

# Effects of dispersion and delay in mathematical models

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## Abstract

Analysis of the mathematical models that can write the influence of the angular momentum and delay in mechanics is suggested: under interaction of many particles, in continuous mechanics, in kinetic theory. Disturber of the ergotic is discussed for the classic equations of continuous environment. The new method of calculation pressure and energy for multicomponent environment was suggested. Non-symmetrical stress tensor is obtained as results of influence of angular momentum for continuous medium. The method for writing of interaction discretion and continuous mediums was suggested. Attention pays on delay for processes discrete mediums. Analytical results are obtained for cases of large gradient. The nucleus of the Navier-Stokes equations is obtained. Equations S.V. Vsallander were received from the kinetic equation.

**Key words:** Angular momentum, delay, conservation laws, non-symmetrical stress tensor, Boltzmann equations, Chapman-Enskog method, conjugate problem Navie-Stokes equations, equations S.V. Vsallander.

## 1 Introduction

An important area of current research is to study the effect of the angular momentum and the delay in the whole mechanics including quantum mechanics. The process is changing so it associated with the appearance of additional forces, which can play the role. These effects may affect at critical and near critical aircraft modes, rockets, various devices, building structures, as well as some natural processes. The value of the additional force is determined by the gradient of the value of physical quantities (density, velocity, momentum) and the structure of the object being studied. In the case of the dynamic formation of the structure, the position of the center of mass is changing, which entails a change of angular momentum of a small perturbation that affects the stability of the structure. In the static case, the angular momentum occurs at non-uniform distribution of parameters. For elementary volume is important that volume is rotated about its center of mass, and the involvement of volume in larger rotation. The definition of the velocity of divergence and the vortex velocity how the of decomposition with respect to an arbitrary point inside the elementary

volume is incorrect, as vortex part is the component of the velocity relative to the axis of inertia of the elementary volume. The mechanics is considering the Lagrange function for non-interacting and collectively interacting particles equally but this is questionable, especially in metallic and ionic bonds. In classical continuum mechanics had an opinion on the small contribution of the angular momentum force against surface forces, as their action has a volume character. In the classical mechanics of a continuous medium, there was an opinion about the small influence of the angular momentum with respect to the contribution of surface forces, since their action is of a bulk nature.

For long bodies and large gradients parameters contribution is significant and can be a cause of instability, leading to changes in the flow structure or destruction of the body. The second important effect-delay [1,2]. Delay in mechanics plays an important role under the relaxation in the case of the commensurability the time of relaxation and delay times. The new proposed option is to consider accounting delay between the time derivative as a limit and final value free path in a rarefied gas and time between collisions. This situation is typical for transition from discrete to continuous environment and is a key issue of mechanics and computational mathematics. For particles without structure is usually considered central of the interaction, i.e. variant, where the momentum cannot play a role in conditions close to equilibrium. For the remaining cases semi-empirical theory is included. In the classical approach, the law of conservation of angular momentum is not constructed. In view of these formulations actually postulated symmetry of any material system of reference and as a result, the symmetry of the angular momentum, the symmetry of the stress tensor and the violation of "continuity" of the medium, while for arbitrary perturbations of the motion of a material point is a non-inertial.

Selecting the conditions of equilibrium of momentums of forces leads to new formulations of equations [3-5]. Therefore, subject to the balance of power we come to a private classical formulation of continuum mechanics. The resulting formulation of conservation laws associates with the recording of the conservation laws for a system that is exchanging the components of physical quantities only by normal convection rate and ignoring all processes within an elementary volume, and the lack of rotation of the volume. Determination of physical quantities in the form of a sum of delta functions and terms of integrals by volume that tends to zero, leads to the same equations. This confirms what we said. Everything that is happening in the volume and with the volume are not considered. This has led to an incomplete accounting of the processes. The accumulated experimental facts led to the hypothesis of the importance of spatial gradients and time derivatives, which also contributed to the change of the momentum. The importance of these effects observed for fluid mechanics and gas plasma, and for the solid. It should be noted that in the kinetic theory (Boltzmann equation), the law of conservation of angular momentum is not executed. The existing representation likely linked to the consideration of an elementary volume as the closed. The effect of the angular momentum of the motion at the equations of the continuum mechanics in [7-9] was studied. The proposed theoretical method of accounting the angular momentum without new empirical constants equations, bases on the fact that in the angular momentum do not have new dimension. Another method was proposed in [10].

