

# Block element forms and factorization methods in cylindrical coordinate systems

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## Abstract

This work shows that being constructed, the block elements have several stages, where they may have different opportunities and functions. It was established that some stages are more opportune for the research aims and for the construction of new block structures. This stage is called packing up the block element and the element is called packed. At another stage, which is called unpacking, the block elements are more opportune for the performing calculations and clear research of solutions. It is shown that analyses of boundary-value problems by existed methods of variable separation, integral transformations and different insertions from the view of the developed theory are aimed at the analysis specifically in the form of the unblocked element. Exactly this restrained the use of these methods in the boundary problems in nonclassical fields, that didn't correspond to the framework of space transformation groups [1], [2]. The method of the block-level element, as it shown in [3], is convergent not only because it unites several approaches but also because it increases opportunities in research of boundary-value problems in nonclassical fields. Keywords: packed block element, factorization, topology, integral and differential factorization methods, exterior forms, block structures, boundary problems.

## 1 Introduction

For method's using it is necessary to make a few steps of transformations connected with engaging of external analysis device, factorization, topology and other branches of mathematics. In this work in collating of brick-element method with existing approaches and more complete revealing of ties with them and also with the view of convenience by its using in practical purposes three stages of transformations by building brick-element are represented which simplify its building and using. Algorithm of method's using on these stages by the example of boundary values in classical and non-classical areas is showed below.

## 2 The factorization method for block element

It is reasonable to call the first stage forming of block element. It comprises statement of boundary value for system of differential equations in partial derivatives with constant coefficients considered in space of slow-growing generalized functions  $\mathbf{H}_s$ . Boundary value for system  $P$  of differential equations with constant coefficients in partial derivatives of random order of differentiation in gibbous three-dimensional area  $\Omega$  in speculation of resolvability without reference to type of boundary value of consideration can be written down in the form

$$\begin{aligned} \mathbf{K}(\partial x_1, \partial x_2, \partial x_3)\varphi &= \sum_{m=1}^M \sum_{n=1}^N \sum_{k=1}^K \sum_{p=1}^P A_{spmnk} \varphi_{p, x_1^{(m)} x_2^{(n)} x_3^{(k)}} = 0, \quad s = 1, 2, \dots, P, \\ A_{sqmnk} &= \text{const}, \quad \varphi = \{\varphi_1, \varphi_2, \dots, \varphi_P\}. \\ \varphi &= \{\varphi_s\}, \quad \varphi(\mathbf{x}) = \varphi(x_1, x_2, x_3), \quad \mathbf{x} = \{x_1, x_2, x_3\}, \end{aligned} \quad (1)$$

On the boundary  $\partial\Omega$  following boundary conditions are assumed:

$$\begin{aligned} \mathbf{R}(\partial x_1, \partial x_2, \partial x_3)\varphi &= \sum_{m=1}^{M_1} \sum_{n=1}^{N_1} \sum_{k=1}^{K_1} \sum_{p=1}^P B_{spmnk} \varphi_{p, x_1^{(m)} x_2^{(n)} x_3^{(k)}} = f_s, \quad s = 1, 2, \dots, s_0 < P, \quad \mathbf{x} \in \partial\Omega \\ M_1 &< M, \quad N_1 < N, \quad K_1 < K. \end{aligned}$$

We can notice that in common with represented above integral method of factorization, boundary value in differential method of factorization is solved precisely if  $\Omega$  is semispace. In case if area  $\Omega$  is gibbose value reduces to solution of system of normally resolvable pseudodifferential equations. With the purpose of systemization of representation differential method of factorization let us a few stages. Reducing of differential equations by Fourier transformation or by other integral transformation to functional equation. Three-dimensional Fourier transformation of form  $\Phi_n(\boldsymbol{\alpha}) = \iiint_{\Omega} \varphi_n(x) e^{i\langle \boldsymbol{\alpha}, \mathbf{x} \rangle} d\mathbf{x} \equiv F\varphi_n$ ,  $\Phi_m = F\varphi_m$ ,  $\langle \alpha^\nu \mathbf{x}^\nu \rangle = \alpha_1^\nu x_1^\nu + \alpha_2^\nu x_2^\nu + \alpha_3^\nu x_3^\nu$

