The problem of damage and high-temperature creep fracture of metals

Alexander R. Arutyunyan, Robert A. Arutyunyan, Regina R. Saitova
r.arutyunyan@spbu.ru, rigastr@yandex.ru

Abstract

Numerous experimental studies on changes of the porosity and density of various metals and alloys due to the formation and development of micropores and microcracks in the process of high-temperature creep are carried out. The results of these studies allow us to consider the density as an integral measure of the structural micro-defects accumulation, and the damage parameter is defined as the ratio of current density to initial density. Taking into account this parameter and the mass conservation law interconnected kinetic equations for creep deformation and damage parameter are formulated. In the case of pure brittle fracture the analytical solutions of these equations are received and the criterion of long-term strength is formulated. The ductile-brittle fracture is also considered. An analytical solution connecting the damage parameter to the value of deformation is obtained. In this case, the creep deformation is calculated approximately. The appropriate choice of the coefficients of the approximate solution allows describing the experimental creep curves.

1 Introduction.

Under the long action of high temperatures and relatively small stresses many metallic alloys and pure metals lose plasticity and fractured as brittle (the phenomenon of thermal brittleness). Because these effects are observed in elements of many important engineering objects, in particular, in power and nuclear, the problem of brittle fractures became a subject of numerous theoretical and experimental researches. In the Kachanov's brittle fracture model [1] the parameter of continuity ψ $(1 \geq \psi \geq 0$ is introduced formally without giving to it a certain physical meaning. In the model of Rabotnov brittle fracture [2, 3] the damage parameter ω $(0 \leq \omega \leq 1)$ is introduced by the ratio $\omega = F_T/F_0$ (F_0 is initial, F_T is total area of pores) and characterize the degree of reduction of cross-section area of the specimen.

To materialize the damage parameter various definitions were offered: the relative size of pores or irreversible change of volume (loosening on Novozhilov's terminology [4]) or density (Arutyunyan [5, 6, 7]). In the paper the parameter of continuity is determined by the ratio $\psi = \rho/\rho_0$ (ρ_0 is initial, ρ is current density) and it is an integral measure of the structural microdefects accumulation during long-term high-temperature loading [8-14]. In the initial conditions $t = 0$, $\rho = \rho_0$, $\psi = 1$, at the fracture time $t = t_f$, $\rho = 0$, $\psi = 0$.

2 Interrelated Rabotnov's equations for the rate of creep and damage.

Robotnov's damage concept is based on the following system of equations for the creep ε and damage parameter ω [3]

$$\frac{d\varepsilon}{dt} = b\sigma^m(1-\omega)^{-q}, \tag{1}$$

$$\frac{d\omega}{dt} = c\sigma^n(1-\omega)^{-r}, \tag{2}$$

where b, c, m, n, q, r are constants, $\varepsilon = \ln(l/l_0)$ is deformation, l_0, l are initial and current length of the specimen.
In the case of pure brittle fracture and small strains can be considered that $F = F_0$, $\sigma = \sigma_0 = const$, and solving the system of equations (1)-(2) we will receive the relation for the creep strain

$$\varepsilon = \frac{k}{m}\frac{t_f^b}{t_f^v}\left[1-\left(1-\frac{t}{t_f^b}\right)^{1/k}\right], \tag{3}$$

where $k = \frac{r+1}{r+1-\mathrm{q}}$, $t_f^b = \frac{1}{c(1+r)\sigma_0^n}$, $t_f^v = \frac{1}{bm\sigma_0^m}$.
Formula (3) is considered as a basic result of the Rabotnov's theory, because using it it is possible to describe the third region of the creep curve, which, in the region of brittle fractures, is completely determined by the damage of material. At the same time, the derivation of this formula is based on the condition $F = F_0$ from which follows $\omega = 0$, i.e. the conception of damage is lose the meaning itself. Further, when the criterion of ductile-brittle fracture is determined using equations (1)-(2), the condition of incompressibility, which is also contrary to the damage conception, is accepted.

