Waves with the negative group velocity in cylindrical shell of Kirchhoff - Love type

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Abstract

Cylindrical shells of different types are the often used models in modern engineering. It is the element of different line tubes, supports, oil rigs and so on. The problems of preventing from damaging of such the constructions, reducing the vibrations of them are the actual problems of modern technique. The exact calculation of such objects from one side needs great computational resources and from another side often mask some important effects. For example the effects of propagating of the waves with negative group velocity better to analyze on the simplest mechanical models which have the exact analytical solution. In report such the analysis is fulfilled on example of infinite thin cylindrical shell of Kirchhoff - Love type. The problem of free oscillations of such the shell is considered. The statement of the problem is considered in the rigorous statement. The dispersion equation is found on the base of exact analytical solution. The propagating waves are analyzed. The exploration of waves with negative group velocity in the neighborhood of bifurcation point of dispersion curves is fulfilled. The analysis of arising effects is fulfilled in terms of kinematic and dynamic variables, and in the terms of energy flux. The relative advantages and disadvantages of these approaches are discussed. The comparison of contributions in the integral energy flux of various mechanisms of energy transmission in the shell is fulfilled. The dependence of subzero energy flux, dynamic and kinematic variables on the relative thickness of the shell, the mode number and other parameters of system is discussed. The possible fields of applicability of the gained effects are established.

1 Statement of the problem

The problem of oscillations of the systems containing cylinder shells is one of the actual problems of modern techniques. It is important to estimate the parameters of vibrations and acoustical fields of such objects in order to provide the construction from damaging, but calculation of these complicated systems demands major computing resources. Therefore the consideration of simple model problems which have exact analytical solution ([1] - [5]) is actual. On these models it is possible to analytically explore main effects and also to use them as the test problems for computing packages.

Let us start considering an infinite cylindrical shell of Kirchhoff–Love type in the cylindrical system of coordinates where the axis $0z$ coincides with axis of the cylinder. The source of an acoustic field in a wave guide is the vibrations of the cylinder shell, caused by the incident wave propagating from the infinite part of the shell. The frequency of this incident harmonic wave is equal to ω. All processes in the shell are supposed to be harmonic with this frequency. The factor $e^{-i\omega t}$ describes the time-dependence and is omitted.

The balance of forces acting on the shell has a view [6]

$$\mathbf{L_w u} = (\mathbf{0}, \mathbf{0}, \mathbf{0})^{\mathbf{t}}. \tag{1}$$

Here following notations are introduced: $\mathbf{u}(\text{'}, \mathbf{z}) = (\mathbf{u_t}, \mathbf{u_z}, \mathbf{u_n})^{\mathbf{t}}$ is the displacement vector of the shell (t is a badge of transposing), $\mathbf{L_w}$ is matrix differential operator of the cylindrical shell of Kirchhoff–Love type

$$\mathbf{L_w} \equiv [L_{ij}] = w^2\mathbf{I} + \mathbf{L}; \quad i, j = 1, 2, 3$$

$$\mathbf{L} = \begin{pmatrix} \alpha_1[\partial_\varphi + \nu_- \widetilde{\partial}_z^2] & \nu_+ \widetilde{\partial}_z \partial_\varphi & \partial_\varphi(1 + 2\alpha^2[1 - \partial_\varphi^2 - \widetilde{\partial}_z^2]) \\ L_{21} & \nu_- \partial_\varphi^2 + \widetilde{\partial}_z^{\,2} & \nu \widetilde{\partial}_z \\ L_{31} & L_{32} & \alpha^2(2\partial_\varphi^2 - 2 + 2\nu \widetilde{\partial}_z^{\,2} - [\partial_\varphi^2 + \widetilde{\partial}_z^{\,2}]^2) - 1 \end{pmatrix} \tag{2}$$

Here $L_{21} = L_{22}$, $L_{31} = -L_{13}$, $L_{32} = -L_{23}$, $\widetilde{\partial}_z = R\partial_z$, $\alpha_1 = 1 + 4\alpha^2$, $\nu_\pm = (1 \pm \nu)/2$, $\mathbf{I}$ is the unit matrix operator.

The following geometrical parameters of the shell are used: R — radius, h — thickness.

Properties of a material of the cylinder are characterized by E, ν and ρ_s - Joung's module, Poisson coefficient and volumetric density accordingly.

The surface density of the shell $\widetilde{\rho}$ ($\widetilde{\rho} = \rho_s h$) and the velocity of median surface deformation waves of the cylindrical shell c_s are introduced $c_s = \sqrt{E/((1 - \nu^2)\rho_s)}$.

