Energy aspects of axisymmetric wave propagation in an infinite cylindrical shell filled with the liquid.

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Abstract

The problem of oscillations of the systems containing pipelines filled with the liquid is one of the actual problems of modern techniques. It is important to estimate the parameters of vibrations and acoustical fields of such objects in order to provide the construction from damaging, but calculation of these complicated systems demands major computing resources. Therefore the consideration of simple model problems which have exact analytical solution ([1] - [5]) is actual. On these models it is possible to analytically explore main effects and also to use them as the test problems for computing packages.

The problem of joint oscillations of infinite thin cylindrical shell with ideal acoustical fluid inside it is considered. The propagating waves and energy flux are analyzed in the system shell-liquid. The comparison of different mechanisms of energy transmission in the shell and input of the energy flux in the water is fulfilled.

1 Statement of the problem

Let us start considering an infinite cylindrical shell filled with an ideal compressible liquid, where the acoustic pressure $P(x, y, z)$ is described by the Helmholtz equation in the cylindrical system of coordinates where the axis $0z$ coincides with axis of the cylinder (see fig. 1). All processes are supposed to be harmonic with frequency ω and independent from angle φ.

$$(\Delta + k^2)P(r, z) = 0, \quad \text{where} \quad k = \omega/c, \quad 0<r<R. \tag{1}$$

The factor $\mathrm{e}^{-i\omega t}$ describes the time-dependence and is omitted. The liquid is supposed to be ideal and compressible. The density is ρ_w, the velocity of sound is equal to c.

Two relations take place on the shell – fluid boundary: kinematic (the adhesion condition)

$$u_n(R, z) = \frac{1}{\rho_w \omega^2} \left. \frac{\partial P(r, z)}{\partial r} \right|_{r=R}; \tag{2}$$

and dynamic (balance of forces acting on the shell)

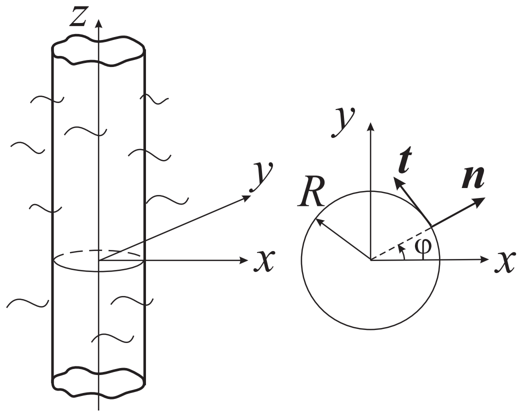


Figure 1: Physical Model

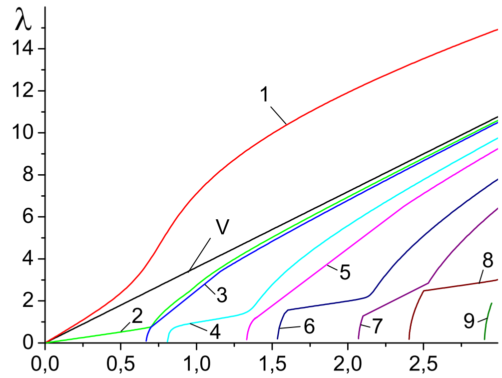


Figure 2: Dispersion curves 1-9

$$\frac{\rho c_s^2}{R^2}\mathbf{L}\vec{u} = (0, P)^t. \tag{3}$$

Here following notations are introduced: $\vec{u} = (u_z, u_n)^t$ (t is a badge of transposing) is the displacement vector of the shell, $\mathbf{L}$ is matrix differential operator of the cylindrical shell of Kirchhoff–Love type

$$\mathbf{L} \equiv [L_{ij}] = w^2\mathbf{I} + \widetilde{\mathbf{L}}, \quad i, j = 1, 2.$$

The following notations are also used: $\mathbf{I}$ is the unit matrix operator, $\mathbf{L}$ and $\widetilde{\mathbf{L}}$ are matrix differential operators. All these operators are represented by the matrixes 2×2. In particular the elements of $\widetilde{\mathbf{L}}$ are as follows:

$$\widetilde{\mathbf{L}} = \begin{pmatrix} \widetilde{\partial}_z^{\,2} & \nu\widetilde{\partial}_z \\ -\nu\widetilde{\partial}_z & \alpha^2(2\nu\widetilde{\partial}_z^{\,2} - \widetilde{\partial}_z^{\,4} - 1) - 1 \end{pmatrix}, \tag{4}$$

where $\widetilde{\partial}_z = R\partial_z$. Here the following geometrical parameters of the shell are used: R is radius, h is thickness. Properties of a material of the cylinder are characterized by E, ν and ρ_s – Young's module, Poisson coefficient and volumetric density accordingly.

