Formulation Of Rheologycal Equations Of The Elastic-Viscous Plastic Medium Taking Into Account The Current Value Of The Lateral Deformation

Alexander R. Arutyunyan, Robert A. Arutyunyan
r.arutyunyan@spbu.ru

Abstract

On the stress-strain diagram, which is obtained in experiments on a simple tension of metal specimens, there is a region of instability due to the formation of the neck. In the theory of plasticity are defined conditions for the transition to an unstable state and appearance of the maximum point on the stress-strain diagram. In the derivation of this condition assumption of incompressibility of the material is accepted. However, this assumption cannot be justified, since in the neck region there are numerous damages (pores, micro-cracks), i.e. the material is compressible. In this paper, the condition for the transition to the unstable state for a compressible plastic medium is formulated. Incompressibility condition is also used in the formulation of nonlinear elastic and viscous-elastic equations generalizing linear models of Hooke and Maxwell.

1 Introduction.

In solid mechanics, the main mechanical characteristics of materials are determined, in particular, from experiments on simple tension. According to the results of measurements of force, current length and diameter of the specimen are calculated the values of true stress $\sigma = P/F = \sigma_0 F_0/F$ and logarithmic strain $\varepsilon = \ln l/l_0$ (P is force, $\sigma_0 = P/F_0$ is engineering stress, l_0, F_0 are initial and l, F are the current length and the cross section area of the specimen). Typical stress-strain diagrams for metallic specimen are shown on Fig. 1.

It is usually assumed that the change in the cross section area of the specimen during deformation can be neglected, then $\sigma \approx \sigma_0$ and stress-strain curves are plotted in $\sigma_0 - \varepsilon$ coordinates (ε is engineering strain). At the point M on Fig. 1 engineering stress reaches maximum, neck is occurs on specimen and the deformation becomes unstable. The drop-down region of the $\sigma_0 - \varepsilon$ curve is the result of a sharp cross section area reduction of the specimen due to the necking. The nature of the neck is determined by the properties of the material and is various for different materials (metals, polymers).

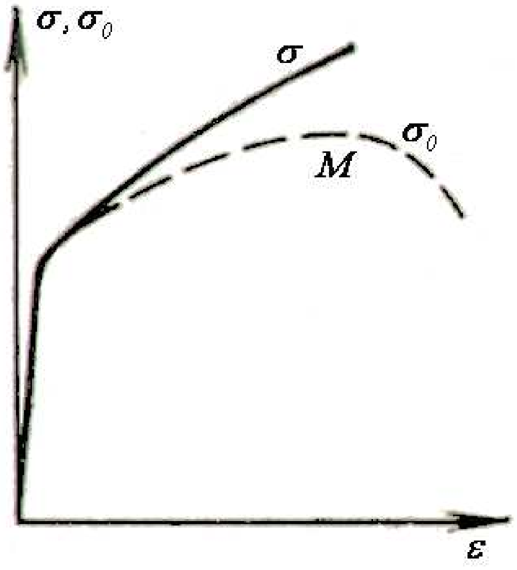

Figure 1: Stress-strain diagrams. Solid line corresponds to tension diagram plotted in true stresses and the dotted line is diagram plotted in engineering stresses.

2 Definition of maximum point achievement on the stress-strain curve for incompressible and compressible plastic medium.

To determine the conditions of the maximum achievement at the point M (Fig. 1) in the literature [1, 2] the material is considered as incompressible, then $l_0F_0 = lF$ and $P = \sigma F = \sigma F_0\, e^{-\varepsilon}$. Differentiating the last expression for the ε we will have

$$\frac{dP}{d\varepsilon} = F_0\, e^{-\varepsilon}\left(\frac{d\sigma}{d\varepsilon} - \sigma\right). \tag{1}$$

Under $\frac{dP}{d\varepsilon} = 0$, from (1) follows the ratio

$$\frac{d\sigma}{d\varepsilon} = \sigma, \tag{2}$$

which is the condition for maximum achievement at the point M (Fig. 1).
Let's note that in derivation of the expression (2) is used the assumption of incompressibility of the material, resulting in a fixed maximum point on the stress-strain diagram. At the same time in real metallic materials during plastic deformation, particularly in the area of instability, the maximum point shifts, numerous damages (pores, cracks) are occurred, so the assumption of incompressibility of the material in the general case cannot be considered as reasonable.
The condition of compressibility is determined by using the current value of the lateral deformation ν: $\nu = -\varepsilon_y/\varepsilon_x = -\varepsilon_z/\varepsilon_x$ (ε_x is longitudinal, ε_y, ε_z are transverse deformations of a cylindrical specimen). Then, taking into account the geometric relation $F_0/F = (l/l_0)^{2\nu}$ [3], we will have

$$P = \sigma F = \sigma F_0 \, e^{-2\nu\varepsilon}. \tag{3}$$

Approximately taking that $\nu = \nu(\sigma_0) = const$ and differentiating (3) for the ε, we receive the following relation

$$\frac{dP}{d\varepsilon} = F_0 \, e^{-2\nu\varepsilon} \left(\frac{d\sigma}{d\varepsilon} - 2\nu\sigma \right), \tag{4}$$