The following dimensionless parameters are put in: $\alpha^2 = \frac{1}{12}(\frac{h}{R})^2$ (the relative thickness of the cylindrical shell) and $w = \omega R / c_s$ (the dimensionless frequency).

2 Determination of the general representation of vibrational field

The solution of the equation (1) is searching in the form

$$\begin{pmatrix} u_t \\ u_z \\ u_n \end{pmatrix} = Ae^{i\lambda z} \begin{pmatrix} \zeta \sin(m\varphi) \\ \xi \cos(m\varphi) \\ \gamma \cos(m\varphi) \end{pmatrix}, \tag{3}$$

believing that $|\zeta|^2 + |\xi|^2 + |\gamma|^2 = 1$. Here following notations are introduced: A, ζ, ξ, γ are arbitrary constants, λ is the the wavenumber which we are looking for.

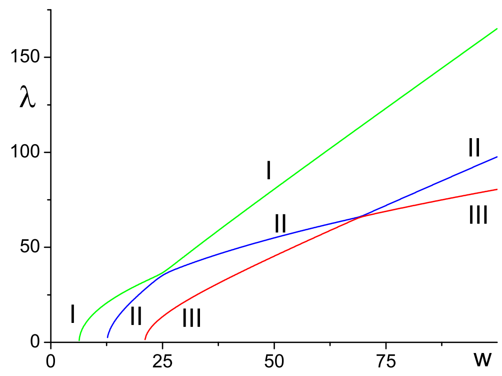

Figure 1: Dispersion curves ($m = 21$).

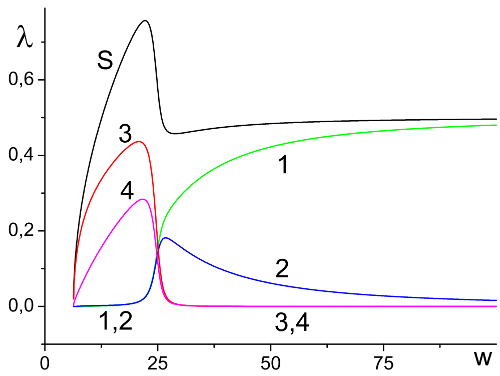

Figure 2: Energy fluxes for the waves from the first dispersion curve ($m = 21$).

After substituting (3) into (1) the following algebraic system is obtained

$$\widehat{\mathbf{L}}_w \mathbf{x} \equiv \left(w^2\mathbf{I} + \widehat{\mathbf{L}}\right)\mathbf{x} = \mathbf{0}; \quad \mathbf{x} = (\zeta, \xi, \gamma)^t \tag{4}$$

Operator $\widehat{\mathbf{L}}_w$ is the Fourier image of operator $\mathbf{L}_w$. The dispersion equation is obtained from the condition of existence of nontrivial solution of this system

$$\det \widehat{\mathbf{L}}_w = 0. \tag{5}$$

We are looking for the real positive solutions of this equation [7] - [8]. If the corresponding set of wavenumbers is founded one can solve the equation (4) and define the previously unknown constants ζ, ξ, γ. After defining constants, the complete solution of the problem in terms of displacements of the shell $\mathbf{u}(\text{'}, \mathbf{z})$ is determined. For the cylindrical shell of Kirchhoff–Love type this equation has three real positive roots $w_i^2 = w_i^2(\lambda^2)$, $i = 1, 2, 3$ which determine three dispersion curves $w_i(\lambda) := \sqrt{w_i^2(\lambda^2)} \geq 0$, (curves 1, 2, 3 in Fig. 1). When $\lambda = 0$ the points of these curves are designated as $w_i^0 = w_i(0)$, $i = 1, 2, 3$, $w_1^0 \leq w_2^0 < w_3^0$, where $w_2^0 = m\sqrt{\nu_-}$;
For certain combination of parameters points w_1^0 and w_2^0 can be coincided (in this case their abscises is equal to $w^0 \equiv m\sqrt{\nu_-}$) [5]. This point will be called the bifurcation point for convenience.

