Starting inertially excited trans-resonant vibration machines with several degrees of freedom of the carrier system

N.P. Yaroshevich, A.V. Sylyvonyuk, I.P. Zabrodets Andriy.Sylyvonyuk@gmail.com, m_yaroshevich@mail.ru

Abstract

Expressions for vibration moments (additional dynamic loading caused by the vibrations of bearing body) during the passage of resonant zone by vibration machines with the flat vibrations of bearing body both with one arbitrarily located vibration exciter and with two self-synchronization vibration exciters for the different modes of starting are got in an analytical form by method of direct division of motions. Using approaches of vibration mechanics of I.I. Blekhman possibilities of improvement of process of running approach of vibration machines with unbalanced vibration exciters are demonstrated by using of methods the "double" (in case of one vibration exciter) and "separate" starting of electric motors (in case of two vibration exciters). It is shown that the first method is based on using semislow vibrations arising in the resonant zone. The necessary condition of the successful using of this method is motion on the rotor of exciter in the moment of the repeated including of engine of rotary-type vibration moment. The conditions when the separate starting is effective are shown. Conclusions and practical recommendations that allow to facilitate starting of vibration machines with an unbalanced drive are pointed.

1 Posing the problem and its connection with the main scientific tasks

Solutions of problems of run-up and run-down of vibrational systems with inertial drive is of considerable interest for vibrational technical devices. When inertial vibroexciter passes the zone of natural frequencies an unset of resonance vibrations is possible which cause both a sufficient rise of dynamic loads on the rotor of electric motor, on elements of machine bearing construction and additional losses of power in the system. So, the start of vibration machine with unbalanced drive needs the power of the drive with sufficiently exceed the power needed for operating in stationary mode (2-5 times as large by some data). In addition to that, in case of large machines with the drive from electric motors of asynchronous type the striking starting current exerts negatively upon the feeding electrical network.

In order to lower the level of vibrations when passing the resonance zone various means are used – from vibroexciters with automatically regulated static moment

of unbalance mass to algorithms with feedback. No doubt, to successful realization of the lasts it is important to have more thorough conception of dynamics of the occurring processes.

2 Analysis of the latest investigations

The survey of investigations, concerning the passing of the resonance zone by inertial vibroexciter one may find in [1-3]. In the last years a number of tasks are solved on the basis of vibrational mechanics approaches, in particular, by using the method of direct separation of motions. In [3] it is shown by the example of the simplest system with linear vibrations of the bearing body and one unbalanced exciter that the important merit of such approach is its comperative simplicity and physical integration of the results.

In work [2] attention is paid to the peculiarity of the motion of the system nearby the resonance – the availability of the so called inner pendulum and its semislow motions, which are physical base of the efficiency of some methods of controlling the starting of vibration machines with inertial exciting of vibration.

A great number of works are dedicated to the use of the phenomenon of selfsynchronization in vibration machines and devices, they are shown in [1, 3], and the latest ones in [4-6]. However, no attention was paid to the dynamics of starting of such vibrations machines. The presented paper is dedicated to generalization and development of the results of works [2, 7-9].

3 Statement of the task

The majority of vibration machines with unbalanced drive may be idealized in the form of a system, consisting with a single lifting rigid body, which may execute planeparallel motion and is connected with stationary base with elastic and damping elements (Fig. 1). As exciters of vibrations of lifting body mostly unbalanced vibroexciters (disbalanced rotors) drived by the electric motors of asynchronous type are used. Motion equations of such system may be written down in the following form (see, for instance, [1-3]):

$$\begin{split} M\ddot{x} + \beta_{x}\dot{x} + c_{x}x &= \sum_{i=1}^{s} m_{i}\epsilon_{i}(\ddot{\phi}_{i}\sin\phi_{i} + \dot{\phi}_{i}^{2}\cos\phi_{i}), \\ M\ddot{y} + \beta_{y}\dot{y} + c_{y}y &= \sum_{i=1}^{s} m_{i}\epsilon_{i}(\ddot{\phi}_{i}\cos\phi_{i} - \dot{\phi}_{i}^{2}\sin\phi_{i}), \\ J\ddot{\phi} + \beta_{\phi}\dot{\phi} + c_{\phi}\phi &= \sum_{i=1}^{s} m_{i}\epsilon_{i}r_{i}(\ddot{\phi}_{i}\cos(\phi_{i} + \delta_{i}) - \dot{\phi}_{i}^{2}\sin(\phi_{i} + \delta_{i})), (s = 1...n) \end{split}$$
(1)

