Mathematical and Experimental Simulation of the Ascending Twisting Flows

S.P. Bautin, I.Yu. Krutova, A.G. Obukhov
IYKrutova@mephi.ru

Abstract

The results of theoretical and experimental studies of the ascending twisting flows encountered in nature in the form of tornadoes and tropical cyclones [1-3] are represented.

Theorems on the existence and the uniqueness of the solutions to specific initial-boundary value problems that, in particular, set the rotation direction of tornadoes, tropical cyclones and fire vortices are proved for the system of gas dynamics equations and the complete system of Navier-Stokes equations.

There are the constructed numerically the solutions of indicated systems of partial differential equations that model the gas flow from the simple planar spiral currents to the three-dimensional nonstationary flows in general. The calculation results are consistent with both the data of natural observations and the results of laboratory experiments.

Results of laboratory experiments on the vertical upwards motion of air along the pipe, which confirmed the occurrence of the twisting in the near-bottom and the vertical parts of the flow in the appropriate direction are given in this investigations.

The results of theoretical and experimental studies of the ascending twisting flows encountered in nature in the form of tornadoes and tropical cyclones [1-3] are proven in this research.

We consider the complete system of Navier-Stokes equations in dimensionless variables under the action of gravity and Coriolis forses [3, 4]:

$$\begin{cases} \rho_t + \mathbf{V}\cdot\nabla\rho + \rho \operatorname{div}\mathbf{V} = 0, \\ \mathbf{V}_t + (\mathbf{V}\cdot\nabla)\ \mathbf{V} + \frac{T}{\gamma\rho}\nabla\rho + \frac{1}{\gamma}\nabla T = \mathbf{g} - 2\mathbf{\Omega}\times\mathbf{V} + \\ +\frac{\mu_0}{\rho}\left[\frac{1}{4}\nabla\left(\operatorname{div}\mathbf{V}\right) + \frac{3}{4}\Delta\mathbf{V}\right], \\ T_t + \mathbf{V}\cdot\nabla T + (\gamma-1)T\operatorname{div}\mathbf{V} = \frac{\varkappa_0}{\rho}\Delta T + \\ +\frac{\mu_0\gamma(\gamma-1)}{2\rho}\left\{\left[(u_x-v_y)^2 + (u_x-w_z)^2 + (v_y-w_z)^2\right] + \right. \\ \left. +\frac{3}{2}\left[(u_y+v_x)^2 + (u_z+w_x)^2 + (v_z+w_y)^2\right]\right\}, \end{cases} \qquad (1)$$

Which for the case of zero viscosity and vanishing thermal conductivity: $\mu_0 = \varkappa_0 = 0$ – the system (1) is the system of gas dynamics equations.

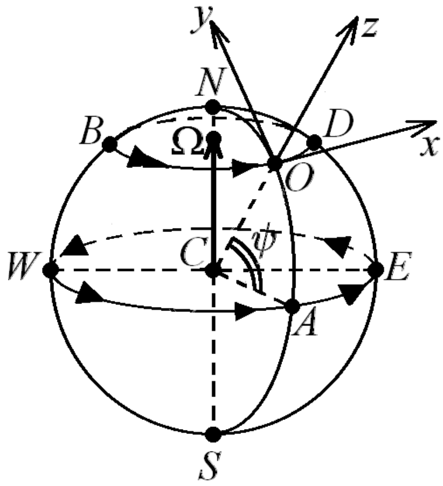

Fig. 1.

The symbols in system (1) are the following: ρ – is the density; u, v, w – are the projections of the velocity vector $\mathbf{V}$ on the axis of Cartesian coordinate system $Oxyz$ (see Fig.1) rotating together with the Earth and the beginning of which lies on the surface of the Earth at the point O on the parallels with the latitude ψ; T – is the temperature; $\mathbf{g}$ – is the acceleration vector of gravity; $a = 2\Omega \sin\psi$; $b = 2\Omega\cos\psi$; Ω – is the modulus of the angular velocity vector $\boldsymbol{\Omega}$ of the Earth's rotation.

The theorem on the smooth flow into the vertical cylinder. For the system of gas dynamics equations there are the specified conditions describing the initial time of the uniform, resting outside the cylinder $\sqrt{x^2+y^2} = r_0$, $r_0 > 0$ gas. Also the smooth radial flow into the cylinder are given. Then this problem has a unique solution in the neighbourhood of the given point $(t = 0, r = r_0, \varphi = \varphi_0, z = 0)$. Here r, φ – are the polar coordinates in the plane xOy.

The properties of the solution of this problem that since the time $t = 0$ in the gas flow occurs the twisting directed in the positive direction in the case of the Northern Hemisphere and the negative direction in the case of the Southern Hemisphere. This direction of rotation corresponds to the direction of air like tornadoes and tropical cyclones. If for the problem $\Omega = 0$, so the twisting does not arize.

The theorem on the smooth heating of the vertical cylinder. For the system of gas dynamics equations there are the specified conditions describing the initial time of the uniform, resting outside the cylinder $\sqrt{x^2+y^2} = r_0$, $r_0 > 0$ gas. Also the smooth heating of this cylinder are given. Then this problem has a unique solution in the neighbourhood of the given point $(t = 0, r = r_0, \varphi = \varphi_0, z = 0)$.

The properties of the solution of this problem that since the time $t = 0$ the twisting directed in the negative direction in the case of the Northern Hemisphere and the positive one in the case of the Southern Hemisphere appears in the gas flow. The air around the fire vortices has such direction of twisting. If for the problem $\Omega = 0$, the twisting in the flow is absent.

The paper provides examples of numerical construction of flows under the action of gravity and Coriolis forces for the system (1) as in the case $\mu_0 = \varkappa_0 = 0$, as well, in the case $\mu_0 \neq 0$, $\varkappa_0 \neq 0$. They are constructed from the simple planar spiral flows in the bottom parts of the being investigated flows to the three-dimensional stationary and notstationary flows in general. In numerical simulation time is set from the beginning of a natural atmospheric vortex before its release to stationary state. The calculation results are consistent with both the data of natural observations of tornadoes and tropical cyclones of varying intensity, and the results of laboratory experiments.

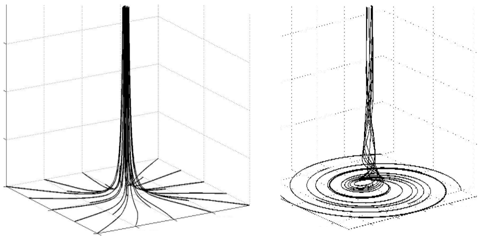

Fig. 2.

The results of laboratory experiments on the vertical upwards motion of air along the pipe, which confirmed the occurrence of twisting in the near-bottom and the vertical parts of the flow in the appropriate direction are represented. Fig. 2 shows the photograph of the room in which the experiment was conducted.

Fig. 3.

Fig. 3 shows the instantaneous streamlines constructed at different times in the simulation flow of the experiment with blowing.

References

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S. P. Bautin, Ural State University of Railway Transport, Yekaterinburg, Russia
I. Yu. Krutova, National Research Nuclear University MEPhI, Moscow, Russia
A.G. Obukhov, Tyumen State University of Oil and Gas, Tyumen, Russia