Biomorphic approach in application to vibration control of continuous systems

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Abstract

One of important applications of the automatic control theory is suppression of vibrations of continuous systems, which have an infinite number of degrees of freedom. The study investigates the modal (or biomorphic) approach to this problem. The specified approach, unlike the local one, implies separate control of the eigenmodes of object, and requires appropriate setting of the control loops. The study analyses methods of identification of the object, which is necessary for correct mode separation in the control system.

The experimental part of the research is devoted to the comparison of local and biomorphic approaches to the problem of suppression of forced flexural vibrations of the metal beam. All control systems created use the same elements — piezoelectric sensors and actuators attached to certain locations on the beam. It is shown that the created modal control system is more efficient than the local ones in cases where it is necessary to suppress vibrations in the frequency range that include several resonance frequencies of the object.

1 Introduction

Vibrations occur in a great variety of mechanical systems during their exploitation. In certain cases, these vibrations can be undesirable or even dangerous for the system and cause failure, damage or unwanted noise. The problem of vibration control is especially complicated in the case of elastic systems with distributed parameters, such as antennas, cables, robot manipulators, different beam and shell structures etc.

The absence of controllability and observability for continuous mechanical systems is due to an infinite number of eigenmodes, each of them being a separate degree of freedom. Of course, the modal stiffness is growing rapidly with increasing mode number, which allows one to neglect the higher modes of the object. However, the presence of higher modes leads to reduced accuracy and stability of the feedback control system. Moreover, the resonance behavior of the elastic object causes phase shifts that limit gain values in the control loops.

Traditionally for controlling continuous systems the local method is used which implies that each sensor is connected to only one corresponding actuator located at the same region of the mechanical object. The phase shifts in this case are to be compensated by means of a sophisticated design of the feedback controllers. The methods of optimal control are widely used for designing controllers [1–5] as well as for determining the optimal spatial configuration of the control system (the mechatronic approach).

Alternatively, there is a modal method [6,7] based on the idea of independent control of different vibration modes of the mechanical system. This method allows one to compensate the phase shift for each mode in the corresponding controller, and it can provide stability of the system for the case of different spatial locations of sensors and actuators. At the same time, the modal system requires sufficient number of sensors and actuators to provide independent control of the modes.

Today the mechatronic systems are developing rapidly, but in the control aspect they are still far from biological systems. Control process in the living organisms has been optimized during millions of years of evolution. The most natural types of motion are inertial motion and motion with eigenmodes, therefore muscles activate these two types of motion. For example, when walking, legs move with the first eigenmode; when running, they move faster with the second eigenmode. Thus, the separate control of eigenmodes of the object can be called biomorphic.

The present study investigates the modal or biomorphic approach to control of continuous systems. Firstly, it analyses methods of identification of the control object, which can be used for correct mode separation in the modal control system. Secondly, it contains experimental part, where the both methods described above are implemented and their efficiency in the problem of suppression of forced bending vibrations of a thin metal beam is analyzed.

2 Mode separation in the biomorphic control system

Consider a system designed to control forced flexural vibrations of a thin cantilevered beam. The control system contains sensors and actuators - piezoelectric patches mounted to both sides of the beam. The actuators cause the bending deformation of the beam at the places where they are located, while the sensors measure this deformation (or curvature) as the beam vibrates. Let there be n sensors and n actuators in the control system. Therefore, this system can control independently not more than n eigenmodes of the beam.

It is obvious that sensors and actuators should be located at special places of the beam, so that they could measure and activate the needed eigenmodes most efficiently. To ensure this efficiency, it is necessary to define the eigenmodes of the beam theoretically or experimentally as the first stage of the control system design.

