DNS of Turbulent Flows in Spherical Layer, Driven by Torsional Oscillations of Boundaries

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Abstract

We have numerically studied the transition to turbulent flow in a layer of a viscous incompressible fluid confined between concentric spherical boundaries performing counter-rotational oscillations relative to the state of rest. The rotation speeds of both spheres were modulated at the same frequency and amplitude with the phase shift ПЃ. We used an algorithm of numerical solution based on a conservative finite difference scheme of the discretization of the NavierвЃYStokes equations in space and semi-implicit RungeвЃYKutta scheme of the third order integration accuracy in time. Discretization in space was performed on grids nonuniform in radial and meridional directions with concentration near the boundaries and equatorial plane. The transition to turbulence was caused by an increase in the amplitude of velocity modulation. Using the concept of instantaneous frequency/phase of flow (based on the building of the analytical signal of the velocity time series by means of Hilbert transform), it is established that the turbulence develops in a limited region of liquid layer, outside which the flow remains laminar. The turbulent region of flow exhibits intermittency of the chaosвЃYchaos type with random alternation of weak and strong turbulence.

1 Introduction

The transition of closed flows in a viscous incompressible fluid layer from laminar to turbulent regimes can take place under the action of temporally nonuniform rotation of boundaries [1]. In a flow of fluid confined between concentric spherical boundaries and driven by their rotation about a common axis, known as the spherical Couette flow (SCF), transitions to turbulence have been previously studied in the case of periodic variations in the velocity of one of the two boundaries. It was established that a change in the frequency of modulation of the rotation speed of the inner or outer sphere can lead to various turbulent flow regimes near the boundary of their formation [2, 3, 4].

Both the results of three-dimensional model calculations [2] and the measurements of flow velocity by laser Doppler anemometry [3] in combination with flow imaging showed that modulation of the rotation speed can lead to the development of temporally non-uniform turbulence. The most frequently observed non-uniformity is intermittency of the cycleвЂYchaos type, representing irregular alternation of the laminar and turbulent regions spreading over the entire spherical layer [2]. With increasing modulation frequency, intermittency of the chaosвЂYchaos type (in the form of randomly alternating regions of weak and strong turbulence) [3] and cycleвЂYchaosвЂYchaos type [4] was observed. The further increase in the frequency leads to the development of temporally uniform turbulence. Our measurements [3, 4] were performed in the region of middle latitudes (i.e., between the equator and pole) close to the outer sphere, which did not allow us to assess whether nonuniform turbulence has been established in the entire spherical layer.

Previously, we have studied [2, 3, 4] the influence of modulation of the rotation speed on the initial periodic flow formed between counter-rotating spherical boundaries. It would be also of interest to consider the possible transition to turbulence directly from the state of rest. [5] investigated the flow around a torsionally oscillating sphere relative to the state of rest. Conclusions concerning the transition to turbulence were based on the data of flow imaging and the results of axisymmetric calculations. However, both these approaches do not allow a final judgment to be drawn on the establishment of a turbulent regime and its characteristics.

The present work aimed at numerically studying the possibility of turbulence formation and characterizing its properties during counterwise rotational oscillations of two spherical boundaries relative to the state of rest. We have considered the case of equal modulation frequencies and amplitudes for both spheres, the angular velocities of which vary with phase shift π.

2 Calculation method and field of study

The flow of a viscous incompressible fluid in a spherical layer is described by a set of the Navier-Stokes and continuity equations,

$$\frac{\partial U}{\partial t} = U \times rotU - grad\left(\frac{p}{\rho} + \frac{U^2}{2}\right) - \nu rotrotU, divU = 0 \quad (1)$$