It reduces to functional equation presentable in form

$$\mathbf{K}(\boldsymbol{\alpha})\Phi = \iint_{\partial\Omega} \boldsymbol{\omega} \equiv \sum_{n=1}^N \iint_{\partial\Omega_n} \boldsymbol{\omega}^n, \quad \mathbf{K}(\boldsymbol{\alpha}) \equiv -\mathbf{K}(-i\boldsymbol{\alpha}_1, -i\boldsymbol{\alpha}_2, -i\boldsymbol{\alpha}_3) = \|k_{nm}(\boldsymbol{\alpha})\|, \quad (3)$$

Here  $\partial\Omega_n$  is orientable element of topological separation of boundary unity  $\partial\Omega$ ,  $\boldsymbol{\omega}^n$  is vector of exterior form built on this element. Here  $\mathbf{K}(\boldsymbol{\alpha})$  is polynomial function matrix of order  $P$ . Vector of exterior forms  $\boldsymbol{\omega}$  has in the function of component two-dimensional functions of the form

$$\boldsymbol{\omega} = \{\boldsymbol{\omega}_s\}, \quad s = 1, 2, \dots, P, \quad \boldsymbol{\omega}_s = P_{12s} dx_1 \wedge dx_2 + P_{13s} dx_1 \wedge dx_3 + P_{23s} dx_2 \wedge dx_3 \quad (4)$$

It relates also to exterior forms  $\boldsymbol{\omega}^n$  on elements  $\partial\Omega_n$ . Operations of exterior forms have indications  $dx_1 \wedge dx_2 = dx_1^1 dx_2^2 - dx_1^2 dx_2^1$ ,  $dx_1 \wedge dx_3 = dx_1^1 dx_3^3 - dx_1^3 dx_3^1$ ,  $dx_2 \wedge dx_3 = dx_2^2 dx_3^3 - dx_2^3 dx_3^2$ . Here vectors of spontaneous coordinate system from the covers of tangent fibration of body surface are introduced. In rectangular coordinate system for tangent vectors of spontaneous element cover are assumed.

$x_1 = \{x_1^1, x_1^2, x_1^3\}$ ,  $x_2 = \{x_2^1, x_2^2, x_2^3\}$ . Coefficients of exterior forms are presented in [4]. Sufficing to stated boundary conditions (5) is reached by introducing in presentation of exterior forms of amounts of solution vector  $\varphi(\partial\Omega)$  and its derivatives along the normal on  $\partial\Omega$  taken from boundary conditions. Presence of derivatives on tangent line is left out of account. Exterior forms contain amounts of solution  $\varphi_n$  and its derivatives on boundary  $\partial\Omega$ . From boundary values (-) functions or derivatives along the normal on boundary are found by trial and error and conversion of nondegenerate matrix and are introduced in corresponding presentations of exterior forms  $\omega$ . The rest of functions or derivatives along the normal must be found from pseudodifferential equations received by transformation of functional equations. Differential factorization of function matrix  $\mathbf{K}(\alpha)$  of functional equation is occurred [4]. Consolidation of the functional equation to the system of pseudodifferential equation is achieved by calculation of residue form of Leray for functions of some complex variables. By such action the automorphism under the mapping of support towards itself is made. Waiving computations, which are in the noticed works were presented, we achieve correlations of the form

$$\sum_{\nu=1}^N \sum_{p=1}^P \iint_{\partial\Omega_\nu} \omega_p^\nu Z_{mp}(z_{s-}^\nu) = 0, \quad s- = 1, 2, \dots, G-, \quad (5)$$