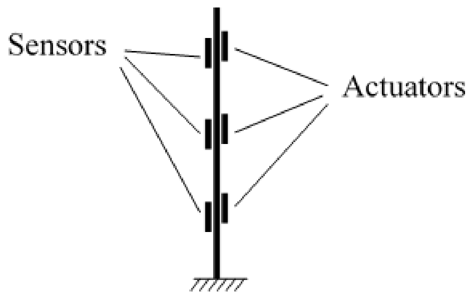

Fig.1. A beam with sensors and actuators

When the sensors and the actuators are mounted to the beam, it is necessary to set the control system, which links them together. We are going to analyze how this system should be organized in order to control the needed n eigenmodes of the beam separately. Consider that m eigenmodes are enough to describe the motion of the beam. The equation of motion of the beam in the form of mode decomposition has the following form:

$$\ddot{q} + \Omega^2 q = Q^c + Q^d, \tag{1}$$

where $q_{m\times 1}$ is a vector of generalized coordinates, corresponding to each of m eigenmodes; $\Omega_{m\times m}$ is a diagonal matrix of the eigenfrequencies; $Q^d_{m\times 1}$ is a vector of external disturbances; $Q^c_{m\times 1}$ is a vector of control influences.
The actuator influences on the modes depends on the control signals:

$$Q^c = \Theta_1 u, \tag{2}$$

where $u_{n\times 1}$ is a vector of control signals at the actuators, and $\Theta_{1m\times n}$ is an influence matrix, defining the influence of each actuator on each of the eigenmodes.
The sensor signals depend on the generalized coordinates q:

$$y = \Theta_2 q, \tag{3}$$

where $y_{n\times 1}$ is a vector of sensor signals, and $\Theta_{2n\times m}$ is a measurement matrix, defining the reaction of each sensor to activation of each eigenmode.
If the sensors and the actuators are located in pairs (at the same places of the beam), matrices Θ_1 and Θ_2 are related:

$$\Theta_1 = \Theta_2^T$$

In the simplest case of proportional control the control signal to the actuators depends on the measured signal as follows:

$$u = \mathcal{K} y \tag{4}$$

The work of the control system is specified by the matrix $\mathcal{K}_{n\times n}$. The system of equations looks as follows:

$$\ddot{q} + \Omega^2 q = \Theta_1 \mathcal{K} \Theta_2 q + Q^d \tag{5}$$

For the separate control of the first n modes of the beam the matrix $\Theta_1 \mathcal{K} \Theta_2$ should be diagonal. To separate the modes it is necessary to satisfy the following conditions:

$$\mathcal{K} = FKT, \qquad \widetilde{\Theta}_1 F = D_1, \qquad T\widetilde{\Theta}_2 = D_2, \tag{6}$$

where $\widetilde{\Theta}_{1n\times n}$ Рч$\widetilde{\Theta}_{2n\times n}$ are square parts of matrices Θ_1 and Θ_2, corresponding to the first n eigenmodes; $K_{n\times n}$ is a diagonal matrix of gain values for each mode; $F_{n\times n}$ and $T_{n\times n}$ are modal matrices (mode synthesizer and mode analyzer), which are defined in such a manner that their multiplication by matrices $\widetilde{\Theta}_1$ and $\widetilde{\Theta}_2$ gives diagonal matrices $D_{1n\times n}$ and $D_{2n\times n}$.
Matrix T is a mode analyzer, it converts the vector of measured sensor signals y into a vector of estimations of the first n generalized coordinates $\hat{q}_{n\times 1}$:

$$\hat{q} = Ty = T\Theta_2 q \tag{7}$$

For the control system working correctly the following condition should be met:

$$T = \widetilde{\Theta}_2^{-1} \tag{8}$$

Matrix K defines the gain values, which convert the vector of estimations of the modes $\hat{q}$ into a vector of influences on each mode $\hat{Q}_{n\times 1}$:

$$\hat{Q} = K\hat{q} \tag{9}$$

Matrix F is a mode synthesizer, it converts the vector of influences to modes $\hat{Q}$ into the vector of control signals to actuators u:

$$u = \mathcal{K}y = FKTy = FK\hat{q} = F\hat{Q}, \qquad Q^c = \Theta_1 u = \Theta_1 F\hat{Q} \tag{10}$$