$$I_{i}\ddot{\varphi}_{i} = L_{i}(\dot{\varphi}_{i}) - R_{i}(\dot{\varphi}_{i}) + m_{i}\varepsilon_{i}(\ddot{x}\sin\varphi_{i} + \ddot{y}\cos\varphi_{i} + r_{i}\ddot{\varphi}_{i}\cos(\varphi_{i} + \delta_{i}) + g\cos\varphi_{i}), \quad (2)$$

where M, J – are correspondingly, mass and moment of inertia of the lifting body as to the axis which passes through its center of gravity; x, y, ϕ – are coordinates,



Figure 1: General diagram of vibrational system with unbalanced vibroexciters.

determining the position of the lifting body; ϕ_i – are the angles of rotation of vibroexciter; r_i and δ_i – are polar coordinates of axes of vibroexciters; m_i , ε_i – are, correspondingly, mass and accentricity of the exciter; I_i – is applied to the shaft of the vibroexciter moment of inertia of the rotating parts of the drive; c_x , c_y , c_{ϕ} – are horizontal, vertical and rotational rigidity of the elastic elements; β_x , β_y , β_{ϕ} – are coefficients of viscous resistance; $L_i(\dot{\phi}_i)$, $R_i(\dot{\phi}_i)$ – is the torque of the electric motor and moment of forces of resistance to rotation; g – is a free-fall acceleration.

4 Exposition of the basic material

To solve the set of equate ions (1), (2) we use the method of direct separation of motions [1, 3]. Set us accept as a zero-order approximation $\varphi_i = \omega t$, $q_i = P_i \sin \omega t + Q_i \cos \omega t$ where $\omega = \omega(t)$ - are slowly and $q_i = x, y, \varphi$ – fast changing time functions. Then it is not complicated to come from the original system of equations of vibroexciters rotors motion (2) to the equations of their rotation in the resonance zone in the form, obtained in [3]:

$$I_{i}\dot{\omega} = L_{i}(\omega) - R_{i}(\omega) + V_{i}(\omega), \qquad (3)$$

where $V_i(\omega) = m_i \varepsilon_i \langle \ddot{x} \sin \phi_i + \ddot{y} \cos \phi_i + r_i \ddot{\phi}_i \cos(\phi_i + \delta_i) \rangle$.

French quotes in (3) point out at averaging for the $T = 2\pi$ by fast time $\tau = \omega t$.

It should be noted that equation (3) differs from classic equation of machine assembly by presence of item $V_i(\omega)$ – vibrational moment which defines the peculiarity of vibrational system conduct. Presence of vibrational moment explains both Zommerfields effect and selfsynchronization of vibroexciters. Determination of the vibrational moment is of main interest.

It should be noted that equation (3) keeps its form, obtained for the system with linear vibrations of the lifting body [3] for the examined more general case as well. Only expression for vibrational moment has more complicated structure, algorithm of its obtaining remains previous, only computing difficulties grow up.

4.1 Vibrational systems with one vibroexciter Zommerfields effect

Certain part of operating at present machines has one unbalanced vibroexciter. It is not complicated to obtain expressions of vibrational moment in the resonance zone for the case of vibroexciter, placed arbitrarily as to centre of masses of the lifting body in plane vibration in the form:

$$V(\omega) = -\frac{(m\varepsilon\omega)^2}{M} \left[\frac{n_x}{B_x^2} + \frac{n_y}{B_y^2} + \frac{Mr^2}{J}\frac{n_\varphi}{B_\varphi^2}\right],\tag{4}$$

$$B_q = \sqrt{(1 - \lambda_q^2)^2 + 4n_q^2}; \lambda_q = \frac{p_q}{\omega}; n_q = \frac{\beta_q}{2M_q\omega};$$

where p_q – are the frequencies of the natural vibration of the system.