The order of given equations and boundary conditions requires a revision. The total consideration of the effects leads to a cumbersome system of equations, and therefore requires the allocation of the major effects in a particular situation. Examples were given showing the contribution of the no symmetric part of the stress tensor in the simplest problems of elasticity theory and boundary layer. Conclusion modified equations for gas based on the kinetic theory, for which it was suggested that the angular momentum need be included as an additional variable; to use a more precise asymptotic approach to Hilbert’s paradox. As already mentioned, the elementary volume can itself rotate around the axis of inertia, or to be involved in a rotary motion. In both cases, the density of the flow across the border is changed to the value  $\frac{d(\rho u)}{dr} \cdot (r' - r) + \dots$  in rotation of the elementary volume. The contribution of other components is small, taking into consideration a little volume and the absence of rotation at the axis. In our opinion there is an inaccuracy in the calculation of the Lagrangian function as a sum mutually interacting particles. The position axis of inertia under equilibrium conditions and non-equilibrium conditions are different, and that leads to the existence of collective effects. Interestingly, the effects of the influence of the angular momentum and changing the position of the center of mass can be important in quantum mechanics when considering the particles decay into three or more particles; when writing potential in the Schrodinger equation. As is known, the equation for the macro parameters can be derived from the Boltzmann equation by the Chapman-Enskog [11-15] method. We give qualitative and quantitative assessment of the impact of using a classic method of the Chapman-Enskog method of the calculation of local-equilibrium distribution function of the macro parameters (density, velocity and temperature), calculated from the zero approximation (of the Euler equations), without correction results using Navier-Stokes equations. The existence of the problem of coordination of macro parameters was pointed Gilbert on solution of the Boltzmann equation by a series expansion in the small parameter. We have proposed an algorithm for matching macro parameters locally equilibrium distribution function [3-5]. In the classical theory believe

$$\int \varphi(\xi) f^0 d\xi = \int \varphi(\xi) f d\xi = \beta ,$$

$\beta$  - macro parameters,

$$f(t, x, \xi) \equiv f_0(t, x, \xi) = n \left( \frac{m}{2\pi kT} \right)^{3/2} \exp \left\{ -\frac{m}{2\pi kT} c^2 \right\} ,$$

$$c^2 = (c_1^2 + c_2^2 + c_3^2) = (\xi - u)^2 ,$$

$$f = f_0 \left[ 1 + \frac{p_{ij} m}{2pkT} c_i c_j - \frac{q_i m}{pkT} c_i \left( 1 - \frac{mc^2}{5kT} \right) \right]$$

and the quantities are determined through the total distribution function  $t$  - time,  $x_i$  - position,  $u_i$  - velocity,  $\tilde{\nu}$  - viscosity,  $\rho$  - density,  $T$  - temperature,  $q$  - heat flux,  $P_{ij}$  - the tensor of viscous pressure,  $X$  - force. The Boltzmann equation is invariant with respect to the choice of macro parameters.

Consequently, the coincidence of Navier-Stokes equations and built equation has a formal character, order of approximation and the parameters in a locally equilibrium distribution function vary. Therefore, when constructing the first approximation in the Chapman-Enskog (Navier-Stokes equations), it seems necessary to clarify the values of density, velocity and temperature for matching orders of approximation. Therefore, in the equations of the first order terms will be responsible for clarification of macro parameters. Throw away they cannot be due to their definitions in the kinetic theory. However, after factoring formal kind of balance function does not change, but the macro parameters are responsible macro parameters Navier-Stokes equations. In the derivation of the first approximation in the Chapman-Enskog made implicitly significant approximation. Part of the terms discarded only after integration over the phase velocity, they differ in arbitrary locations. It does not take into account that in this case the integrals for  $f\xi(\rho u)$  flow ( $\rho u$ ) and speed on the density of the product in the first approximation, differ from each other (analogue divergent and non-divergent difference schemes). In view of the difference obtained by the conservation equations S. V. Vallander [16, 17]. The paper summarizes the results of the study. For the multi-component gas and gas with rotational and vibrational degrees of freedom offered another form of calculation of the average values of pressure, temperature and energy. The effect of correlation of zero and first approximations of the Boltzmann equation for writing continuum mechanics equations.

## 2 Equations

The equations of motion, energy and angular momentum have been obtained earlier, but the use as force equilibrium condition does not require the calculation of the moment. Therefore, in the classical theory of equations conservation of angular momentum is not used explicitly. The modified equation

$$\begin{aligned} \frac{\partial \rho u_i}{\partial t} + \frac{\partial}{\partial x_i} \left( \rho u_j + P_{ij} + x_i \frac{\partial P_{ij}}{\partial x_i} \right) - \frac{X_i}{m} \rho &= 0 . \\ \frac{\partial}{\partial t} \rho \left( \frac{3}{2} RT + \frac{1}{2} u^2 \right) + \frac{\partial}{\partial x_i} \left[ \rho u_j \left( \frac{3}{2} RT + u^2 \right) \right] + \\ + \frac{\partial}{\partial x_i} x_i \frac{\partial}{\partial x_j} \left[ \rho u_j \left( \frac{3}{2} RT + \frac{1}{2} u^2 \right) + u_k P_{kj} + q_j \right] &= 0 \\ \frac{\partial \vec{r}}{\partial x} \times \vec{p}_x + \frac{\partial \vec{r}}{\partial y} \times \vec{p}_y + x_j \frac{\partial}{\partial x_j} \left( \vec{P}_j \right) &= M_I \end{aligned}$$

Where  $t$  - time,  $x, y, z$  - coordinates,  $\rho$  is the density,  $P_{ij}$  - stress tensor,  $u$  - velocity,  $q$  - heat flow,  $R$  - gas constant.