To provide the separate mode control it is necessary to satisfy the condition:

$$F = \widetilde{\Theta}_1^{-1} \tag{11}$$

In this case the system of equations for the first n modes is the following:

$$\ddot{q} + \Omega^2 q = Kq + Q^d \tag{12}$$

The equations are separate since the matrix K is diagonal. Generally, it is possible to specify individual control laws for each of the eigenmodes. In this case the elements of the matrix K will be the functions of the complex variable s:

$$k_i = k_i(s)$$

3 Defining the modal matrices F and T

First, the modal matrices F and T can be calculated theoretically. To do this, it is necessary to know the eigenmodes of the beam and the locations of the sensors and the actuators on the beam. This allows one to calculate matrices $\widetilde{\Theta}_1$ and $\widetilde{\Theta}_2$. One needs to invert these matrices in order to obtain the desired matrices F and T. However, for most accurate determination of the desired matrices one needs to conduct certain experiments.
Each column of the measurement matrix $\widetilde{\Theta}_2$ corresponds to one of the beam eigenmodes and shows, in what proportions sensors measure this eigenmode. In order to obtain this information experimentally it is necessary to induce resonant vibrations

of the beam at each of the n resonance frequencies. At each resonance one needs to measure the amplitude of signals of each sensor. Thus one obtains matrix $\widetilde{\Theta}_2$.
Analogously, each row of the influence matrix $\widetilde{\Theta}_1$ corresponds to one of the eigenmodes and shows, in what proportions actuators excite this mode. In order to obtain this information experimentally it is necessary to induce resonant vibrations of the beam by each actuator at each of the n resonance frequencies and measure the intensity of beam vibrations. This is the way to obtain matrix $\widetilde{\Theta}_1$.
Modal matrices F and T are then calculated by inverting matrices $\widetilde{\Theta}_1$ and $\widetilde{\Theta}_2$. After that, one needs to perform verification of the obtained matrices F and T. For matrix T it means that at resonant modes vector of estimations of the generalized coordinates $\hat{q} = Ty$ should be zero except for only one component corresponding to the resonance frequency excited. In order to verify matrix F one needs to ensure that each control loop excites only one corresponding eigenmode and does not excite the others even if the excitation have resonance frequency. That means in fact that the vector of control influences on modes $Q^c = \Theta_1 F \hat{Q}$ matches the vector of desired influences on modes $\hat{Q}$.
However, sometimes it can be difficult to carry out all the experiments at each of the n resonance frequencies, especially if the number of sensors and actuators is redundant. Moreover, difficulties can occur with inverting of the experimentally obtained matrices. So let us assume that we want to limit ourselves to controlling independently only the first k eigenmodes of the object. Consider an algorithm of gradual increasing the number of eigenmodes used to control the system, or gradual optimization algorithm, for the mode synthesizer F. For the mode analyzer T this procedure is similar.
So, it is necessary that each column of the matrix F (denote them as F_i) is orthogonal to each row of the matrix $\widetilde{\Theta}_1$ (denote them as θ_i) except for one row with the same number. Then the following condition will be met (6):

$$\widetilde{\Theta}_1 F = D_1,$$

where D_1 is a diagonal matrix.
At the first step we want to control only the first eigenmode of the beam, and we are looking for only the first column of the mode synthesizer matrix F_1. The other columns are zero. Experiments at the first resonance frequency give the first row of the influence matrix θ_1. Vector F_1 is found from the condition of maximum correspondence to the vector θ_1. This means that the first control loop have the maximum influence at the first eigenmode:

$$\max \theta_1^T F_1 \tag{13}$$

In addition, the normalization condition for the vector F_1 should be satisfied:

$$F_1^T F_1 = 1 \tag{14}$$

The method of Lagrange multipliers for this problem gives the following result:

$$J = \theta_1^T F_1 - \lambda(F_1^T F_1 - 1) \tag{15}$$

$$\frac{\partial J}{\partial F_1} = \theta_1 - \lambda F_1 = 0 \tag{16}$$

Then,

$$F_1 = \frac{1}{\lambda}\theta_1, \qquad \lambda^2 = \theta_1^T \theta_1 \tag{17}$$

At the second step of the algorithm the first and the second columns of matrix F are calculated. In order to do this the first and the second rows of matrix $\widetilde{\Theta}_1$ are obtained experimentally. Along with the conditions of maximum correspondence and normalization there are two orthogonality conditions: the first control loop should not excite the second eigenmode, while the second one should not excite the first mode. All this requirements have the following form:

$$\max \theta_1^T F_1, \max \theta_2^T F_2; \qquad F_1^T F_1 = 1, F_2^T F_2 = 1; \qquad \theta_2^T F_1 = 0, \theta_1^T F_2 = 0 \tag{18}$$