Here, if q = x, y, then $M_q = M$, if $q = \phi$, then $M_q = M_{h^2}^{\rho^2}$; in addition to that, $n_{\phi} = \frac{\beta_{\phi}}{2J\omega}$. One can see that all items in formula (4) are negative. Hence, vibrational moment

is always braking one, that is, it is an additional dynamic load upon the rotor of the engine, its dependence from frequency is of resonance character and, therefore, an essential braking exercion is manifested in comparatively narrow range of natural frequencies. In addition to that, rapid growth of value $V(\omega)$ at approaching to resonance just explains the possible sticking of frequency in the process of starting (Zommerfields effect) and, as consequence, the necessity of overrated (from starting conditions) power of the drive of postresonance vibromachines. Such conclusion follows from diagramic presentation of dependences $L(\omega)$ and $M_{sum} = R(\omega) + V(\omega)$ (fig. 2), abscissas of intersection points correspond to possible stationary modes (curves L describe statical characteristics of electric engines. Stability of motions is easily determined geometrically by the sign and values of slope angles tangent to curves $L(\omega)$ and M_{sum} . It is evident that right slopes of resonance curve cannot be realized. According to the figure, the presence of several resonance peaks of the curve of vibrational moment may lead to the emergence (as compared with the system of linear vibration of the lifting body) points of curves intersection. So, there exists a possibility of several stationary modes of motions, having different angular velocities (up to seven, four of them may be stable). However, there are only two, different in lessence modes of motion: sticking (curves 1) of the system with engine of deficient power in the resonance zone (motor) on having come in the process of running to this mode, would not be able to overcome the resonance peak and far postresonance mode with frequency of electric motor. If the motor power is sufficient, then, as a rule, after some breaking in the resonance zone, the system rapidly (upsetting) passes to far postresonance modes of motion (curves 2).

So, to reach by the exciter the working frequency, the moment of the motor should overcome vibrational moment $V(\omega)$ during its running. According to (4), maximal (peak) value of moment $V(\omega)$ is as much large as a damping of n_q becomes less and higher of the own vibrations of system p_q . Hence, it is important not to overrate the value of rigidity of elastic elements; the use of elastic suspension may be effective; it is possible to lower resonance peak values of vibrations as well as the power of the drive by installation of dampers of maximal vibrations. Expression



Figure 2: Stationary modes of rotation of vibroexciter: 1 - sticking in resonancezone; 2 - far postresonance mode

(3) may be presented in the form of the sum of partial vibration movements ν_q , which characterize the impact of vibrations, corresponding to each of the generalized coordinates: $V(\omega) = \sum_{q=x,y,\varphi} \nu_q$ where $\nu_q = \frac{1}{2} F a_q \sin \gamma_q$; $\sin \gamma_q = -\frac{2n_q}{B_q}$; $a_q = \frac{m\epsilon}{M_q B_q}$; $F = m\epsilon \omega^2$. It is natural that maximal breaking exertion is effected by partial vibrational moment which corresponds to the highest natural frequency of vibrations p_q , so it is often enough to use damper of only suck vibrations.

It is clear from formula (4) that start of vibromachine at the absence of working load is more complicated than at its presence; that to make the start easier it is advisable to install vibroexciter in the centre of masses of the system or as close to it, as possible. So, the breaking vibrational moment, resonance vibrations and, correspondingly, the necessary power of the motor are sufficiently less for centredrilled system (fig. 3, a) than, for instance, for the diagram shown in fig. 3, b (in the first case the last component in formula of vibrational moment (4) disappears). It should also be noted that rapid (with frequency 2ω) vibrations of vibrational moment do not take place in such system in the steady mode, which is favorable for the durability of the system.

On the other hand, taking into account the fact that the value of the vibrational moment depends, first of all, on the velocity of running of the rotor of vibroexciter, to make easier the start, engines with higher starting moment are recommended (it facilitates, also, the solution of the problem of lifting the unbalanced mass at first half-turn). At prescribed static moment unbalance mass should be designed with minimal moment of inertia. So, constructions of vibroexciters with laid on unbalanced mass are more preferable for changing the amplitude of vibrations, then those, having regulated static moment. In addition to this, it is recommended to exclude from the construction (if they are available) synchronizers, mechanical transmissions and so on, using the phenomena of selfsynchronization, employing controlled electric drive.