which are supplemented by the conditions of no slip and nonpercolation at the boundaries. In a spherical coordinate system with radial (r), polar (θ), and azimuthal (φ) directions, these conditions take the following form: $u_\varphi(r = r_k) = \Omega_k(t) r_k \sin(\theta), u_r(r = r_k) = 0, u_\theta(r = r_k) = 0, k = 1, 2$ where the subscripts 1 and 2 refer to the inner and outer sphere, respectively; U, p, and ρ are the fluid velocity, pressure, and density; u_φ, u_r, and u_θ are the azimuthal, radial, and polar components of the velocity; r_k and Ω_k are the radius and angular velocity of rotation of the corresponding sphere; and ν is the kinematic viscosity of fluid in the layer. The angular velocities of rotation of the spheres are periodically varied as $\Omega_1(t) = A\sin(2\pi ft + \psi_1), \Omega_2(t) = A\sin(2\pi ft + \psi_2), \psi_1 - \psi_2 = \pi$, where A and f are the amplitude and frequency of modulation, and ψ_1 and ψ_2 are the corresponding initial phases.

By analogy with [2], we have used the computation algorithm based on a conservative finite difference scheme of discretization for the Navier-Stokes equations and a semi-implicit third-order accurate RungeвЂYKutta scheme for integration with respect

to the time [6]. The spatial discretization has been performed on non-uniform grids with decreasing cell size toward the boundaries and the equatorial plane. The ratio of the maximum to minimum cell size was 3, and the total number of nodes was $5.76 \cdot 10^5$.
Model calculations have been performed for the following set of dimensional parameters: $\nu = 5 \cdot 10^{-5} m^2/s, r_1 = 0.075m, r_2 = 0.15m$, and $f = 0.51Hz$. The character of flow depends on the values of A, f, r_1, r_2, and $\psi_1 - \psi_2$. By analogy with [5], the dimensionless parameters of similarity are introduced as Reynolds numbers for the inner and outer spheres, $Re_1 = 2Ar_1/2\pi f\delta$, and $Re_2 = 2Ar_2/2\pi f\delta$ (where $\delta = (2\nu/2\pi f))^{1/2}$ and relative thickness of layer $\beta = (r_2 - r_1)/r_1 = 1$. The dependence on the phase difference reduces to relation $\Omega_1(t) + \Omega_2(t) = 0$. For the parameters indicated above, we have $\delta = 5.64 \cdot 10^{-3} m$, which corresponds to seven nodes of the computational grid. Time series of the flow velocity were recorded at two points. Point 1 is situated near the inner sphere at the equator ($r = 3.55\delta, \theta = 1.586$), while point 2 is situated near the outer sphere at middle altitudes ($r = 2.84\delta, \theta = 1.033$)

3 Results

At $Re_1 \leq 55$, the flow is symmetric relative to the axis of rotation and equatorial plane and only the modulation frequency is present in the flow velocity spectrum. As Re_1 increases, the flow loses stability and becomes asymmetric relative to the rotation axis. Near the inner sphere, the secondary flow is asymmetric relative to the equatorial plane and its wave number in the azimuthal direction is $m = 1$. Near the outer plane, the flow is quasi-symmetric relative to the equatorial plane and $m = 2$. The velocity spectrum exhibits a second frequency, which is much smaller than the modulation frequency. With further growth in Re_1, the flow becomes turbulent and the level of the low-frequency $(0.01 - 0.1Hz)$ region in the velocity spectrum increases by no less than two orders of magnitude. In a narrow region near the equator, the flow retains a spatial structure characteristic of secondary flow (Figure 1), while chaotization is observed at middle latitudes.
Let us quantitatively characterize the degree of flow chaotization in terms of correlation dimension D determined using the relation [7] $C(r) \sim r^D$, $C(r) = \lim_{m\to\infty} \frac{1}{m^2} \sum_{i,j=1}^{m} H(r^{(p)} - |x_i^{(p)} - x_j^{(p)}|)$, where H is the Heaviside function, $r^{(p)}$ is the distance in p - dimensional space, and $x(t_i), x(t_i + \tau), \ldots, x(t_i + (p-1)\tau)$ is a point in the p - dimensional space that characterizes the state of the system at time moment t_i. The $x(t_0 + kt)$ series represents a discrete record of the flow velocity at interval Δt. Thus, D is the slope of the plot of $logC(r) = f(log(r))$. Here, we calculate the values of dimensions D_1 and D_2 using time series of u_ϕ recorded at points 1 and 2, respectively, and the value of dimension D_A calculated for A_{3D} [6], which refers to the entire volume of flow:

$$A_{3D}^2 = (3/4\pi)(r_2^2 - r_1^2)^{-1} \int_0^{2\pi} d\varphi \int_0^{\pi} d\theta \int_{r_1}^{r_2} (U - U_{2D})^2 r^2 \sin(\theta) dr \tag{2}$$

where U_{2D} is the axisymmetric velocity component defined as $U_{2D} = (2\pi)^{-1} \int_0^{2\pi} U d\varphi$
The length of records was no less than $5.9 \cdot 10^4$ points for $\tau = 300s$. The magnitude of D_A is plotted in Figure 2 (curve 1).

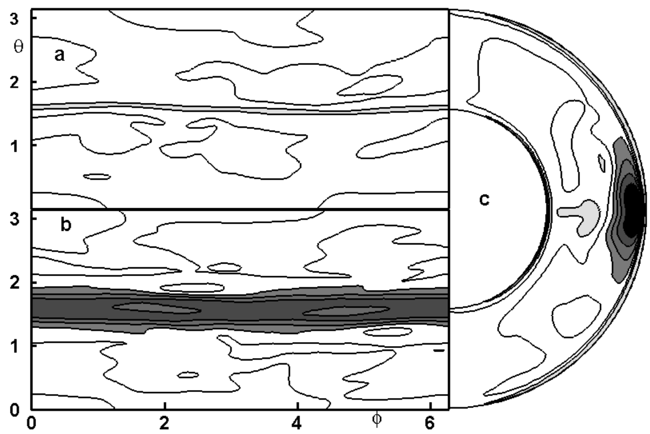

Figure 1: Equilevel lines $u_\varphi[m/s]$: (a) in the $\varphi - \theta$ plane near inner sphere (point 1), (b) in the $\varphi - \theta$ plane near outer sphere (point 2)), (c) in the meridional plane passing through the axis of rotation. $Re_1 = 72$, $u_{\varphi max} = 0.1, u_{\varphi min} = 0.4$, and $\triangle u_\varphi = 0.1$. White areas correspond to $u_\varphi \geq 0.1$; increasing intensity of background corresponds to decreasing u_φ. The moment of time corresponds to an increase in the rotation velocities of $0.2s$ upon passage through the zero level.

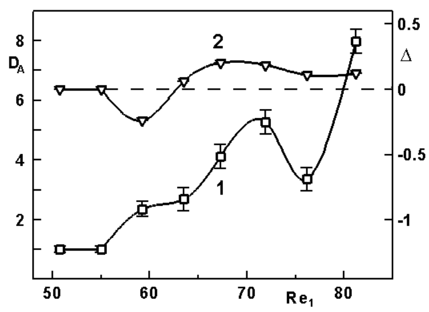

Figure 2: Plots of (1) D_A and (2) Δ vs. Re_1. Vertical bars indicate the possible spread of correlation dimensions for various τ.

Taking into account that, in spherical layers at constant speeds of rotation, the correlation dimension at a threshold of the chaotic flow regime formation falls in the interval $3.5 \leq D \leq 4$ [8, 9], we may conclude that transition to turbulence in the case under consideration takes place at $Re_1 \approx 65$ (Fig. 2, curve 1). At constant boundary conditions, the dimension is independent of their position of flow velocity recording [8, 10]. In order to check for this condition, let us consider the value of $\Delta = (D_1 - D_2)/D_A$. Maximum deviations of Δ from zero are observed in the transition region $55 \leq Re_1 \leq 65$ (Figure 2, curve 2). For all turbulent regimes, this value is $\Delta \geq 0$, which quantitatively characterizes a more pronounced stochasticity of flow at middle latitudes as compared to the near equatorial region, as is manifested by the flow structure (Figure 1). The dependence of the correlation dimensions on the point of determination suggests that the influence of rotational oscillations of the boundaries on the flow is differently manifested in various regions. This circumstance does not allow the flow upon transition to turbulence to be considered as a unified dynamical system.