For the first column F_1 we obtain the following equations:

$$J = \theta_1^T F_1 - \lambda_1 (F_1^T F_1 - 1) - \lambda_2 \theta_2^T F_1; \qquad \frac{\partial J}{\partial F_1} = \theta_1 - \lambda_1 F_1 - \lambda_2 \theta_2 = 0 \tag{19}$$

Then,

$$F_1 = \frac{1}{\lambda_1}\theta_1 - \frac{\lambda_2}{\lambda_1}\theta_2, \qquad \lambda_2 = \frac{\theta_2^T \theta_1}{\theta_2^T \theta_2}, \qquad \lambda_1^2 = \theta_1^T \theta_1 - \frac{(\theta_2^T \theta_1)^2}{\theta_2^T \theta_2} \tag{20}$$

For the second column F_2 the result is analogous:

$$F_2 = \frac{1}{\hat{\lambda}_1}\theta_2 - \frac{\hat{\lambda}_2}{\hat{\lambda}_1}\theta_1, \qquad \hat{\lambda}_2 = \frac{\theta_1^T \theta_2}{\theta_1^T \theta_1}, \qquad \hat{\lambda}_1^2 = \theta_2^T \theta_2 - \frac{(\theta_1^T \theta_2)^2}{\theta_1^T \theta_1} \tag{21}$$

Thus, the same procedure can be repeated for the next steps of the algorithm up to the n-th step. At k-th step for each vector F_i the conditions of maximum correspondence and normalization are formulated along with $k-1$ conditions of orthogonality. This algorithm allows one to carry out the identification without any mathematical model of the object getting all necessary data from the certain limited number of experiments.

4 Experimental set-up

A control object is an aluminium beam 70 cm long with the cross-section 3×35 mm. It is disposed vertically and fixed at one point at the distance of 10 cm from the lower end. The external loading is a base excitation applied by means of a piezoelectric stack actuator which is a part of the fixation constructionas it is shown in *figure 2*. The stack actuator **A** is connected to the beam through a flexible plate **B**. It allows the fixation point to move in single direction along the stack and prohibits to move in other directions. Through this plate two screws **C** and **D** at the top and the bottom of the stack actuator perceive the weight of the beam releasing the actuator itself from this weight. Longitudinal vibrations of the actuator cause bending vibration of the beam.