Figure 3: Diagrams of vibromachines: a) with centrally installed vibroexciter; b) with shifted vibroexciter

Manifestation of Zommerfields effect during the run-up of vibromachine is visually demonstrated by the results of numerical modelling, obtained for vibrational system (fig. 3, a) with parameter M = 330 kg; $J = 8.02 \text{ kg} \cdot \text{m}^2$; $c_y = c_x = 4.5 \cdot 10^5 \text{ N/m}$; $c_{\varphi} = 2.8 \cdot 10^4 \text{ N} \cdot \text{m}$; m = 10 kg; $\varepsilon = 0.036 \text{ m}$, electric engine – asynchronous, with frequency of rotation $n_c = 1500 \text{ rpm}$, of power P = 1.5 kW. According to fig. 4, at passing the natural frequencies zone (t = 0.15 - 0.4 s) dynamic load upon the rotor of electric engine grows sufficiently (curve 1); one can see that the value of vibrational moment is larger, than in stationary mode several times as much and its maximal vibrations are compatible with starting moment of the engine.

Correspondingly, the velocity of running of the rotor of exciter shows down intensively up to short-term stabilization of the frequency of rotation (curve 2), in addition to that, maximal resonance vibration of the lifting body are excited. Just after passing the resonance the value of vibrational moment decreases sufficiently fast and its vibrations cover positive zone, that is it becomes rotating in some moments of time. Then their damping takes place as to small negative level (determined by resistance to the vibrations of the lifting body); the amplitude of vibrations of lifting body decreases as fast and the value of rotating moment of the engine changes from starting to nominal value (curve 3).

As it follows from the diagrams of velocity of rotation of vibroexciter for cases of different powers of driving electric motor (fig. 5) at replacing motor of power P = 1.5 kW with motor of power P = 2.2 kW, slowing down of velocity of exciter in resonance zone is practically absent (curve 3) while its steady postresonance mode of operating becomes impossible (curve 2 – sticking of angular velocity in postresonance zone).

4.2 Double start of vibrational machines with unbalanced drive

In practical use of such machines the so called method of double starting is applied for lowering the level of vibrations during passing the resonance frequencies. Its technical realization is rather simple. Method consists in switching-off and next



Figure 4: Changing in time: 1 - of engine moment; 2 - of vibrational moment; 3 - of vibroexciter velocity



Figure 5: Changing in time the vibroexciter velocity: 1 - P = 1.5 kW; 2 - P = 1.1 kW (sticking of velocity); 3 - P = 2.2 kW



Figure 6: Changing in time: 1 – of vibrational moment; 2 – of motor moment (P = 1.1 kW): a) switching-off of the motor in the resonance zone, $t_{sw.-off} = 0.3 \text{ s}$; b) double starting of the engine, $t_{sw.-off} = 0.3 \text{ s}$, $t_{rep.sw.-on} = 0.48 \text{ s}$

switching-on the electric motor in the resonance zone in predetermined moment of time. Theoretical grounding of the method with account of standpoints of vibrational mechanics facilitates its wider use. The basis of the method lies in two existing appropriatenesses of motion of the system close to the region of manifestation of Zommerfields effect: the first one – at switching-off the motor in the resonance zone vibrational moment effecting the rotor of vibroexciter becomes positive, that is, rotating (it follows from the basic equation of vibrational mechanics (3), written down for the case of stationary mode); the second one – availability of so called inner pendulum and its semislow motions. So, using the method of direct separation of motions and accepting as the first approximation $\varphi_1 = \varphi_1^{(1)} = \omega t + \psi$, $\mathbf{q} = \mathbf{q}^{(0)} + \mathbf{q}^{(1)}$, for general system (fig. 1) in case of one vibroexciter it is not complicated to obtain equation of semislow vibrations of velocity of rotor in the form [2]:

$$\ddot{\Psi} + 2n_1\dot{\Psi} + B\sin\Psi - P\sin^2\frac{\Psi}{2} = 0, \qquad (5)$$

for the system under consideration $B = \sum_{q=x,y,\phi} b_q$; $b_q = \frac{(m\epsilon\omega^2)^2}{2MI} \frac{p_q^2 - \omega^2}{(p_q^2 - \omega^2)^2 + 4n_q^2\omega^4}$; $P = \sum_{q=x,y,\phi} \rho_q^2$; $\rho_q = \frac{(m\epsilon\omega^2)^2}{MI} \frac{p_q^2 - \omega^2}{(p_q^2 - \omega^2)^2 + 4n_q^2\omega^4}$; $2n_1 = k/I$; k - is a coefficient of damping.