3 The system of equations for the creep rate and the continuity parameter.

To overcome these contradictions in this paper a system of equations for the rate of creep and damage, based on the continuity parameter $\psi = \rho/\rho_0$, is proposed. Let's consider the following system of equations

$$\psi^\beta\frac{d\varepsilon}{dt} = B\sigma^m, \tag{4}$$

$$\psi^\alpha\frac{d\psi}{dt} = -A\sigma^n, \tag{5}$$

where B, A, α, β are constants.
Taking into account the mass conservation law $\rho_0 l_0 F_0 = \rho l F$, from which follows the relation $\sigma = \sigma_0\psi e^\varepsilon$, these equations can be written in the form

$$\frac{d\varepsilon}{dt} = B\sigma_0^m \psi^{m-\beta} e^{m\varepsilon}, \tag{6}$$

$$\frac{d\psi}{dt} = -A\sigma_0^n \psi^{n-\alpha} e^{n\varepsilon}. \tag{7}$$

The system of equations (6)-(7) can be solved approximately, for example, for the case of purely brittle fracture and small deformations, when the approximations $e^{m\varepsilon} \approx 1$, $e^{n\varepsilon} \approx 1$ can be considered. In this case, taking into account the initial conditions $t = 0$, $\psi = 1$, $\varepsilon = 0$, we can receive the following analytical solutions

$$\psi = [1 - (\alpha - n + 1)A\sigma_0^n t]^{\frac{1}{\alpha-n+1}}, \tag{8}$$

$$\varepsilon = \frac{B\sigma_0^{m-n}}{A\gamma} \left\{1 - [1 - (\alpha - n + 1)A\sigma_0^n t]^{\frac{\gamma}{\alpha-n+1}}\right\}, \tag{9}$$

where $\gamma = m - \beta + \alpha - n + 1$.
On Fig. 1 the curves $\psi(t)$ according the formula (8) for different values of parameter α ($\alpha = 6$ - curve 1, $\alpha = 4$ - curve 2 and $\alpha = 2$ - curve 3) are shown. In the calculations the following values of coefficients were used: $A = 1 \cdot 10^{-9}$ $[MPa]^{-2}$, $\sigma_0 = 100\, MPa$, $n = 2$.

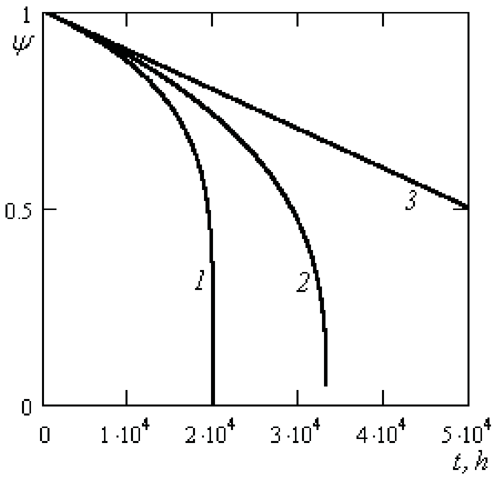

Figure 1: The curves $\psi(t)$ according (8) for different values of parameter α: $\alpha = 6$ - curve 1, $\alpha = 4$ - curve 2 and $\alpha = 2$ - curve 3.

From equations (6)-(7) the exact solution for function $\psi(\varepsilon)$ can be received. Dividing (7) to (6), we will obtain the following equation

$$\frac{d\psi}{d\varepsilon} = -\frac{A}{B}\sigma_0^{n-m}\psi^{n-\alpha-m+\beta}e^{(n-m)\varepsilon}. \tag{10}$$

Using the initial condition $\psi = 1$, $\varepsilon = 0$ and solving (10) we receive

$$\psi = \left[1 + \frac{A\sigma_0^{n-m}(1 - n + \alpha + m - \beta)}{B(n-m)}(1 - e^{(n-m)\varepsilon})\right]^{\frac{1}{1-n+\alpha+m-\beta}}. \tag{11}$$

On Fig. 2 the curves $\psi(\varepsilon)$ according (11) for different values of parameter α ($\alpha = 6$ - curve 1, $\alpha = 4$ - curve 2 and $\alpha = 2$ - curve 3) are shown. In the calculations the following values of coefficients were used: $A = 1 \cdot 10^{-12}$ $[MPa]^{-2}$, $B = 5 \cdot 10^{-17}$ $[MPa]^{-4}$, $\sigma_0 = 100\,MPa$, $n = 2$, $m = 4$, $\beta = 1$.