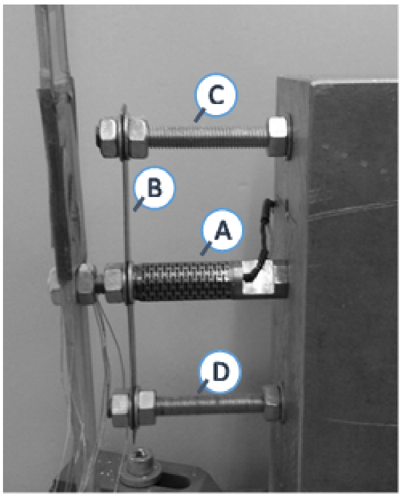

Fig.2. The fixation construction of the beam

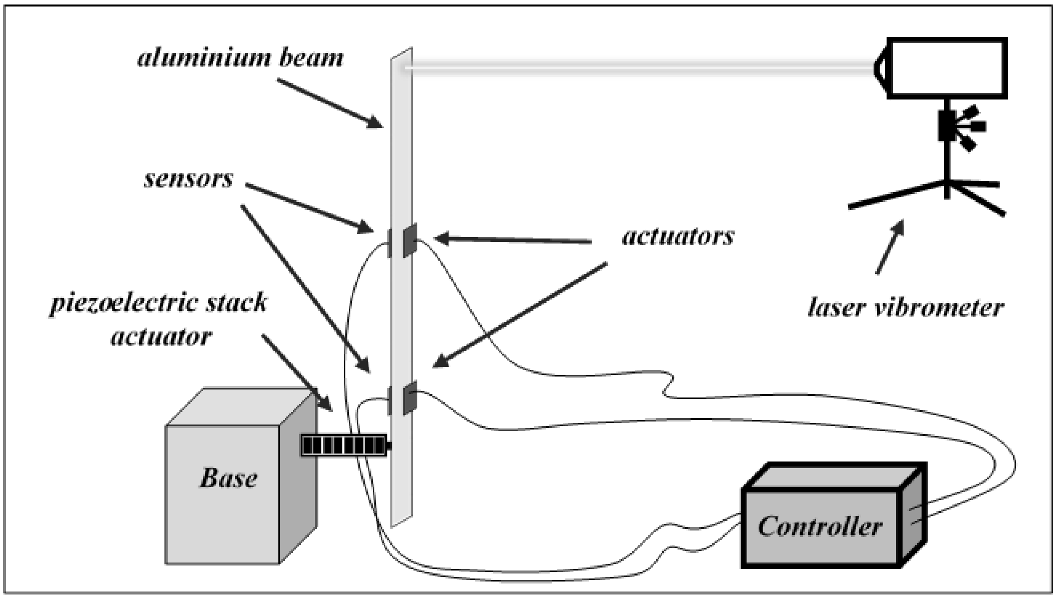

Fig.3. The scheme of the experiment

The control system includes two piezoelectric sensors and two actuators attached in pairs to the beam as it is shown in *figure 3*. Once attached, they keep the same positions for all the control systems created. The actuators change the curvature of the beam depending on the control signal applied, while the sensors measure this curvature as the beam vibrates [8,9]. Actuators and sensors are connected through a controller.

The purpose of the control system is to suppress forced resonance vibrations of the beam with the first and the second resonance frequencies. These frequencies correspond to the first and the second bending modes of the beam. Therefore, sensors and actuators are attached to special locations on the beam, so that they can most efficiently measure and influence these particular modes.

In order to design local and modal control systems with the best performance frequency methods of the Automatic Control Theory are used. To monitor the efficiency of the control systems obtained the vibration amplitude of the upper endpoint

of the beam is measured by the laser vibrometer. This choice is caused by the fact that the vibration amplitude of this point is the greatest one among all points of the beam for the vibration modes under consideration.
To work with the highest efficiency the piezoelectric sensors and actuators should be placed at the areas where the curvature of the modes to be controlled takes the maximum values. In order to satisfy this condition the first and the second bending modes of the beam are analyzed theoretically and experimentally (*figure 4*).

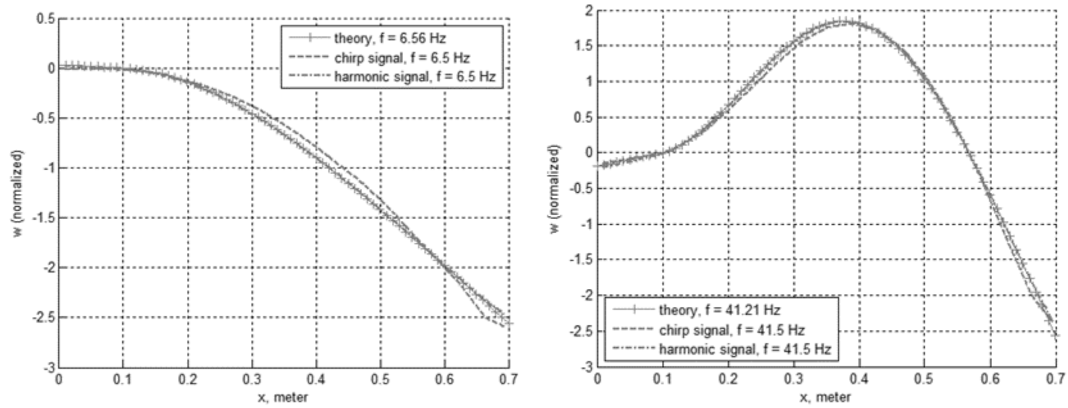

Fig.4. The first and the second bending modes of the beam

As a result, the location of the first pair of piezoelectric patches was defined to be near to the fixation point ($10.5cm \leq x \leq 16.5cm$), and the second pair was placed not far from the beam center ($37.5cm \leq x \leq 43.5cm$).