The value $\mathbf{q} = \sqrt{|\mathbf{B}|}$ is frequency of small free vibrations of the inner pendulum on condition of slow changing of the frequency of rotation of rotor $\boldsymbol{\omega}$. It should be noted that at $\mathbf{B} < 0$, the stable position of the inner pendulum simply changes; it is look like it turns [2]. The effect of appearance of semislow vibrations in the resonance zone may be observed in the fig. 5-7. In addition to that, according to fig. 6, a (curve 1) semislow (with frequency $2\mathbf{q}$) vibrations of vibrational moment take place after switching-off of the motor with regard to the shifted to the positive side level.



Figure 7: Changing in time of the velocity of vibroexciter (P = 1.1 kW): 1– switching-on the motor in the resonance zone, $t_{sw.-off} = 0.3 \ s$; 2 – double start of the motor, $t_{sw.-off} = 0.3 \ s$, $t_{rep.sw.-on} = 0.48 \ s$

Fig. 6, b and fig.7 demonstrate the possibility of realization of running ang coming to the mode of rotation with frequency, close to nominal of the motor of unsufficient power (P = 1.1 kW) with the help of method of double starting as one can see, the necessary condition of successful use of the method is, first of all, effect upon the rotor of vibroexciter in the moment of repeated switching-on of the motor (in figures $t_{rep.sw.-on} = 0.48 \text{ s}$) of rotating vibrational moment commensurable with its starting moment. The abovementioned condition is not complicated to realize with the help of modern means of controlling the electric motors. Applied recommendations to switch off the motor in the moment of growing of intensive resonance vibrations of the lifting body and at once (in a period of time of semiperiod of semislow vibrations t = 2/q) switch it on again.

4.3 Vibrational systems with two selfsynchronizing exciters. Separate starting

Many modern vibrational machines, in particular, screens and platforms with directed vertical (horizontal) vibrations are realized by the diagram, shown in fig. 8. Expressions for vibrational moments influencing in resonance zone upon the rotors of exciters rotating in opposite directions are presented in the form:

$$V_{i}(\omega) = -\frac{1}{2} \frac{(m\varepsilon\omega)^{2}}{M} \frac{n_{y}}{B_{y}^{2}},\tag{6}$$



Figure 8: Diagram of vibrational machine with selfsynchronizing exciters

Vibrational machines with selfsynchronizing exciters permit the possibility of separate (in turns) start of electric motors, however, it is not applied in practice. Using the approach under consideration, it is possible to demonstrate possibility advantages of such start.

It is not complicated to establish that vibrational moment in case of running only one of vibroexciters will equal $V_{separ}(\omega) = \frac{1}{2}V(\omega)$, where $V(\omega)$ is determined by formula (4). It follows from analysis (4) that if natural frequencies of vibrational system p_q differ sufficiently enough (it may always be reached by the choice of elastic elements), then with the grows of frequency ω in the process of running each item (except the one corresponding to $\omega \approx p_q$) will be disregardly small. Taking into account the fact that ratio $Mr^2/2$ is for the dynamic system in consideration, sufficiently less than unity it is possible to come to the following estimation of the value of vibrational moment, functioning in the resonance zone in case of separate start of its electric motors: $V_{separ}(\omega) \approx \frac{1}{2}V_i(\omega)$. So, by corresponding choice of the parameters of the system at separate start of motors it is possible to attain the decrease of resonance vibrational moments and, as a result, to attain all connected with this possible of improvement of dynamic and power characteristics of vibromachines.

In favor of decrease of vibrational moments at separate start of selfsynchronizing vibroexciters are the facts that, owing to the differences between their phases, some collateral vibrations of the lifting body occur and that it is necessary at more precise determination of vibrational moment of the exciter under consideration, to take account of the effect of other exciters. That is, at calculation of vibrational moment it should be presented in the form of the sum of two items, one of which (being determined above) represents additional load, caused by losses of power at vibrations, and the second (noticeably less in the resonance zone, as a rull) is caused by the influence of other vibroexciters. It should be noted, that the second item represents redistribution of power between the vibroexciters. Formulas for determination of its value for many of vibrational systems may be found in specialize literature [1, 3]. The positive effect may be magnified by installation of damper of vertical vibrations. Besides, in case of using separate start of motoes, the decrease (almost twice as much) of starting currents is rather important.