from which is follows the condition for maximum achievement

$$\frac{d\sigma}{d\varepsilon} = 2\nu\sigma. \tag{5}$$

For an incompressible material $\nu = 1/2$ and the relation (5) will coincide with the formula (2). According to the formula (5) the position of the maximum point M on the stress-strain curve will be vary depending on the material state.
In the case of elastic-plastic media $d\varepsilon = d\varepsilon^e + d\varepsilon^p$ (ε^e, ε^p are the components of the elastic and plastic deformation, $\varepsilon^e = \sigma/E$, E is Young modulus). The ratio between stress and deformation can be determined by the following equation

$$\frac{dl}{l} = \frac{d\sigma}{E} + \varphi(\sigma) d\sigma. \tag{6}$$

Integrating equation (6), we will receive

$$\ln \varepsilon = \frac{\sigma}{E} + \int_0^{\sigma} \varphi(\sigma) d\sigma. \tag{7}$$

In the general case, inserting at (6) a relation $\sigma = \sigma_0 \, e^{2\nu\varepsilon}$, we can obtain the equation written through the value of the lateral deformation ν. For different values of ν, we can plot a non-monotonic $\sigma_0 - \varepsilon$ diagrams and, thus, to describe experimental curves for metallic materials in engineering stress-strain coordinates. Further, this approach is applied for the case of rigid-plastic Ludwig medium with nonlinear hardening

$$\sigma = \sigma_T + b\varepsilon^m, \tag{8}$$

where σ_T is the yield stress, b, m are constants.
Let's write equation (8) through σ_0

$$\sigma_0 = (\sigma_T + b\varepsilon^m) \, e^{-2\nu\varepsilon}. \tag{9}$$

The theoretical curves according formula (9) for $\sigma_T = 200 \; MPa$, $b = 5 \cdot 10^2 \; MPa$, $m = 0,5$ and for different ν values are shown on Fig. 2.

3 Formulation of nonlinear equations for compressible elastic and elastic-viscous medium.

Next, let's formulate nonlinear equations for compressible viscoelastic medium based on the linear relations of Hooke's law and viscous Newton media. It is well known

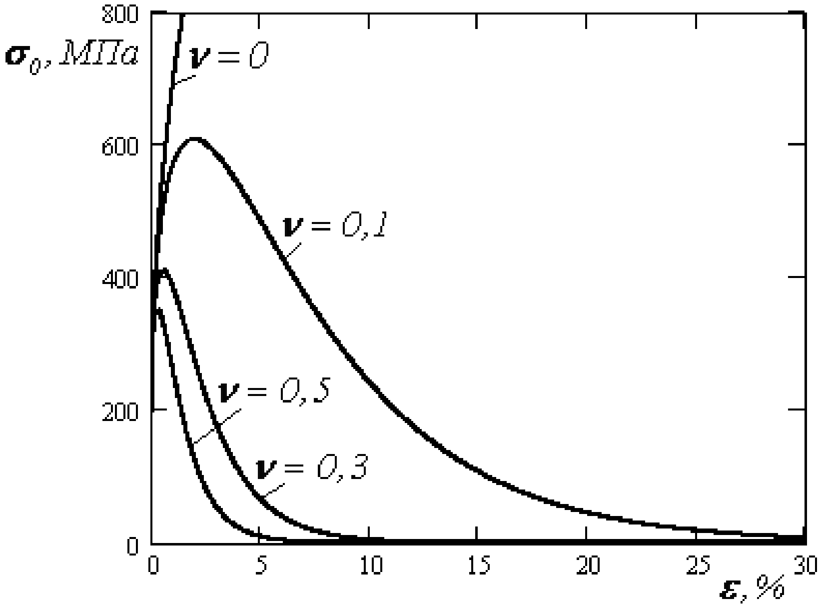

Figure 2: The theoretical curves according formula (9).

that using the linear relations the mechanical behavior of materials, in particular, polymers can be described only in limited temperature and force actions. In general case, we should operate the non-linear rheological equations. Under the proposed approach, will be considered the behavior of a compressible elastic-viscous medium, which generalized linear models of Hooke and Maxwell.

In the case of Hooke's law, nonlinear version of the equations for the elastic medium written using the current value of the transverse deformation has the form

$$\sigma_0 = E\varepsilon e^{-2\nu\varepsilon}. \tag{10}$$

Using equation (10) the nonlinear effects observed in experiments on simple tension can be described. When $\nu = 1/2$ relation (10) is described the behavior of an incompressible nonlinear medium. When $\nu = 0$ the medium is linearly elastic. Intermediate cases will correspond to the elastic materials with different mechanical properties. Diagrams $\sigma_0 - \varepsilon$ for different values of lateral deformation coefficient are shown on Fig. 3.