The surface density of the shell ρ (ρ=$\rho_s h$) and the velocity of median surface deformation waves of the cylindrical shell c_s are introduced $c_s = \sqrt{E/(1-\nu^2)\rho_s}$.

The following dimensionless parameters are put in: $\alpha^2 = \frac{1}{12}\left(\frac{h}{R}\right)^2$ (the relative thickness of the cylindrical shell) and $w = \omega R/c_s$ (the dimensionless frequency).

The source of an acoustic field in a wave guide is the vibrations of the cylinder shell, caused by the incident wave propagating from infinite part of the shell. The frequency of this incident harmonic wave is equal to ω. All processes in the system shell – liquid are supposed to be harmonic with this frequency.

2 Determination of the general representation for acoustic and vibrational fields

The exact expression for displacements of the shell can be derived only after defining a field in the medium. So we come to the boundary problem with the Helmholtz equation. Further it is more convenient to involve the new vector $(u_z, P)^T$

$$\begin{pmatrix} u_z \\ u_n \end{pmatrix} = \mathbf{M} \begin{pmatrix} u_z \\ P \end{pmatrix}, \quad \text{where} \quad \mathbf{M}= \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{\rho_w \omega^2} \frac{\partial}{\partial r}\Big|_{r=R} \end{pmatrix}, \tag{5}$$

then the equation (3) can be rewritten in the form

$$\mathbf{S} \begin{pmatrix} u_z \\ P \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \equiv \mathbf{0}, \quad \text{where} \quad \mathbf{S}{=}\mathbf{LM}-\mathbf{N}, \quad \mathbf{N}{=}\frac{w^2}{\rho\omega^2} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \tag{6}$$

The solution of the equation (3) is searching in the form

$$\begin{pmatrix} u_z \\ P \end{pmatrix} = A e^{i\lambda z} \begin{pmatrix} \xi \\ \gamma J_0 \left(r\sqrt{k^2-\lambda^2} \right) \end{pmatrix}, \tag{7}$$

believing that $|\xi|^2 + |\gamma|^2{=}1$. Here following notations are introduced: J_0 is Bessel function with index 0, and A, ξ, γ are arbitrary constants, λ is the the wavenumber which we are looking for. It can be noted that if $k{<}\lambda$ then Bessel function J_0 is converted to I_0 function.
After substituting (7) into (3) the following algebraic system is obtained

$$\widehat{\mathbf{S}}\mathbf{x}{=}\mathbf{0}, \quad \text{where} \quad \mathbf{x}{=}(\xi, \gamma)^T \tag{8}$$

Operator $\widehat{\mathbf{S}}$ is the Fourier image of operator $\mathbf{S}$. The dispersion equation is obtained from the condition of existence of nontrivial solution of this system

$$\det \widehat{\mathbf{S}}{=}0 \tag{9}$$

We are looking for real positive solutions of this equation. If the corresponding set of wavenumbers is founded one can solve the equation (8) and define the previously unknown constants ξ, γ. After defining constants, the complete solution of the problem in terms of displacements of the shell $\vec{u}(z)$ and pressure $P(r, z)$ in the liquid is determined.

3 Energy streams in the system shell-liquid

As it was mentioned above all processes in the liquid and shell are supposed to be harmonic with frequency ω. It is convenient to average the energy streams on period of oscillations $T = 2\pi/\omega$. The integral energy stream Υ_{liq} in the liquid along axes z through the cross-section of the cylinder can be written in the form

$$\Upsilon_{\text{liq}} = \frac{\omega}{2}\frac{1}{2\rho\omega}\int_0^{2\pi} d\varphi \int_0^R \operatorname{Im}\left(\bar{P}\frac{\partial P}{\partial z}\right) r dr. \tag{10}$$

The integral stream of the energy along axes z through the cross-section of the cylinder shell has a view