3 Energy streams in the shell

As it was mentioned above all processes in the shell are supposed to be harmonic with frequency ω. It is convenient to average the energy streams on period of oscillations $T = 2\pi/\omega$. The integral stream of the energy along axes z through the cross-section of the cylinder shell has a view

$$\Pi = \frac{\omega}{2}\int\limits_0^{2\pi} \operatorname{Im}\left(\mathbf{u^4}, \mathbf{Fu^4}\right)_{\mathbf{C}^4} R d\varphi = \Pi_t + \Pi_z + \Pi_n + \Pi_m, \tag{6}$$

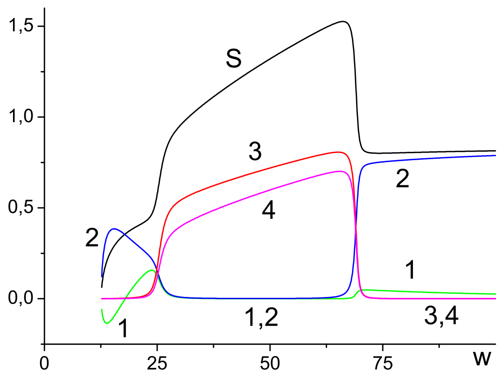

Figure 3: Energy fluxes for the waves from the second dispersion curve ($m = 21$).

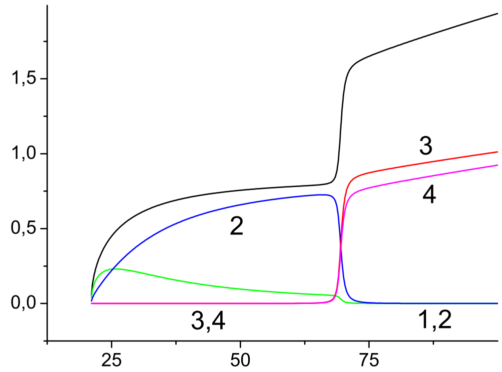

Figure 4: Energy fluxes for the waves from the third dispersion curve ($m = 21$).

$$\left\{\begin{array}{c}\Pi_t\\ \Pi_z\\ \Pi_n\\ \Pi_p\end{array}\right\} = \pi\rho c_s^2\frac{\omega}{2}\,\mathrm{Im}\left\{\begin{array}{c}(-\alpha_1\nu_-\widetilde{\partial}_z u_t - \nu_-\widetilde{\partial}_\varphi u_z + 2\alpha^2(1-\nu)\partial_\varphi\widetilde{\partial}_z u_n)\overline{u}_t\\ (-\nu\partial_\varphi u_t - \widetilde{\partial}_z u_z - \nu u_n)\overline{u}_z\\ \overline{u}_n\alpha^2(-2\partial_\varphi\widetilde{\partial}_z u_t + ((2-\nu)\partial_\varphi^2 - \nu + \widetilde{\partial}_z^2)\widetilde{\partial}_z u_n)\\ \alpha^2(-2\nu\partial_\varphi u_t + \nu(\partial_\varphi^2 - 1)u_z + \widetilde{\partial}_z^2 u_n)(-\widetilde{\partial}_z\overline{u}_n)\end{array}\right\}, \quad (7)$$

where $\mathbf{u^4} = (\mathbf{u_t}, \mathbf{u_z}, \mathbf{u_n}, -\mathbf{R}\partial_\mathbf{z}\mathbf{u_t})^\mathbf{t}$ is the vector of generalized displacements, $\mathbf{F}$ is the matrix differential operator 4×4 [5]. Here letters t, z, n, p marked tangential(rotating), longitudinal, normal and momentum components of energy flux Π and components of generalized vector $\mathbf{u^4}$.
In the particular case of axisymmetric rotating movements ($m = 0$) of the shell the integral energy flux Π^0 of it consists of unique component Π_t^0 and is equal to

$$\Pi^0 = \Pi_t^0 = 2\pi\rho c_s^2\frac{\omega}{2}|A|^2\beta; \quad \beta = w\sqrt{\alpha_1\nu_-} \quad (8)$$

4 Numerical calculations

Formulas (6)-(8) can be used for obtaining the normalized energy stream in the shell and it components

$$S = \Pi/\Pi^0, \qquad S_{t,z,n,m} = \Pi_{t,z,n,m}/\Pi^0. \quad (9)$$

The following values of parameters of the system are assumed for calculations ν=0.28, h/R=0.05 that corresponds to thin shell made of steel. Figures 1 - 4 are calculated for the mode $m = 21$, others for $m = 63$. Dimensionless frequency w is plotted along abscise axis on all figures.
The regular case (absence of bifurcation point) is considered at first. In Fig. 1 the dependence of dimensionless wavenumber $\lambda := \lambda R$ with respect to dimensionless

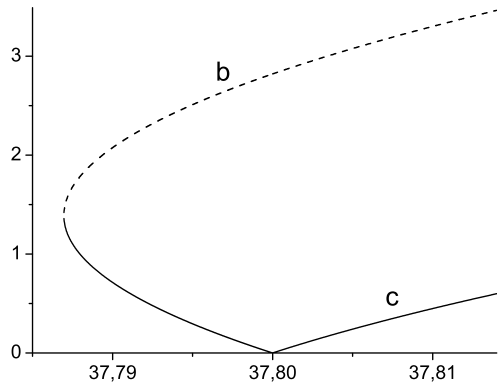

Figure 5: Dispersion curves ($m = 63$).

