Deformation and divergence of the moving beams made from thermoelastic materials

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Abstract

The problem of stability and out-of-plane deformation analysis is considered for an axially moving elastic web modelled as a beam (undergoing cylindrical deformation). The beam is under homogeneous pure mechanical in-plane tension and thermal strains corresponding to the thermal tension and bending. In accordance with the static approach of stability analysis the problem of out-of-plane thermomechanical divergence (buckling) is reduced to an eigenvalue problem which is analytically solved. This problem corresponds to the case of in-plane thermomechanical tension and zero thermal bending. The general case of deformations induced by combined thermomechanical bending and tension is reduced to nonhomogeneous boundary-value problem and analyzed with the help of Fourier series.

1 Introduction

The study of the mechanical behavior of axially moving elastic systems has attracted the attention of researchers for a long time beginning with Skutch [1]. Studies in this direction written in the English language began to appear half a century later starting with those by Sack [2], Archibald and Emslie [3], Miranker [4]. Other classic studies of moving elastic systems include those by Mote [5] - [8], Thurman and Mote [9], Simpson [10], Mujumdar and Douglas [11], Pramila [12], [13], Wickert and Mote [14]. Recent studies include Wang et al. [15], Banichuk et al. [16] - [18]. Wide exposition of obtained results in domain of mechanics of moving materials is presented in the books by Marynowski [19] and Banichuk et al. [20]. An extensive literature review can also be found in these books.

2 Basic relations

Let us consider a beam which occupies the domain ($0 \le x \le l$, $-h/2 \le z \le h/2$, $0 \le y \le b = 1 \ll l$) in the rectangular coordinate system xyz and moves axially at a constant transport velocity V_0 in the x-direction. The transverse (out-of-plane) displacement is described by the function $w = w(x,t)$. By considering the wave equation in the co-moving coordinate and using transformations from the

Lagrange derivatives to the Euler derivatives of transverse displacements we can write the equation for small transverse vibrations in the following form [20]:

$$m\frac{d^2w}{dt^2} = \mathcal{L}^M(w) - \mathcal{L}^B(w), \tag{1}$$

where m is the mass per unit length of the beam and

$$\mathcal{L}^M(w) = T\frac{\partial^2 w}{\partial x^2} = \left(T_0 - \frac{Eh}{1-\nu}\varepsilon_\theta\right)\frac{\partial^2 w}{\partial x^2}, \tag{2}$$

$$\mathcal{L}^B(w) = -\frac{\partial^2 M}{\partial x^2} = D\left[\frac{\partial^4 w}{\partial x^4} + (1+\nu)\frac{\partial^2 \kappa_\theta}{\partial x^2}\right]. \tag{3}$$

Here $\mathcal{L}^M$, $\mathcal{L}^B$ are operators on w, $T_x = T$ is the in-plane tension, E is the Young modulus, ν is the Poisson ratio, M is the bending moment, D is the bending rigidity, h is the thickness of the beam. The quantities ε_θ and κ_θ are the generalized thermal strains corresponding to the thermal tension and bending of the beam defined as

$$\varepsilon_\theta = \frac{1}{h}\int_{-h/2}^{h/2}\alpha_\theta\theta dz, \quad \kappa_\theta = \frac{12}{h^3}\int_{-h/2}^{h/2}\alpha_\theta\theta z dz, \tag{4}$$

where α_θ is the coefficient of linear thermal expansion.
In the stationary case the behavior equation takes the form

$$\left(mV_0^2 + \frac{Eh}{1-\nu}\varepsilon_\theta - T_0\right)\frac{d^2w}{dx^2} + D\left[\frac{d^4w}{dx^4} + (1+\nu)\frac{d^2\kappa_\theta}{dx^2}\right] = 0 \tag{5}$$

with simply supported boundary conditions

$$w = 0, \quad \frac{d^2w}{dx^2} + (1+\nu)\kappa_\theta = 0, \quad x = 0, l. \tag{6}$$