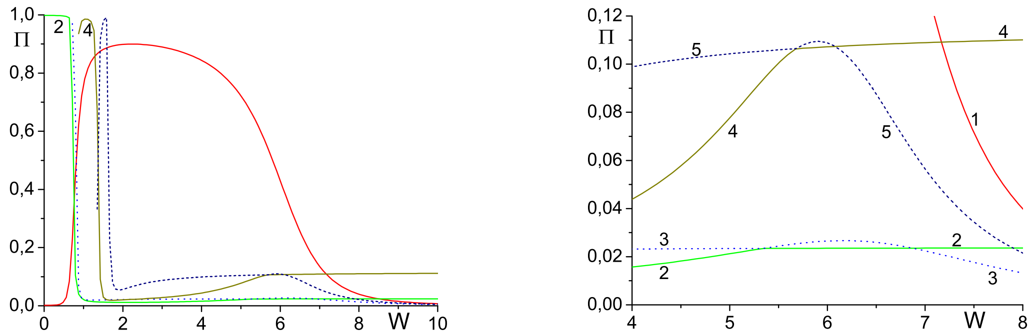
$$\Upsilon_{cyl} = \frac{\omega}{2}\int_0^{2\pi} \operatorname{Im}\left(\vec{u}^3, \vec{f}^3\right)_{\mathbf{C}^3} R d\varphi = \Upsilon^z_{cyl} + \Upsilon^n_{cyl} + \Upsilon^m_{cyl}, \quad \text{where} \tag{11}$$

$$\vec{f}^3 = \mathbf{F}\vec{u}^3 = \begin{pmatrix} f_z \\ f_n \\ f_p \end{pmatrix}, \quad \mathbf{F} = \frac{\rho c^2}{R}\begin{pmatrix} \widetilde{\partial}_z & \nu & 0 \\ 0 & -\nu & -\widetilde{\partial}_z \\ 0 & 0 & \nu - \widetilde{\partial}_z^{\,2} \end{pmatrix}$$

$$\vec{u}^3 = \begin{pmatrix} u_z \\ u_n \\ -\widetilde{\partial}_z u_n \end{pmatrix}, \quad \begin{Bmatrix} \Upsilon^z_{cyl} \\ \Upsilon^n_{cyl} \\ \Upsilon^m_{cyl} \end{Bmatrix} = -\pi R\omega \operatorname{Im}\begin{Bmatrix} \overline{u}_z(\widetilde{\partial}_z u_z + \nu\overline{u}_n) \\ -\nu\overline{u}_n u_n + \overline{u}_n\widetilde{\partial}_z^{\,2} u_n \\ \widetilde{\partial}_z\overline{u}_n(\nu\widetilde{\partial}_z u_n - \widetilde{\partial}_z u_n) \end{Bmatrix} \tag{12}$$