Here  $\alpha_{3s\pm}^\nu \equiv z_{s\pm}^\nu(\alpha_1^\nu, \alpha_2^\nu)$  laid above positive and under negative semiplanes nulls of determinant of function matrix  $\mathbf{K}(\alpha)$ . Built system is pseudodifferential equations. Admitted that demanded by problem to solve integral equations are derived from pseudodifferential equations and solved and the result is introduced to outer forms, the second stage of block element, named packing, is accomplished. Packed brick element has compact form.  $\varphi(\mathbf{x}^\nu) = \frac{1}{8\pi^3} \iint_{-\infty}^{\infty} \mathbf{K}^{-1}(\alpha) \iint_{\partial\Omega} \omega e^{-i(\alpha^\nu \mathbf{x}^\nu)} d\alpha_1^\nu d\alpha_2^\nu d\alpha_3^\nu$ ,  $\mathbf{x}^\nu \in \Omega$ ,  $\mathbf{K}^{-1}(\alpha) = \mathbf{K}_r^{-1}(\alpha_3^\nu) \mathbf{K}^{-1}(\alpha_3^\nu, -)$ . Here functions matrix  $\mathbf{K}_r(\alpha_3^\nu)$ ,  $\mathbf{K}(\alpha_3^\nu, -)$  are the results of differential factorization of functional matrix  $\mathbf{K}(\alpha)$  by argument  $\alpha_3^\nu$  besides, determinant of the first contains nulls only on the upper semiplane, the second is on the lower one. In this pattern it represents topological manifold with the edge. In such form it is convenient for formation of brick structure for building non-homogeneity, hollows, and cracks, fixed and deformable inclusions couplings with deformable and fixed bricks in it. All of it is achieved by the formation of equality relations between given brick and similar adjoined one. For it on the basis of equivalence relations quotient topology of Cartesian product of brick elements  $\check{Y}$  supporter and topological space of vector-function built on them is building. An attribute of right built up brick element is the opportunity of calculation in model of solution of one integral by the method of theory of residues. It is called opening or unpacked of block element. Calculated this integral we achieve following presentation of opened brick element.

$$\varphi(\mathbf{x}^\nu) = \frac{1}{4\pi^2} \iint_{-\infty}^{\infty} \sum_s [\mathbf{A}_+(\alpha_1^v, \alpha_2^v, z_{s+}^v) e^{-i(z_{s+}^v x_3^v + \alpha_1^v x_1^v + \alpha_2^v x_2^v)} + \mathbf{B}_-(\alpha_1^v, \alpha_2^v, z_{s-}^v) e^{-i(z_{s-}^v x_3^v + \alpha_1^v x_1^v + \alpha_2^v x_2^v)}] d\alpha_1^v d\alpha_2^v \quad (6)$$

Here matrixes- functions  $\mathbf{A}_+(\alpha_1^v, \alpha_2^v, z_{s+}^v)$ ,  $\mathbf{B}_-(\alpha_1^v, \alpha_2^v, z_{s-}^v)$  depend on specified boundary conditions, form of  $\Omega$  area and also on qualities of matrixes- function

**K(α)** . Considering the form of opened brick element (6) it is easy to notice in it the form of representation, in which solutions of common differential equations and also equations in partial differential quotient with constant coefficients with the method of exponential substitution or separation of variables are searched. [5], [6] on the base of it, it could be said that all practice of mentioned methods appliance is summarized to researches of opened or unpacked block elements. As a result scope of available for solution problems narrowed. It may be used only for classic areas denial of which excluded the opportunity to solve boundary problem with mentioned methods.

### 3 The packed block elements for cylindrical domain

For illustration of mentioned above let us look at examples. Example 1. In the layer  $\Omega$  with parallel limits of thickness  $b - a$  boundary problem for partial differential equation following form is placed:

$$\frac{\partial^2 \varphi}{\partial x_1^2} + \frac{\partial^2 \varphi}{\partial x_2^2} + \frac{\partial^2 \varphi}{\partial x_3^2} - r^2 \varphi = 0, \quad |x_1| \leq \infty, \quad |x_2| \leq \infty, \quad a \leq x_3 \leq b \quad (7)$$