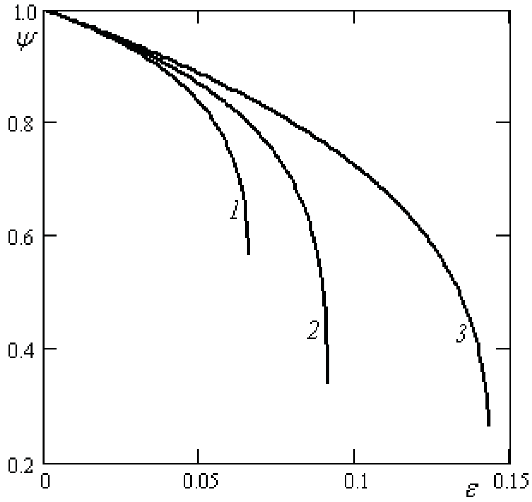

Figure 2: The curves $\psi(\varepsilon)$ according (11) for different values of parameter α: $\alpha = 6$ - curve 1, $\alpha = 4$ - curve 2 and $\alpha = 2$ - curve 3.

As can be seen from Fig. 1 and 2 the damage curves for formulas (8) and (11) are identical.
In scientific literature there are many experimental data on porosity changes (damage) of different metals and alloys on creep conditions. For comparison with the corresponding theoretical curves we chose the most characteristic experimental data for pure copper, aluminum and nickel [8, 9, 10, 12]. On Fig. 3 theoretical curves of density changes according (11) for $\alpha = 6$ and experimental points of density changes for pure copper in the process of high-temperature creep at 500° C [8] are shown. From this figure it follows that the experimental points are described well by straight line and have the general character for different metals tested under various temperature and force conditions [9, 10, 12]. These results allow us to consider the damage parameter $\psi = \rho/\rho_0$ as universal characteristic of porosity accumulation in the creep process [14]. In the calculations the following values of coefficients were used: $A = 3 \cdot 10^{-9}$ $[MPa]^{-2}$, $B = 7 \cdot 10^{-12}$ $[MPa]^{-4}$, $\sigma_0 = 100\,MPa$, $n = 2$, $m = 4$, $\beta = 1$.
Taking the fracture condition $t = t_f$, $\psi = 0$, from (8) we obtain the creep fracture criterion

$$t_f^b = \frac{1}{(\alpha - n + 1) \cdot A\sigma_0^n}. \tag{12}$$

When $\alpha = 2n$ the criterion (12) coincides with the Kachanov-Rabotnov criterion. In Fig. 4 in the double logarithmic coordinates are shown the creep fracture curves according to (12) for different values of the coefficient α ($\alpha = 6$ - curve 1, $\alpha = 4$ -

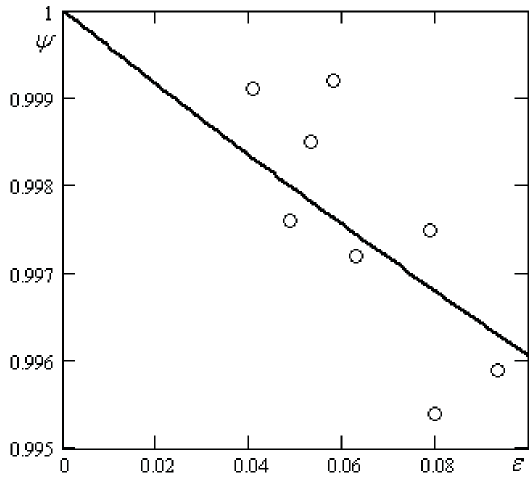

Figure 3: Theoretical curves of density changes (solid line) for $\alpha = 6$ and experimental points of density changes of pure copper during creep under 500° C [8] (circle points).

curve 2 and $\alpha = 2$ - curve 3). In the calculations the following values of coefficients were used: $A = 1 \cdot 10^{-9}$ $[MPa]^{-2}$, $n = 2$.

