Pseudo 3D hydraulic fracturing model with account for vertical viscous dissipation

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Abstract

A great number of hydraulic fracturing (HF) models exists which are differentiated with complexity and physical accuracy. Most straightforward 2D models (PKN, KGD, radial models) are reasonable while express analysis or first estimations fulfilling only as they are restricted with geometrical shapes significantly and consequently are not of interest in HF design treatment. At the same time, fully 3D models or planar 3D models are most accurate ones from physical point of view but cause great times costs in numerical simulation.

Pseudo 3D (P3D) models play a role of compromise between two previously examined cases. Key results of P3D model fracture growth in multi-layered lithology with focus on proppant transport mechanism and tip-screen out are examined in [1] . To reduce the consequences of assumptions various efforts to enhance P3D models are made ([2], [3], [4], [5], [6]). In particular, pressure profile is assumed to be one-dimensional (along the fracture growth direction) within the P3D model. Consequently, the fluid flow in lateral direction is not examined, viscous dissipation is ignored in this direction and it turns into fracture height overestimating (toughness regime is observed only). To enhance the model authors in [5] account for viscous dissipation via so-called apparent fracture toughness that depends on propagation velocity on both lower and upper fracture tips. However, the case of symmetric three-layers lithology is investigated only.

The purpose in the present project is to generalize the concept of apparent fracture toughness for multi-layered lithology with arbitrary properties. Numerical results present significant difference in fracture width profile between classic P3D model and enhanced one (about 10 % of difference can be achieved). The enhancement is demonstrated in the frame of the problem of unwanted breakthrough layers that may lead to water or gas coning breakthrough.

1 Introduction

Enhanced pseudo-3D (EP3D) hydraulic fracture model is based on cell-based P3D model with non-equilibrium height growth. EP3D model aims to account for viscous dissipation in vertical growth which is essential as break-through into high permeability layers takes place. Cell-based P3D model accounts for height growth mechanism in multi-layered lithology. Its enhanced modification represents the fracture as a series of connected cells with a plane strain and with account for excessive pressure inside each cell. The fracture opening profile and height are calculated analytically for the given piece-wise lithology (stress intensity, YoungвҐЄs modulus, PoissonвҐЄs ratio and fracture toughness). Initial growth state of each cell is examined via PKN model while its further growth takes place in accordance with P3D model.

2 Non-equilibrium height assessment

In the presence of significant vertical fluid flow within the fracture (accompanied with instant breakthrough into neighboring layers) the equilibrium height model is not reasonable. To bypass this problem the non-equilibrium growth model is applied: the excessive pressure is considered to depend on fracture tip velocity. The relation for this dependence is obtained at [7] and is given as follows

$$P_{net} = E' \frac{2\sqrt{2}(2+n)}{\pi(2-n)} \left[\frac{K}{nE'h^n} \left(\frac{cos[(1-\beta)\pi]}{sin(\beta\pi)} \right)^{n+1} \left(\frac{2n+1}{n(2+n)} \right)^n \right]^{1/(2+n)} v_{\pm}^{n/(2+n)}, \tag{1}$$

where u_{tip} - is a fluid velocity at the fracture tip, n and K are coefficients of power-law rheology, $\beta = 2/(2+n)$ is an auxiliary coefficient, h is a fracture height, $E' = E/(1-\nu^2)$, E is Young modulus, is Poisson coefficient.
The relationship governing net pressure P_{net} and stress intensity factor K_{Ic} is the following [8]

$$P_{net} = \frac{K_{Ic}}{\sqrt{\pi h/2}}. \tag{2}$$

The stress intensity coefficients at upper and lower fracture of tips $K_{I_{u/l}}$ are the following

$$K_{I_{u/l}} = \sqrt{\frac{\pi h}{2}}(p-\sigma_n+\rho_f g(h-\frac{h}{2}\pm\frac{h}{4}))+\sqrt{\frac{2}{\pi h}}\sum_{i=1}^{n-1}(\sigma_{i+1}-\sigma_i)(\frac{h}{2}\arccos(\frac{h-2h_i}{h})\pm\sqrt{h_i(h-h_i)}) \tag{3}$$

where p is a pressure averaged along the fracture, h is a distance between lower fracture tip and its center, h_i is a distance between lower fracture tip and upper boundary if the i-th layer, σ_n and σ_i are stress at upper fracture tip and stress at i-th layer, respectively.
The workflow for non-equilibrium height assessment includes the following steps:

1. to estimate zero-order magnitude of both lower and upper fracture tip velocities for a given cell;
2. to find new positions of both lower and upper fracture tips for a given time step using received velocities;
3. using relations (3) to estimate stress intensity factors (SIF) at fracture tips with account for fracture toughness at those layers which are intersected with fracture tips;
4. using relations (1) and (2) to define such tip velocities that give us the values of SIF obtained at the previous step;
5. to accept received velocities and to repeat the whole workflow till the stability is respected.