. On the limits (-) boundary conditions are accepted

$$\begin{aligned} \varphi(x_1, x_2, x_3) &= f_1(x_1, x_2), \quad x_3 = a, \\ \frac{\partial \varphi(x_1, x_2, x_3)}{\partial x_3} &= f_2(x_1, x_2), \quad x_3 = b \end{aligned} \quad (8)$$

Applied to equations and boundary conditions double transformation in  $x_1, x_2$  in form

$$\begin{aligned} \Phi(\alpha_1, \alpha_2, x_3) &= \iint_{-\infty}^{\infty} \varphi(x_1, x_2, x_3) e^{i(\alpha_1 x_1 + \alpha_2 x_2)} dx_1 dx_2, \\ \varphi(x_1, x_2, x_3) &= \frac{1}{4\pi^2} \iint_{-\infty}^{\infty} \Phi(\alpha_1, \alpha_2, x_3) e^{-i(\alpha_1 x_1 + \alpha_2 x_2)} d\alpha_1 d\alpha_2 \end{aligned}$$

Achieve one-dimensional boundary problem

$$\frac{\partial^2 \Phi}{\partial x_3^2} - k^2 \Phi = 0, \quad \Phi(\alpha_1, \alpha_2, a) = F_1, \quad \Phi'(\alpha_1, \alpha_2, b) = F_2, \quad k^2 = \alpha_1^2 + \alpha_2^2 + r^2, \quad (9)$$

Similar  $F_1, F_2$  are obtained Fourier transformation of right parts of boundary conditions For building of boundary problem's solution use exponential substitution and receive solution in the form

$$\Phi(\alpha_1, \alpha_2, x_3) = c_1 e^{kx_3} + c_2 e^{-kx_3}, \quad (10)$$

Complying with the given solution of boundary problem receives constant describing solution as following:

$$c_1 = \frac{F_1 k e^{-kb} + F_2 e^{-ka}}{2kch k(b-a)}, \quad c_2 = \frac{F_1 k e^{kb} - F_2 e^{ka}}{2kch k(b-a)} \quad (11)$$

Inserted value (11) into (10) and made double Fourier inversion following parameters  $\alpha_1, \alpha_2$  receive a solution of boundary problem. Let us apply method of block element in the  $\Omega$  area for solution of this boundary problem. No complicated constructions lead to the functional equation with using of outer formula in form

$$\begin{aligned} (\alpha_3^2 + k^2) \Phi_0(\alpha_1, \alpha_2, \alpha_3) &= \int_a^b d\omega, \quad \omega(x_3, \alpha_3) = \left( \frac{\partial \Phi}{\partial x_3} - i\alpha_3 \Phi \right) e^{i\alpha_3 x_3}. \\ (\alpha_3^2 + k^2) \Phi_0(\alpha_1, \alpha_2, \alpha_3) &= \Phi'(\alpha_1, \alpha_2, b) e^{i\alpha_3 b} - \Phi'(\alpha_1, \alpha_2, a) e^{i\alpha_3 a} - \\ &\quad - i\alpha_3 \Phi(\alpha_1, \alpha_2, b) e^{i\alpha_3 b} + i\alpha_3 \Phi(\alpha_1, \alpha_2, a) e^{i\alpha_3 a} \end{aligned} \quad (12)$$

Inserted in right part (11) given boundary conditions (9) achieve equation as (-)

$$\Phi_0(\alpha_3) = (\alpha_3^2 + k^2)^{-1} [F_2 e^{i\alpha_3 b} - \Phi'(a) e^{i\alpha_3 a} - i\alpha_3 \Phi(b) e^{i\alpha_3 b} + i\alpha_3 F_1 e^{i\alpha_3 a}] \quad (13)$$

Here for formula simplification of function  $\Phi(\alpha_1, \alpha_2, a), \Phi(\alpha_1, \alpha_2, b), \Phi(\alpha_1, \alpha_2, x_3)$ , two first arguments are omitted and all actions are accomplished towards the third one i.e. their notations  $\Phi(a), \Phi(b), \Phi(x_3)$ , are accepted. Considering correlation in local coordinate system and requested accomplishment of automorphism - reflection of segment on yourself, we reach such pseudodifferential equation.