Let's consider a rheological Maxwell model for nonlinear elastic-viscous medium. With this aim, the classical linear Maxwell's equation

$$\frac{d\varepsilon}{dt} = \frac{1}{E}\frac{d\sigma}{dt} + \frac{\sigma}{\eta} \tag{11}$$

will be written through the stress σ_0, considering that the material is compressible and introducing the current value of the lateral deformation

$$\frac{d\varepsilon}{dt} = \frac{2\sigma_0\nu}{E}\, e^{2\nu\varepsilon}\frac{d\varepsilon}{dt} + \frac{\sigma_0}{\eta}\, e^{2\nu\varepsilon}. \tag{12}$$

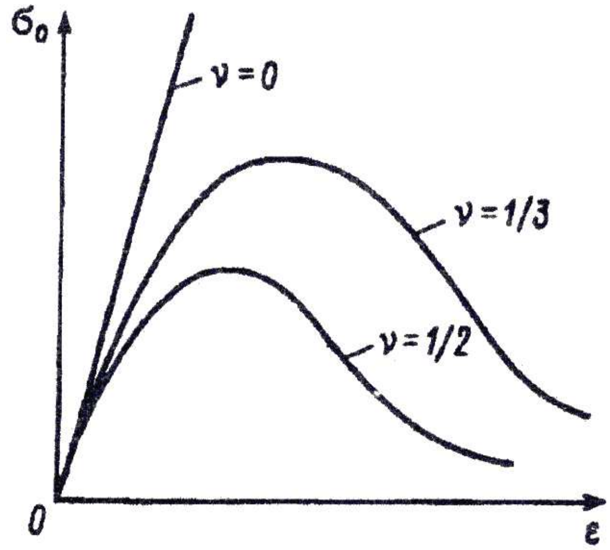

Figure 3: Diagrams $\sigma_0 - \varepsilon$ for different values of lateral deformation coefficient: incompressible non-linear medium ($\nu = 1/2$), linear-elastic medium ($\nu = 0$).

From equation (11) when $\sigma = const$ is follows a linear dependence for creep deformation $\varepsilon = \frac{\sigma}{\eta} t$, which is known to be bad fit with the experimental results. Solving the equation (12) the nonlinear equations for creep strain can be obtained. Considering that $\nu = \nu(\sigma_0) = const$ from the solution of equation (12) with the initial conditions $t = 0$, $\varepsilon = 0$, we can obtain

$$t = \eta \left(\frac{1 - e^{-2\nu\varepsilon}}{2\nu\sigma_0} - \frac{2\nu\varepsilon}{E} \right). \tag{13}$$

The theoretical creep curves for a given stress level and different values of ν according to the formula (13) are shown on Fig. 4.
For $\nu \to 0$ from (13) follows the linear Maxwell relation for creep deformation. In the general case, creep curves are nonlinear and qualitatively described the corresponding experimental curves.

4 Conclusion.

In the article, the condition for the transition to instable state in the region of necking for compressible metal specimen is formulated. For an incompressible material Hill was first who described this condition. The effect of compressibility is determined by using the current value of the lateral deformation. Taking into account this coefficient the rheological equations for compressible media are obtained, which generalized well-known equations for an incompressible plastic, elastic and elastic-viscous medium. Analytical solutions were obtained, constructed the corresponding theoretical stress-strain curves and creep curves dependent on the current value of lateral deformation coefficient. In particular, it is shown that for a compressible material in the region of instability, the maximum point is shifted. In the case of

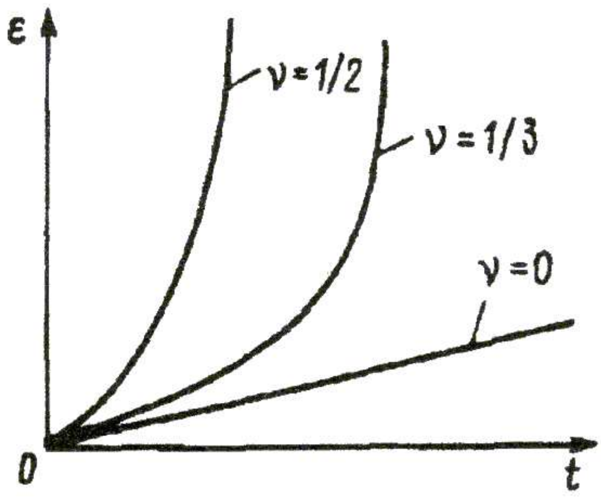

Figure 4: Theoretical creep curves for different values of ν according to the formula (13).

the Hill solution the maximum point of the stress-strain curve is fixed.

Acknowledgements

Financial support of the Russian scientific Fund (Grant N 17-79-20361) is gratefully acknowledged.

References

[1] Hill R. Mathematical theory of plasticity. M: Gostekhizdat. 1956. 408p. (in Russian).

[2] Kachanov L.M. Fundamentals of fracture mechanics. Moscow: Nauka. 1974. 311p. (in Russian).

[3] Arutyunyan R.A. The problem of deformation aging and prolonged fracture in mechanics of materials. St.-Petersburg: Publishing house of St. Petersburg state University. 2004. 253p. (in Russian).

Alexander R. Arutyunyan, Robert A. Arutyunyan, Universitetskii pr., 28, Faculty of Mathematics and Mechanics Sankt-Petersburg State University, Sankt-Petersburg, Petrodvoretz, 198504, Russia.