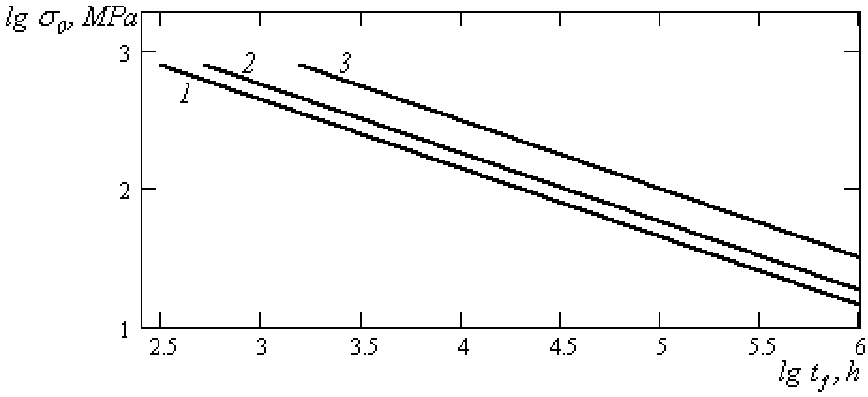

Figure 4: Curves of long-term strength under criterion (12): $\alpha = 6$ - curve 1, $\alpha = 4$ - curve 2 and $\alpha = 2$ - curve 3.

In Fig. 5 are shown the theoretical creep deformation curves according to the relation (9) for different values of the coefficient α ($\alpha = 6$ - curve 1, $\alpha = 4$ - curve 2 and $\alpha = 2$ - curve 3). As can be seen from this figure, the system of equations (6)-(7) is able to describe the third phase of creep curves, which is determined by the processes of damage accumulation. In the calculations the following values of coefficients were used: $A = 1 \cdot 10^{-12}$ $[MPa]^{-2}$, $B = 5 \cdot 10^{-17}$ $[MPa]^{-4}$, $\sigma_0 = 100\,MPa$, $n = 2$, $m = 4$, $\beta = 1$.

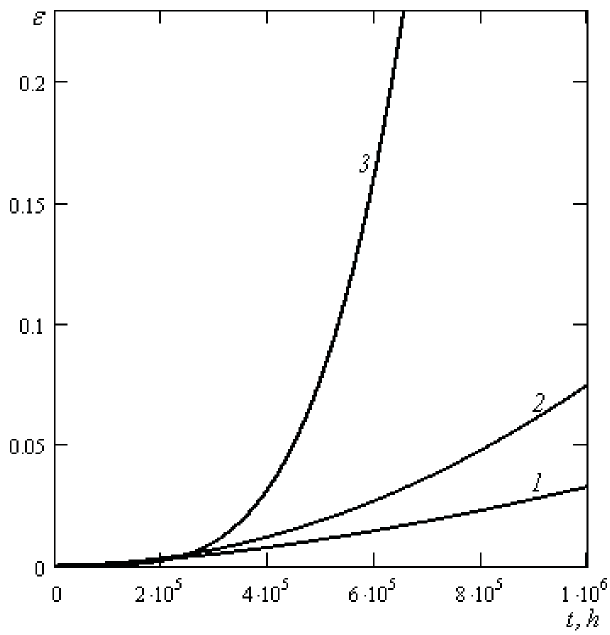

Figure 5: The theoretical creep deformation curves according to the relation (9) for different values of the coefficient α: $\alpha = 6$ - curve 1, $\alpha = 4$ - curve 2 and $\alpha = 2$ - curve 3.

Acknowledgements

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Alexander R. Arutyunyan, Robert A. Arutyunyan, Regina R. Saitova, Universitetskii pr., 28, Faculty of Mathematics and Mechanics St.-Petersburg State University, St.-Petersburg, Petrodvoretz, 198504, Russia.