In order to confirm this hypothesis, let us consider fragments of a record of the azimuthal velocity upon the transition to turbulence at $Re_1 = 72$ (Figure 3). As can be seen, the behavior of u_φ near the inner sphere at the equator (curve 1) is close to periodic with a weak irregular modulation of the amplitude, while flow near the outer sphere at middle altitudes (curve 2) becomes turbulent. Thus, the flow is spatially nonuniform so that the turbulence develops in a limited region and remains laminar outside this region. The degree of temporal nonuniformity can be determined from analysis of the behavior of instantaneous phase and frequency of the signal [11] by analogy with the procedure used in [4]. According to this, instantaneous phase $\Psi(t)$ of velocity $x(t)$ is defined as $\Psi(t) = \arctan(y(t)/x(t))$, where $y(t)$ is the orthogonal complement to $x(t)$, which is calculated as the Hilbert transform of the $x(t)$ series [11]. Derivative $\chi(t) = \partial\Psi(t)/\partial t$ is then the instantaneous frequency of modulation. Consider the difference of instantaneous phases and frequencies between the rotation speed of the inner surface and flow velocity at a given point of the flow. As can be seen, the phase difference at point 1 remains constant in time (Figure 3, curve 3) while the frequency difference exhibits regular variations (curve 6). Thus, the flow at point 1 is fully synchronized with rotation of the boundary. At the same time, these parameters at point 2 near the outer sphere vary with time. By analogy with [4], it is possible to reveal regions of weak turbulence, where strong synchronization also makes the phase difference constant and the frequency difference exhibits no jumps. In regions of strong turbulences and weak synchronization, the phase difference varies with the time and the frequency strongly deviates from average values. Regions with different character of turbulence are randomly alternating in the time, which leads to the conclusion that there is intermittency of the chaos-chaos type.

4 Conclusions

Thus, an increase in the amplitude of counterwise rotational oscillations of spherical boundaries leads to the formation of flow with random alternation of weakly and strongly turbulent regions in a part of the layer of fluid confined between these boundaries. At the same time, in other parts of the turbulent flow, the character of

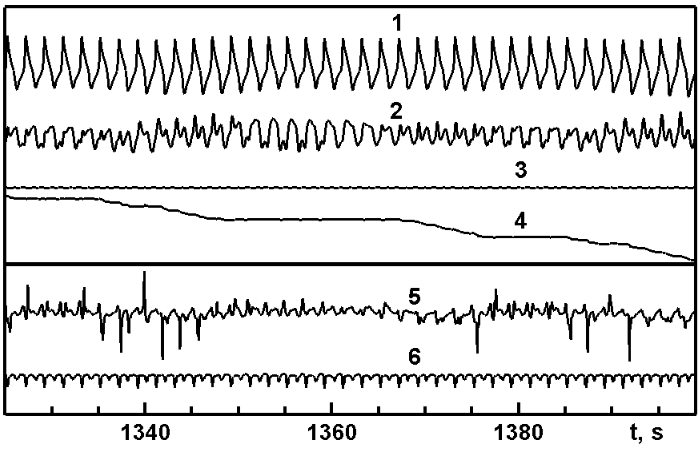

Figure 3: *1,2* time series of azimuthal flow velocity u_φ and differences of instantaneous *3,4* phases and *5,6* frequencies between the rotation speed of the inner sphere $\Omega_1(t)$ and flow velocity u_φ; *1,3,6* near the inner and *2,4,5* near the outer spheres vs. time t for $Re_1 = 72$. The dashed contours indicate regions of stronger turbulence.

flow can be different. It may be suggested that the approach described above, which is based on the concept of phase/frequency differences, can be used for determining boundaries both between the turbulent/nonturbulent regions of flow and between regions of temporally uniform/nonuniform turbulence.

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