As a result both lower and upper fracture tip velocities are estimated. Further new positions of grid nodes are estimated that define a new non-equilibrium height.
To get the fracture growth dynamics in addition t geo-mechanical relations one is to examine non-newtoninan liquid flow inside the fracture channel. The local conservation of fluid flow along the fracture channel with continuous cross section $A(x, t)$ is defined as follows

$$\frac{\partial A}{\partial t} + \frac{\partial A v}{\partial x} + q_l = 0,$$

where q_l is Carter's losses which is defined as follows [9]

$$q_l = \frac{2\widetilde{C}_l h}{\sqrt{t - \tau(x)}}$$

where τ is the time at which leak-off velocity was exposed, $\widetilde{C}_l$ is an effective leak-off coefficient, defined as weighted for all lithology layers

$$\widetilde{C}_l = \frac{1}{h_f} \int C_l dz.$$

The system is closed with following initial (initial zero opening profile) and boundary conditions (flow rate at the inlet and no flow condition at the fracture tip)

$$w(x, t = 0) = 0, \qquad q(x = 0, t) = q_0(t), \qquad q(x = L, t) = 0.$$

3 Numerical results

The system of equations is solved numerically using explicit gradient iteration scheme based on Newton's method. At each time step the hydraulics solution in iterative manner converges to the truth one by updating geo-mechanical parameters. This approach ensures the growth of numerical stability and reasonable linear convergence of numerical solution. Besides, the time step is adjusted in such a way the time cost of calculation keeps reasonable.

The strategy of space and time grid construction is chosen as follows. The number of grid points in vertical direction is fixed, its meshing is uniform. As for the lateral direction grid the idea is based on auto-growing calculation domain. At initial time the domain represents three sequential cells: well cell, fracture cell, tip cell. In the first and last cells the boundary conditions for slurry rate and zero flow rate are set, respectively. A new fracture cell is being added if the fracture opening at this cell width exceeds a certain level. Spatial grid cells are situated in the region where the fracture physically exists only while in most cases the calculation domain is pre-defined and consequently unwanted cells are involved. The time grid is non-uniform. The time derivative is approximated by the implicit first order Euler scheme while the fluxes at the cell faces are approximated with backward scheme. In addition, the fracture geometry changing is defined in accordance with criteria conditions for its growth.

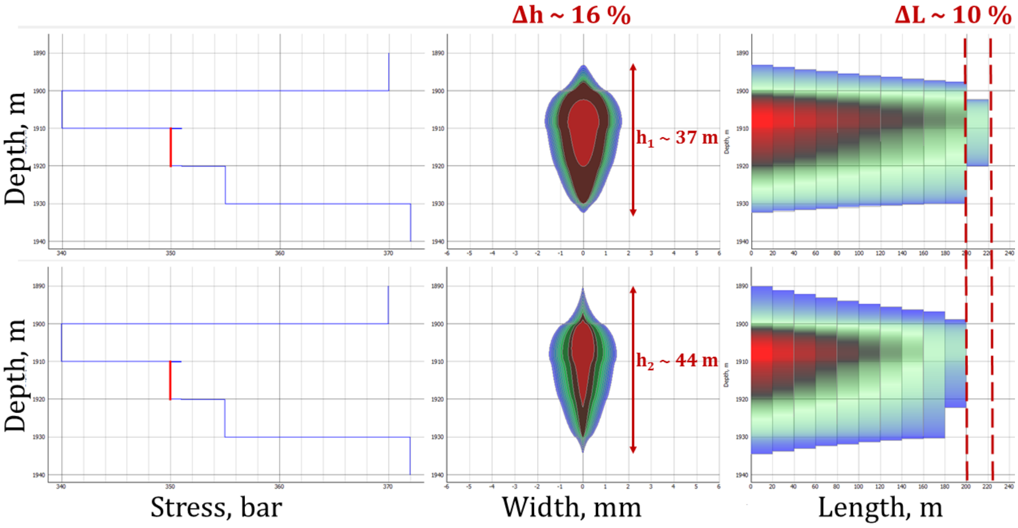

Figure 1: Lithology, fracture width profile, side view: upper picture - non-equilibrium height growth, lower picture - equilibrium height growth

On the picture above two cases are examined: equilibrium height growth and non-equilibrium one with the same inflow regime, liquid properties and lithology ($T = 10\ min.$ - pumping period, $q = 3.5 m^3/min.$– pump rate, K and n are 0.05 and 0.6, respectively). The set of parameters for the cases is $H = 10m$ for the height of reservoir layer, $\mu = 0.05 Pa \cdot s$ for the fluid viscosity, $\nu = 0.3$ for the Poisson's ratio, $E = 30GPa$ for the value of Young's module and $K_{Ic} = 0.5MPa$ for the fracture toughness. The Carter's leak-off coefficient is the same for all layers, $C_l = 10^{-5}\ m \cdot s^{-0.5}$.

Neglecting viscous regime dissipation in vertical growth makes the height overestimated by 16 % while the fracture length is underestimated by 10 %. The account for viscous dissipation in lateral growth is vitally crucial in the context of breakthrough problems (in particular, water or gas coning).

4 Discussion

The solution for joint problem of fracture growth within multi-layered lithology and non-newtonian flow inside is developed. Enhanced cell-based pseudo-3D model is applied as it accounts for fracture height growth with viscous dissipation and represents relatively reasonable agreement between the complexity and accuracy.
Sequentially, we fulfilled benchmarking analysis: comparing equilibrium and non-equilibrium height growth models. As it turns out a significant deviation between two examined cases is observed (of order 10 %). Thereby, the risks of water or gas breakthrough may be over-estimated significanty if neglecting with viscous-dissipation in vertical growth.

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