5 Local control system

The working scheme of the controller for the local control system is shown in *figure 5*. Here y_1 and y_2 mean the signals measured by the sensors, while u_1 and u_2 are the control signals being sent to the actuators. In order to design such a system one needs to determine transfer functions $R_1(s)$ and $R_2(s)$, which establish the relation between measured and control signals.

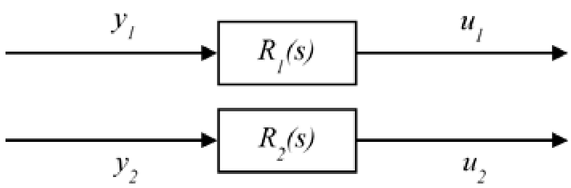

Fig.5. The scheme of the controller for the local control system

To design the transfer functions for the controller the logarithmic characteristics of the open-loop system are analyzed. The transfer function for each control loop is chosen in order to provide the most effective vibration suppression at the first and the second resonance frequencies of the beam. At the same time, special attention is paid to ensure stability of the systems obtained.
As a result two most effective local control systems were created. The first system shows the optimal performance at the first resonance frequency, while the second

one gives the best result at the second resonance. Frequency response functions in the vicinity of the first and the second resonance frequencies measured for the upper point of the beam are shown in *figure 6* for the first and the second local control systems, respectively. The black curve represents the uncontrolled system while the other curves correspond to the different gain values in control loops.

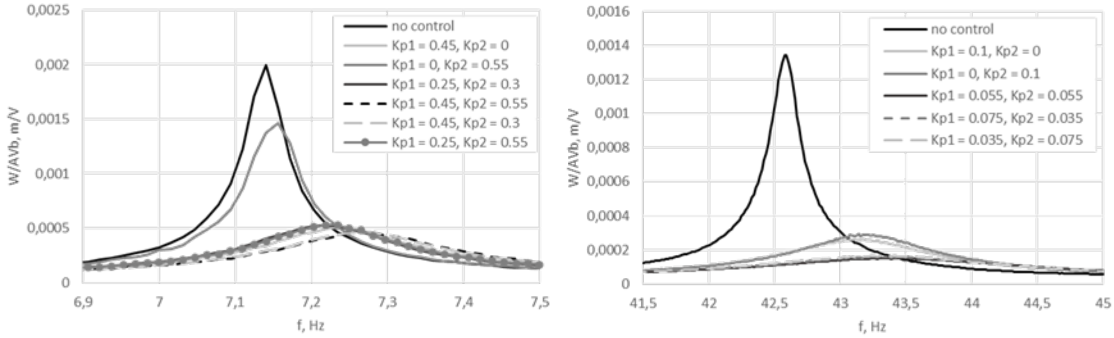

Fig.6. FRF of the local system #1 at the first resonance and the local system #2 at the second resonance

The first control system is effective at the first resonance, providing the decrease of the vibration amplitude by 77% (using gain values $K_p^{(1)} = 0.45$, $K_p^{(2)} = 0.55$), but it fails to suppress the second resonance. On the contrary, the second system has great performance at the second resonance (reduction of the amplitude is 88% with gain values $K_p^{(1)} = K_p^{(2)} = 0.055$), but at the first resonance it is not so good (only 45% reduction). The attempts to create a local system that could suppress both resonances with good efficiency were not successful.

6 Modal control system

The scheme of the controller for the modal or biomorphic control system is shown in *figure 7*. As before, y_1 and y_2 are the measured signals, while u_1 and u_2 are the control signals. In the control system all the sensors measure and all the actuators affect both first and second vibration modes of the beam, therefore the mode analyzer T and the mode synthesizer F are used to perform a linear transformation of signals in order to separate the modes. To design a modal system one needs to define these matrices and to determine the transfer functions $R_1(s)$ and $R_2(s)$.