$$\begin{cases} kF_1 e^{-k(b-a)} + F_2 - \Phi'(a) e^{-k(b-a)} - k\Phi(b) = 0, \\ -kF_1 + F_2 e^{-k(b-a)} - \Phi'(a) + k\Phi(b) e^{-k(b-a)} = 0, \end{cases} \quad (14)$$

Their decision gives the follow meanings of sought expressions.

$$\Phi'(a) = \frac{-kF_1 \operatorname{sh}k(b-a) + F_2}{\operatorname{ch}k(b-a)}, \quad \Phi(b) = \frac{kF_1 + F_2 \operatorname{sh}k(b-a)}{k \operatorname{ch}k(b-a)}$$

Inserting these meanings in formula and applied triple access Fourier, we reach presence of decision- block element boundary problem in layer in the form of

$$\begin{aligned} \varphi(x_1, x_2, x_3) &= \frac{1}{8\pi^3} \iiint_{R^3} e^{-i(\alpha \mathbf{x})} (\alpha_3^2 + k^2)^{-1} [ \{ k^2 F_1 e^{i\alpha_3 a} \operatorname{sh}k(b-a) + i\alpha k F_1 \operatorname{ch}k(b-a) e^{i\alpha_3 a} - \\ &\quad - k F_2 e^{i\alpha_3 a} \} + \{ -i\alpha e^{i\alpha_3 b} F_2 \operatorname{sh}k(b-a) + F_2 k \operatorname{ch}k(b-a) e^{i\alpha_3 b} - k F_1 i\alpha e^{i\alpha_3 b} \} ] \\ &\quad [ k \operatorname{ch}k(b-a) ]^{-1} d\alpha_1 d\alpha_2 d\alpha_3, \quad (\alpha \mathbf{x}) = \alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3, \quad a \leq x_3 \leq b \end{aligned} \quad (15)$$

Received expression, representing topological object - manifold with boundary, is called packed block element. Similar packed block elements for other layers can conjugate with each other very simply, if they are in such form. According to topology, it is necessary to make factor - topology and determined functions on them, taking into account equivalence relation of boundary condition. In this simplest case, transit of boundary condition in areas of pasting together of layers is enough. In packed block element, if operations of building are carried out rightly, integral of parameter is always calculated according to theory of residues. Accomplished this act, we get uncovered or unpacked block element, which has form

$$\begin{aligned} \varphi(x_1 x_2 x_3) &= \frac{1}{4\pi^2} \iint_{R^2} e^{-i(\alpha_1 x_1 + \alpha_2 x_2)} \times \\ &\quad \{ (f_1 k e^{-kb} + f_2 e^{-ka}) e^{kx_3} + (f_1 e^{kb} k - f_2 e^{ka}) e^{-kx_3} \} 2k \operatorname{ch}k(b-a) \end{aligned}$$

Decision coincides with decision, built by means of exponential substitutions. As mentioned above, precisely these unpacked block elements were built by means of separation of variables, integral transformations, difference substitutions. They are slightly simpler, than packed block element, containing in it, but, as the follow example show, have more restricted set of boundary tasks, which can be researched by these methods. Below here is comparatively simple example of boundary task, which is accessibly for research by means of block element, and not solvable by listed approaches. Example 2. Learn considering above boundary task for differential equation (1) in domain  $\Omega$ , occupying cylinder in direction of axis  $0x_3$  with triangular section in plane  $x_1 0x_2$ , having tops in point  $(0, 0)$ ,  $(x_{01}, 0)$ ,  $(0, x_{02})$ ,  $x_{01} > 0$ ,  $x_{02} > 0$ . Outer normal line to side  $x_{01}x_{02}$  makes up  $\nu$  with axis  $0x_1$ , but distance of this side from beginning of coordinate is  $p$ . In this case in accepted coordinate system, which we will call absolute, coordinate of top of triangle have expressions