3 Divergence problem

Suppose that

$$T = T_0 - \frac{Eh}{1-\nu}\varepsilon_\theta \tag{7}$$

and $\kappa_\theta = 0$. Then, using (5) with dimensionless variables $\tilde{x} = x/l$ (the tilde will be omitted) and introducing the auxiliary function

$$\psi(x) = \frac{d^2w}{dx^2}, \quad 0 \le x \le 1 \tag{8}$$

and the quantity

$$\lambda = \frac{l^2}{D}\left(mV_0^2 + \frac{Eh}{1-\nu}\varepsilon_\theta - T_0\right) \tag{9}$$

we formulate the eigenvalue problem

$$\frac{d^2\psi}{dx^2} + \lambda\psi = 0, \quad \psi(0) = 0, \quad \psi(1) = 0. \tag{10}$$

Integrating the behavior equations and taking into account the formulated boundary conditions we find the unknown shape function $w = C\sin(j\pi x)$, $j = 1, 2, 3, ...$ (C - arbitrary constant) and the correspondent critical velocity

$$\left(V_0^{div}\right)_j^2 = \frac{j^2\pi^2 D}{ml^2} + \frac{T_0}{m} - \frac{Eh}{m(1-\nu)}\varepsilon_\theta. \tag{11}$$

Thus, we observe that the shape of the eigenmode coincides with the membrane (with $D = 0$ and $\kappa_\theta = 0$) eigenmode regardless of the value of the bending rigidity D and the thermal strain ε_θ, but the bending rigidity and thermal strain contribute additional terms to the divergence speed.

4 Deformations under combined thermomechanical bending and tension

Using dimensionless coordinate x ($0 \le x \le 1$) and assuming that $\kappa_\theta = const \ne 0$, $\varepsilon_\theta = const$ we consider an axially moving beam subjected to tension and bending and represent the behavior equation in the form

$$\frac{d^2}{dx^2}\left[\frac{d^2 w}{dx^2} + \lambda w + (1+\nu)\,\kappa_\theta\right] = 0, \quad 0 \le x \le 1, \tag{12}$$

where λ is given by (9). Integrating (12) with boundary conditions (6) we obtain the second order differential equation for the displacement function:

$$\frac{d^2 w}{dx^2} + \lambda w + (1+\nu)\,\kappa_\theta = 0, \quad 0 \le x \le 1. \tag{13}$$

To solve this equation we represent the desired solution $w(x)$ and the quantity $(1+\nu)\kappa_\theta$ as the Fourier series

$$w = \sum_{n=1,3,...}^{\infty} a_n \sin n\pi x, \tag{14}$$

$$(1+\nu)\kappa_\theta = \frac{4(1+\nu)\kappa_\theta}{\pi} \sum_{n=1,3,...}^{\infty} \frac{1}{n}\sin n\pi x. \tag{15}$$

Substituting (14), (15) into (13) and perfprming elementary operations we obtain the equation

$$\sum_{n=1,3,...}^{\infty} \sin n\pi x \left\{ a_n\left[\lambda - (n\pi)^2\right] + \frac{4(1+\nu)}{n\pi}\kappa_\theta \right\} = 0 \tag{16}$$

leading to the expressions for unknown coefficients

$$a_n = \frac{4(1+\nu)\kappa_\theta}{n\pi\left[(n\pi)^2 - \lambda\right]}, \quad n = 1, 3, ... \tag{17}$$

5 Conclusions

In this paper the deformation and stability of an axially moving elastic panel were considered. The panel was travelling at a constant velocity between the rollers modelled by the corresponding boundary conditions. Small transverse elastic displacements of the panel were described by a fourth-order differential equation that includes the terms expressing the action of in-plane tension and out-of-plane centrifugal forces and the terms taking into account a thermomechanical action. We investigated separately two important cases: the divergence of the panel loaded by in-plane thermomechanical tension and the deformation of the beam subjected to combined thermomechanical bending and tension. In both cases the considered problems have been studied analytically and the solutions have been found in an explicit form.

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