The parameters r, m, β can be determined using the experimental recession curves by standart deviations (for the material in Fig. 1, $r = 2.5$, $\beta = 0.37$, $m = 0.09$): r, β are dimensionless; m characterizes the values of strain energy and has the dimensionality of energy; r is related to the degree of softening; β specifies the slope of the softening curve vs. the initial loading curve. When the strain energy reaches the value of the parameter m, the material begins to return to the initial loading curve. The case $m = 0$ is unrealistic, because complete softening occurs in this case at indefinitely small deformations.

The influence of these parameters on the loading curve obtained for softened material was studied in a series of numerical experiments with MATLAB. The analysis indicates that the Ogden-Roxburgh model used in such commercial packages as ANSYS and ABAQUS is not nearly adequate enough for describing specific features associated with a softening effect encountered in rubbers. In particular, this model cannot be applied to describe the anisotropy of the softening effect. What is more, in the case of high strain data scattering a single set of parameters describes the Mullins effect with significant error. In order to obtain softening curves needed for precise computations, the Ogden-Roxburgh model was modified by changing parameters to special functions fitted to particular loading curves for a wide range of deformations.

The method proposed todescribe the Mullis effect (including changes produced in the Ogden-Roxburgh model) was implemented in the package ANSYS. The developed algorithms made it possible to perform computational experiments devoted to virtual modeling of the motion of a car tyre on a road with and without taking into account rubber softening. The experiments demonstrated that the degree of softening was different in these two cases. Therefore, the stress-strain state of the tyre should be evaluated at each point of its diameter with account for different degree of softening (Fig. 2). The mechanical properties of the tyre were assumed on condition that the car tyre was manufactured from the composite material.



a b

Figure 2: Distribution of the left stretch tensor fields found with (a) and without (b) taking into account the Mullins effect

Conclusions

Computational experiments show that the softening effect in real rubber materials has a profound influence on the stress-strain state of a car tyre and therefore it should be taken into account when developing the design of a car tyre.

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