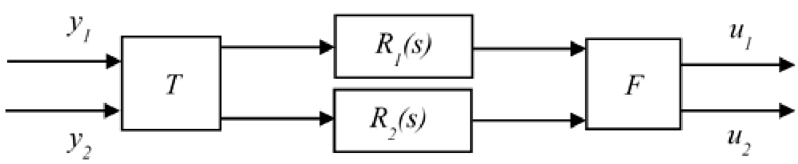

Fig.7. The scheme of the controller for the modal control system

At the first stage of creating a modal system mode analyzer T and synthesizer F are to be specified properly. As a result of this work, two requirements should be observed: the first control loop should not affect and react to the activation of the second bending mode of the beam, and the second one should not affect and react to the activation of the first mode. The matrices T and F are first evaluated theoretically based on the known mode shapes and the measured frequency response functions of the control object. It is known that first and the second actuator excite the first mode in the ratio of approximately 3 to 1 with the same sign, and they excite the second mode in the ratio of 1 to 1 with the opposite sign. The same can be said about the sensors. Therefore, the influence matrix $\widetilde{\Theta}_1$ and the measurement matrix $\widetilde{\Theta}_2$ are estimated as follows:

$$\widetilde{\Theta}_1^{(th)} = \left(\widetilde{\Theta}_2^{(th)}\right)^T = \begin{pmatrix} 0.75 & 0.25 \\ -0.5 & 0.5 \end{pmatrix} \tag{22}$$

Consequently, an estimation for matrices T and F is calculated according to the formulas (8) and (11):

$$F^{(th)} = \left(\widetilde{\Theta}_1^{(th)}\right)^{-1} = \begin{pmatrix} 1 & -0.5 \\ 1 & 1.5 \end{pmatrix}, \qquad T^{(th)} = \left(\widetilde{\Theta}_2^{(th)}\right)^{-1} = \begin{pmatrix} 1 & 1 \\ -0.5 & 1.5 \end{pmatrix} \tag{23}$$

Then, the matrices are to be specified more precisely by performing a set of additional experiments. Matrices $\widetilde{\Theta}_1$ and $\widetilde{\Theta}_2$ are defined experimentally:

$$\widetilde{\Theta}_1^{(exp)} = \begin{pmatrix} 0.76 & 0.25 \\ -0.52 & 0.49 \end{pmatrix}, \qquad \widetilde{\Theta}_2^{(exp)} = \begin{pmatrix} 0.76 & -0.51 \\ 0.24 & 0.49 \end{pmatrix} \tag{24}$$

Thus, matrices T and F have the following form:

$$F = \left(\widetilde{\Theta}_1^{(exp)}\right)^{-1} = \begin{pmatrix} 0.98 & -0.49 \\ 1.02 & 1.51 \end{pmatrix}, \qquad T = \left(\widetilde{\Theta}_2^{(exp)}\right)^{-1} = \begin{pmatrix} 0.99 & 1.03 \\ -0.49 & 1.53 \end{pmatrix} \tag{25}$$

Then the transfer functions are designed in order to provide the optimal performance of each modal control loop at the corresponding resonance frequency. The efficiency of the control systems obtained is limited by the requirement of their stability.
Frequency response functions in the vicinity of the first and the second resonance frequencies measured for the upper point of the beam for the created modal control system are shown in *figure 8.* The black curve represents the uncontrolled system while the other curves correspond to the different gain values in control loops.

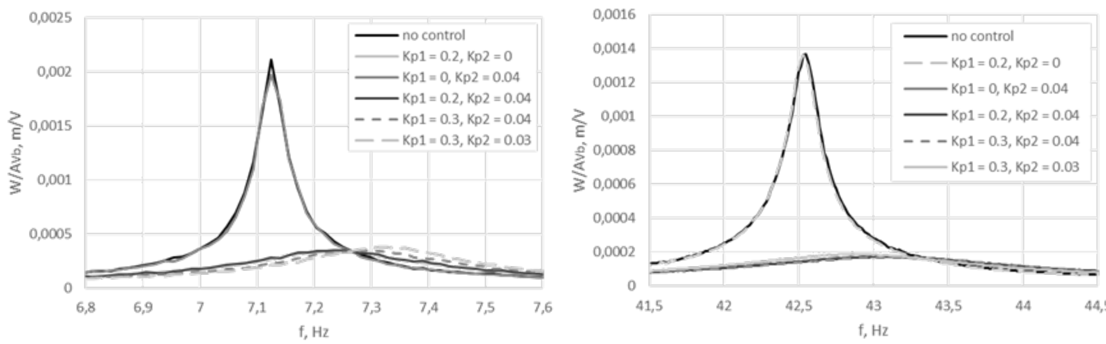

Fig.8. FRF of the modal system at the first and the second resonances

The graphs show that the modal system is very effective at both resonance frequencies, providing the reduction of the vibration amplitudes by 84% and 87%, respectively (the gain values in the control loops are $K_p^{(1)} = 0.2$ and $K_p^{(2)} = 0.04$).