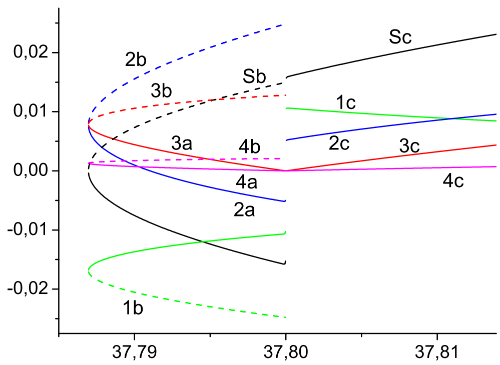

Figure 6: Integral energy fluxes and its components ($m = 63$).

frequency w is shown (these curves are marked by digits I, II, III). The veering (quasiintersection) of the curves I and II is well noted at $w \approx 25.0$. The veering is usual situation for the systems consisting of several subsystems. In our case it is caused by interaction of the different type of movements of the shell. It can be analyzed by energy fluxes components in Fig. 2 - Fig. 4.

The integral energy fluxes (curves S) and their components S_t, S_z, S_n, S_p (curves 1, 2, 3, 4) for the waves from dispersion curves in Fig. 1 are shown in Fig. 2 - Fig. 4 correspondingly according to formulas (9). The dominating of bending component in the wave from the first dispersion curve is changed to dominating of rotational component and dominating of longitudinal component in the wave from the second dispersion curve is changed to dominating of bending component. The interesting fact is that rotational component in the second dispersion curve is negative in the neighborhood of bearing point. There are no visible veering points for the third dispersion curve in Fig. 1 in the neighborhood of $w \approx 25.0$ but energy flux components "feel" the change of the wave character. The dominating of rotational component in Fig. 4 is changed to dominating of longitudinal component in this point.

It can be noticed that the veering of second and third dispersion curves is occurred at $w \approx 70$. The dominating of bending component in the wave from the second dispersion curve is changed to dominating of longitudinal component and dominating of longitudinal component in the wave from the third dispersion curve is changed to dominating of bending component.

In Fig. 5 the case of bifurcation point is considered. Two dispersion curves $w_1(\lambda)$ and $w_2(\lambda)$ (they will be called left and right brunches correspondingly) have the same bearing point $w = 37.80$. The behavior of the wave from the left dispersion curve differs from others significantly. This dispersion curve consists the section b with positive group velocity and smaller section a with negative group velocity. Right brunch consists of the singular section c with positive group velocity.

For these sections of the curves the integral energy fluxes (curve S) and their components S_t, S_z, S_n, S_p (curves 1, 2, 3, 4) are shown in Fig. 6. On these figures the curves with letters a, b, c in designations are corresponded to the sections of

dispersion curves in Fig. 5. For convenience the dependencies of the wave processes corresponding to the left dispersion curve are shown only for the frequencies less than bifurcation one.
Fig. 5 illustrates the fact that group velocity for the waves from both brunches in the neighborhood of bifurcation point is not equal to zero, have the opposite sign and equal module. It well corresponds with the fact that integral energy fluxes and their components have the opposite sign in this point (Fig. 6). Moreover the negative character of the integral energy flux is realized due to the negative character of the longitudinal and rotating components of it with dominating of rotating one. By contrast to this both integral fluxes (and its components) and group velocity are tending to zero if $\lambda \to 0$ in regular case. The exceptional case is when the dispersion curve is starting from the point $(w, \lambda) = (0, 0)$ $(m = 0)$ [5].
The numerical analysis shows that specific character of the waves with negative group velocity is their quick switching in the energy transmission process on the long waves (group velocity is not equal to zero at the point of their bearing).
The energy flux analysis [9] - [10] gives additional opportunities to investigate different components of vibrating and energy fields and their cross influence to each other [4] - [5].

Acknowledgements

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