$$x_{10} = \frac{p}{\cos \nu}, x_{20} = \frac{p}{\sin \nu}$$

. Applied to boundary task transformation Fourier on parameter  $x_3$  we get two-dimension boundary task in section  $x_1 0x_2$ . We will consider, that on reference bounders, sides of triangle  $x_{01}0$ ,  $0x_{02}$ ,  $x_{02}x_{01}$ , right local coordinate system of coordinates  $x_1^\pi 0^\pi x_2^\pi$ ,  $x_1^{0,5\pi} 0^{0,5\pi} x_2^{0,5\pi}$ ,  $x_1^{0,5\pi-\nu} 0^{0,5\pi-\nu} x_2^{0,5\pi-\nu}$ , are built, at that coordinate axis with index 1 have direction along the sides, but with index 2 - on outer normal line. The upper indexes in accepted indication characterized degree of rotation of local coordinate system concerning to absolute. On mentioned above sides of triangle, in such consistency, boundary conditions are assigned in form of functions  $g_1(x_1^\pi)$ ,  $g_2(x_1^{0,5\pi})$ ,  $g_3(x_1^{0,5\pi-\nu})$  which are considered sufficiently smooth. Inserting boundary task in topology structure, described above, and accomplished acts, connected with method of block element, as in the first items and example by analogy, we reach to packed element, having form

$$\begin{aligned} \varphi(x_1, x_2, x_3) &= \frac{1}{8\pi^3} \iiint_{R^3} e^{-i(\alpha x)} \frac{i}{(\alpha_1^2 + \alpha_2^2 + k^2)} [ F_1(\alpha_1^\pi)(\alpha_{2-}^\pi - \alpha_2^\pi) + F_2(\alpha_1^{0,5\pi})(\alpha_{2-}^{0,5\pi} - \alpha_2^{0,5\pi}) \\ &\quad + F_3(\alpha_1^{0,5\pi-\nu})e^{i\alpha_2^{0,5\pi-\nu}p}(\alpha_{2-}^{0,5\pi-\nu} - i\alpha_2^{0,5\pi-\nu}) ] d\alpha_1 d\alpha_2 d\alpha_3 \quad (16) \\ \alpha_{2\pm}^\pi &= \pm i\sqrt{(\alpha_1^\pi)^2 + k^2}, \quad \alpha_{2\pm}^{0,5\pi} = \pm i\sqrt{(\alpha_1^{0,5\pi})^2 + k^2}, \quad \alpha_{2\pm}^{0,5\pi-\nu} = \pm i\sqrt{(\alpha_1^{0,5\pi-\nu})^2 + k^2} \\ x_1 &= x_1^{0,5\pi-\nu} \sin \nu + x_2^{0,5\pi-\nu} \cos \nu, \quad x_2 = -x_1^{0,5\pi-\nu} \cos \nu + x_2^{0,5\pi-\nu} \sin \nu \\ \alpha_1 &= \alpha_1^{0,5\pi-\nu} \sin \nu + \alpha_2^{0,5\pi-\nu} \cos \nu, \quad \alpha_2 = -\alpha_1^{0,5\pi-\nu} \cos \nu + \alpha_2^{0,5\pi-\nu} \sin \nu, \\ \alpha_1^{0,5\pi-\nu} &= \alpha_1 \sin \nu - \alpha_2 \cos \nu, \quad \alpha_2^{0,5\pi-\nu} = \alpha_1 \cos \nu + \alpha_2 \sin \nu \\ x_1^{0,5\pi} &= x_2, \quad x_2^{0,5\pi} = -x_1, \quad x_1^\pi = -x_1, \quad x_2^\pi = -x_2 \\ \alpha_1^{0,5\pi} &= \alpha_2, \quad \alpha_2^{0,5\pi} = -\alpha_1, \quad \alpha_1^\pi = -\alpha_1, \quad \alpha_2^\pi = -\alpha_2 \end{aligned}$$