7 Conclusions

The present research highlights application of the local and the modal or biomorphic approaches to active vibration control of continuous elastic systems. Firstly, it focuses on the problem of identification within the modal approach. This identification means the correct calculation of mode analyzer and mode synthesizer matrices. The algorithm of gradual optimization has been proposed, which allows one to control only the needed number of eigenmodes of the object.

Secondly, local and modal approaches were compared experimentally for the problem of suppression of bending vibrations of a thin metal beam. For both methods under consideration efficient control systems have been created. However, the local system failed to suppress efficiently vibrations at both resonances, showing the optimal performance either at the first or at the second resonance frequency. On the contrary, the modal system succeeded in reducing vibrations at both resonance frequencies efficiently. Thus, the advantage of the modal approach over the local one has been demonstrated for the cases where it is necessary to suppress vibrations in the frequency range containing multiple resonance frequencies of the control object. The result obtained can be explained by the fact that, in contrast to the local one, in the modal system each control loop corresponds to a particular vibration mode and can be designed to provide optimal performance at the corresponding resonance frequency.

Further investigation is devoted to development of the modal method and its generalization to a wider class of mechanical systems and different types of control elements. Further experimental study is aimed at implementing the modal approach and comparing it with the local one for systems with larger number of sensors and actuators. In addition, the problems of interest are to develop the methods of automatic identification and to modify the methods of biomorphic control in order to improve stability and robustness of modal control systems.

References

[1] G.E. Stavroulakis, G. Foutsitzi, E. Hadjigeorgiou, D. Marinova, C.C. Baniotopoulos, Design and robust optimal control of smart beams with application on vibrations suppression//Advances in Engineering Software 36 (2005) 806—813

[2] W. Shia, M. Alsarheed, E. Schuster, M.L. Walker, J. Leuer, D.A. Humphreys, D.A. Gates, Multivariable model-based shape control for the National Spherical Torus Experiment (NSTX) // Fusion Engineering and Design 86 (2011) 1107—1111

[3] G.E. Stavroulakis, D.G. Marinova, E. Hadjigeorgiou,G. Foutsitzi, C.C. Baniotopoulos, Robust active control against wind-induced structural vibrations J. Wind Eng. Ind. Aerodyn. 94 (2006) 895—907

[4] N.R. Fisco, H. Adeli, Smart structures: Part II — Hybrid control systems and control strategies // Scientia Iranica, Transactions A: Civil Engineering 18 (2011) 285—295

[5] I. Kucuk, et al., Optimal control of a distributed parameter system with applications to beam vibrations using piezoelectric actuators, Journal of the Franklin Institute. (2012), http://dx.doi.org/ 10.1016/j.jfranklin.2012.10.008

[6] L.A. Gould and M.A. Murray–Lasso. On the modal control of distributed parameter systems with distributed feedback. In Transactions on Automatic Control, volume 11, page 79. IEEE, 1966.

[7] A.K. Belyaev, V.V. Kotov, V.A. Polyanskiy, N.A. Smirnova, Biomorphic control in problem on active suppression of vibrations (in russian), Vestnik St. Petersburg University: Mathematics, Mechanics, Astronomy, 1, 96—106 (2014)

[8] H. Irschik, M. Krommer, A.K. Belyaev, K. Schlacher. Shaping of piezoelectric sensors/actuators for vibrations of slender beams: coupled theory and inappropriate shape functions //Journal of intelligent materials, systems and structures, Vol. 9 — July 1998.

[9] M. Krommer, H. Irschik. An electromechanically coupled theory for piezoelastic beams taking into account the charge equation of electrostatics //Acta Mechanica. Springer-Verlag. 2002. P.141-158.

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