It should be noted that somewhat excessive power of electric drive is recommended for easing the start in case of vibromachines with two selfsynchronizing Starting inertially excited trans-resonant vibration machines with several degrees of freedom of the carrier system



Figure 9: Trajectories of the centre of masses of lifting body: a – synchronous (ordinary) start of motors; b – start of one motor (separate start)

exciters. In addition to that, effect of vibrational support of rotation of unbalanced exciter in steady state should be used, working with one switched off motor. Especially as mode of vibrational support of rotation is the most stable for the dynamic system under consideration. It follows from comparison (4) and (6) that dynamic load upon the rotors of electric motors and, correspondingly, the necessary total power of electric drive in case of vertical vibrations of the working part of vibromachine will be sufficiently less than at elliptic trajectory.

Results of simulation confirm the advantages of separate start. Thus, for instance, according to fig. 9 ($Mr^2/J = 0.52$), in case of such start resonance amplitudes of vibration of masses centre of the lifting body are sufficiently less than those at synchronous start of the motors. In addition to that amplitudes of horizontal and turning resonance vibrations grow. However, their amplitudes are far from maximal values of amplitudes of vertical vibrations.

5 Findings

Thus, the majority of mechanisms of behavior of inertial vibroexciters and vibrational system in general at passing the resonance zone may be explained on the basis of approaches of vibrational mechanics. On this ground practical conclusions and recommendations improving dynamic and power characteristics of vibromachines with unbalanced drive may be obtained. Methods of double and separated start are effective for easing the start of postresonance vibromachines with unbalanced drive.

References

[1] Blekhman I.I. Theory of vibration processes and devices. Vibration mechanics and vibration technology. The Publishing House "Ore and Metals", 2013. – 640p.

- [2] Blekhman I.I., Indieytshev D.A., Fradkov A.I. Slow motions in systems with inertial excitation of oscillations // Problems of machine building and reliability of machines. – Vol. 1, 2008. pp. 25-31. (in Russian).
- [3] Blekhman I.I. Vibrational Mechanics, World Scientific, Singapore, 2000.
- [4] Li Y., Li H., Wei X., Wen B. Self-synchronization theory of a vibrating system with a two-stage vibration isolation frame driven by two motors Zhendong yu Chongji, Dongbei Daxue Xuebao, Journal of Northeastern University. – Vol. 35, 2014. – pp. 836-840.
- [5] Zhang X., Wen B., Zhao C. Vibratory synchronization and coupling dynamic characteristics of multiple unbalanced rotors on a mass-spring rigid base, Journal of Mechanical Science and Technology. – Vol. 28, 2014. – pp. 249-258.
- [6] Franchuk V.P., Savluk N.V. Applying of self-synchronisation in mountain vibration machines, Vibrations in technics and technology. – Vol. 1(33), 2004. – pp. 12-14.
- [7] Blekhman I.I., Yaroshevich N.P. Transition regimes in inertially excited transresonant vibration devices with several degrees of freedom of the carrier system / Nonlinear problems of theory of oscillation and theory of control Vibrational Mechanics. – SPb.: Nauka, 2009. –pp. 215-238.
- [8] Blekhman I.I., Vasilkov V.B., Yaroshevich N.P. On Some Opportunities for Improving Vibration Machines with Self-Synchronization Inert Vibration Exciters. Journal of Machinery manufacture and reliability, Vol. 42 (3), 2013.– pp. 192-195
- [9] Yaroshevich N.P., Sylyvonuk A.V. About some features of dynamic acceleration of vibration mashines with self-synchronisation inertion vibroexciters, Scientific Bullietin of National Mining University, Vol. 4 (136), 2013. – pp. 70-75.
- N.P. Yaroshevich, 75 Lvivska Str., Lutsk, 43018, Ukraine
- A.V. Sylyvonyuk, 75 Lvivska Str., Lutsk, 43018, Ukraine
- I.P. Zabrodets, 75 Lvivska Str., Lutsk, 43018, Ukraine