Here vector  $\{ F_1(\alpha_1^\pi), F_2(\alpha_1^{0,5\pi}), F_3(\alpha_1^{0,5\pi-\nu}) \}$  is result of some linear continuous reflection of vector  $\{ g_1(x_1^\pi), g_2(x_1^{0,5\pi}), g_3(x_1^{0,5\pi-\nu}) \}$  in space of transformation Fourier on basis vector. Unpacked block element by means of calculation one integral, it is possible in rightly packed block element, we reach to uncovered block

element, which is presented in form

$$\begin{aligned} \varphi(x_1, x_2, x_3) = \frac{1}{4\pi^2} \iint_{R^2} \langle [ F_2(\alpha_1^{0,5\pi}) \exp [ -i(\alpha_1^{0,5\pi} x_1^{0,5\pi} + \alpha_{2+}^{0,5\pi} x_2^{0,5\pi} + \alpha_3 x_3) ] d\alpha_1^{0,5\pi} - \\ - F_1(\alpha_1^\pi) \exp [ -i(\alpha_1^\pi x_1^\pi + \alpha_{2+}^\pi x_2^\pi + \alpha_3 x_3) ] d\alpha_1^\pi - \\ - F_3(\alpha_1^{0,5\pi-\nu}) \exp [ -i(\alpha_1^{0,5\pi-\nu} x_1^{0,5\pi-\nu} + \alpha_{2+}^{0,5\pi-\nu} x_2^{0,5\pi-\nu} + \alpha_3 x_3) ] d\alpha_1^{0,5\pi-\nu} \rangle d\alpha_3 \end{aligned} \quad (17)$$

By calculating of integrals, it is necessary to claim the same point  $(x_1, x_2, x_3)$ , which situates in inside of triangle, to examine and Jacobean determinants by substitution of variable in integrals to take into account. The authors don't know works, in which decision of mentioned boundary task was accomplished by means of separation of variables or other analytical approach.

## 4 Conclusion

The difference between packed and uncovered block element consists, of that in packed form block element, which is capable boundary tasks, not yielding to other methods, can uniformly conjugate with other block elements of such or other dimensions and build packed block family, keeping singly topology structure. In his turn unpacked block element gives pictorial presentation about decision's character and allow learning its properties, to pick out features of localization's zone. In particular because of enumerated above, it was succeed in building of model of starting earthquake, revealed dangerous zones of stress concentration [7], [8].

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## References

- [1] Gelfand I.M., Graev M.I., Pyatetskiy- Shapiro I.I. "Representation theory and automorphic functions". M. Nauka. 1698. 512 p.
- [2] Vilenkin N. Ya. YSpecial functions and representation theory of groupsY. M. Nauka, 1991. 576p.
- [3] Babeshko V.A., Evdokimova O.V, Babeshko O.M., Gorshkova E.M., Zaret-skaya A.V., Muhin A.S., Pavlova A.V. "Convergent Properties of Block Elements"/// Doklady Physics. - 2015. - V. 60. - No.11 . - P. 515-518. DOI: 10.1134S1028335815110099
- [4] Babeshko V.A., Evdokimova O.V., Babeshko O.M. YAbout block element in applicationsY.// Physical mesomechanics. 2012. T.15.ε1. C. 95-103.
- [5] Kurant R. YControl with partial derivativeY M.Mir, 1964.832p.

## REFERENCES

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- [6] Tihonov A.N., Samarskiy A.A. *Equations of mathematical physics*. M. Nauka, 1996. 724p.
- [7] Babeshko V.A., Evdokimova O.V., Babeshko O.M. *The Problem of Physical and Mechanical Precursors of an Earthquake: Place, Time, Intensity*/// *Doklady Physics*. - 2016. - V. 61. - No.2 . - P. 92-97. DOI: 10.1134/S1028335816020099
- [8] Babeshko B.A., Evdokimova O.V., Babeshko O.M. *Topological methods in the problem of earthquake forecast*/// *Ecological Bulletin of Scientific centers BSEC Cooperation*. 2015,  $\epsilon$ 2, C. 8-13.

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