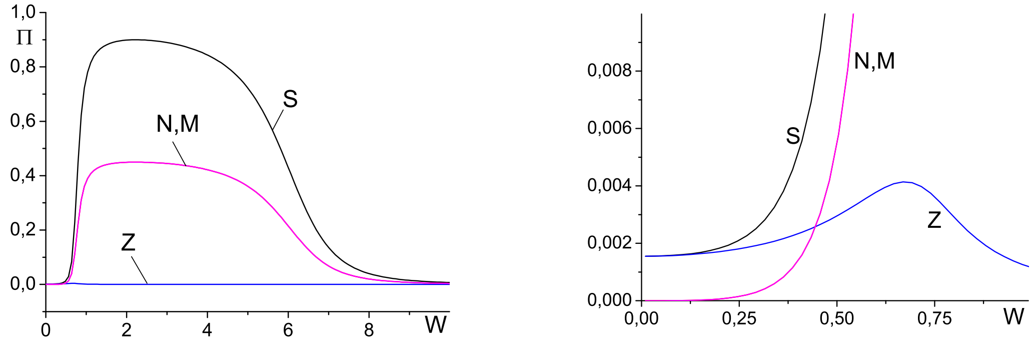
4 Numerical calculations

The formula (11) can be used for obtaining the normalized energy stream Π in the shell and it components $\Pi = \Upsilon_{\text{cyl}}/(\Upsilon_{\text{cyl}} + \Upsilon_{\text{liq}})$, $\Pi^{z,n,m} = \Upsilon^{z,n,m}_{\text{cyl}}/(\Upsilon_{\text{cyl}} + \Upsilon_{\text{liq}})$. On figures the curves corresponding Π, Π^z, Π^n, Π^m are marked by letters S, Z, N, M. The pressure P and vectors of generalizes displacements $\vec{u}^3$ and forces $\vec{f}^3$ are also normalized: $P := P/|P|$, $\vec{u}^3 := \vec{u}^3/|\vec{u}^3|$, $\vec{f}^3 := \vec{f}^3/|\vec{f}^3|$. On figures the curves corresponding components vectors $\vec{u}^3$, $\vec{f}^3$ (12) are marked by letters Z, N, M.
The following values of parameters of the system are assumed for calculations ν=0.3, c_s/c=3.6, ρ_s/ρ_w=7.8, h/R=0.05 that corresponds to interaction of water with shell made of steel.
On fig.2 the dependence of dimensionless wavenumber $\lambda := \lambda R$ with respect to nondimensional frequency w is shown for the first nine dispersion curves (these curves are marked by numbers 1-9). The multiple veering (quasiintersection) of these curves is well noted. It is caused by interaction of the two waveguides (liquid and cylindrical shell).
The behavior of the wave from the first dispersion branch differs from others significantly. On figure 3a and 3b is well seen that wave energy flux from the first dispersion branch is significant in more wide diapason of frequencies. The banding component in energy flux is dominated in it (fig. 4a,b).
The veering of the dispersion curves is well corresponds with the changing of the specific weight of different components of the energy flux (fig. 2 and fig.4).



a) b)

Figure 3: Normalized energy flux of the waves in the shell from first five dispersion branches (curves 1-5)(a); in greater scale (b)



a) b)

Figure 4: Energy fluxes in the shell for the wave from the first dispersion branch (a); in greater scale (b).

Pressure and energy flux in the wave from first dispersion curve are concentrated near the shell surface (curves 1a, 1b, 1c on fig. 5 and curve 1 on fig. 6). This dependence is most distinctive in the diapason of frequencies where the velocity of the waves in the shell is less then in the liquid infinite space. Unlike it the pressure and energy flux in the liquid in next dispersion branches are concentrated near the center of the cross section of the cylinder (curves 2a, 2b, 2c on fig. 5 and curves 2-7 on fig. 6). The general tendency is increasing of the role of the center part of the cross section for propagating of the energy in the system via increasing of the branch number or frequency increasing.

Even and odd branches (since second branch) have different behavior near the shell surface (fig. 6). While pressure and energy flux of even branches are near zero (dotted lines), the normal component of displacement vector is near zero for the odd branches (dashed lines). It corresponds to the wave guide with free and rigid surface correspondingly. But this dependence can be changed to opposite (fig.3b) due to the veering of the dispersion curves (fig.2).

Another speciality is that energy flux in even branches (corresponding to the "free surface") is stabilized on high frequencies (fig. 3b)).

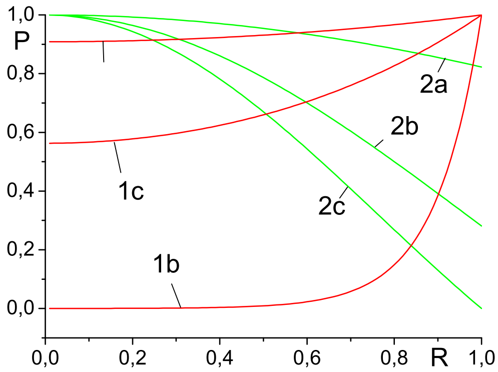


Figure 5: Pressure in the waves via radius from two first dispersion branches (1a,2a - w=0.25, 1b,2b - w=2.5, 1c,2c - w=10.0)

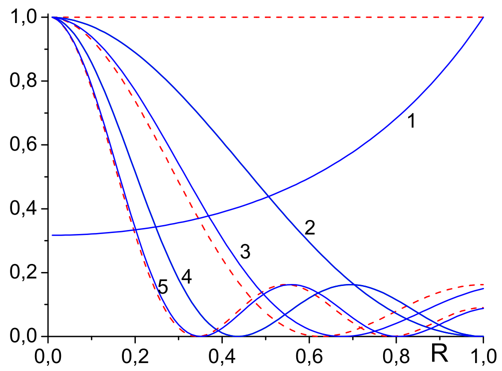


Figure 6: Energy flux of the waves from first seven dispersion branches via radius for w=10.0 (curves 1-5)

Acknowledgements

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