This equation is used to determine the degree of no symmetry of the stress tensor. The issue arose when writing the law of conservation of density. We will try to get it from the phenomenological principles. The modified equation for the density was obtained from the kinetic theory in the form of

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_i}{\partial x_i} + \frac{\partial}{\partial x_i} \left( x_i \frac{\partial \rho u_i}{\partial x_i} \right) = 0 ,$$

From Fig. we can see that velocity  $u = \omega \times (r' - r)$  is the velocity with respect to the point M quasi solid movement around the axis r without forward speed. The point M may itself be in rotation around the axis of inertia. For elementary volume formula  $u = \omega \times (r' - r)$  means that the rotation occurs around the axis of inertia, but the axis of rotation can lie outside the volume. Therefore, we obtain for elementary volume

$$\int_{(s)} (\nabla \rho u (r' - r_c))_n ds = \int_{(s)} \text{div} (\nabla \rho u) (r' - r_c) dv .$$

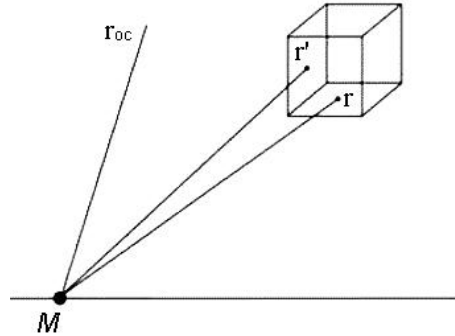


Figure 1: Elementary volume for the density

The degree of no symmetry of tensor derived from the law of conservation of momentum (in projections) ( $\sigma = \tau$ ). Designations is standard.

$$y \left( \frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} \right) - z \left( \frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{zy}}{\partial z} \right) + \sigma_{zy} - \sigma_{yz} = 0 ,$$

$$x \left( \frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} \right) - z \left( \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{yx}}{\partial y} + \frac{\partial \sigma_{zx}}{\partial z} \right) + \sigma_{zx} - \sigma_{xz} = 0 ,$$

$$x \left( \frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{zy}}{\partial z} \right) - y \left( \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{yx}}{\partial y} + \frac{\partial \sigma_{zx}}{\partial z} \right) + \sigma_{yx} - \sigma_{xy} = 0 .$$

The procedure of calculation is to calculate the degree of no symmetry the stress tensor by the last equations and substitution of these values in the rest of the equations. The equation of state remain the same because as they are the higher-order corrections. Interestingly, some elasticity theory provisions in „ses force in case no symmetric stress tensor. For example, we have for two opposite sides of an elementary volume of its direction of principal stresses  $tg 2\theta_1 = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$  ,  $tg 2\theta_2 = \frac{2\tau_{yx}}{\sigma_x - \sigma_y}$  and since  $\tau_{xy} \neq \tau_{yx}$  we get different results. Thus, at each point we have main its direction of stresses.

### 3 Delay Effects

In the kinetic theory when considering the role of delay should deal with the question of what measures the experiment: the instantaneous values or averaged. If the

experiment was dealing with averages, it is important to choose the time and scope of averaging. At the agreed time, in this case to take into account the delay is not necessary, except in cases of commensurable of relaxation times and delay, otherwise it is necessary to bear in mind the following:

The mean free path of molecules of the  $i$ -th group relative to the molecules of the  $j$ -th group is equal in classical mechanics

$$\lambda_{ij} = \frac{\xi_i}{\sigma_{ij} n_j g_{ij}} .$$

The mean free path of molecules

$$\bar{\lambda} = \frac{\sum_i^k \xi_i n_i}{\frac{1}{2} \sum_{i,j=1}^k \sigma_{ij} n_i n_j g_{ij}} .$$

The mean velocity of molecules

$$\bar{g} = \frac{1}{2n^2} \sum_{i,j=1}^k n_i n_j g_{ij} .$$

The mean time

$$\bar{\tau} = \frac{\bar{\lambda}}{\bar{g}} .$$

Taking this into account, the Boltzmann equation can be written in the form

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + c_i \cdot \frac{\partial f}{\partial r_i} + c_i \cdot \frac{\partial}{\partial r_i} r_j \frac{\partial f}{\partial r_j} - \frac{F}{m} \frac{\partial f}{\partial c_i} = I$$

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \bar{\tau} \frac{\partial^2 f}{\partial t^2} + c_i \cdot \frac{\partial f}{\partial r_i} + c_i \cdot \frac{\partial}{\partial r_i} r_j \frac{\partial f}{\partial r_j} - \frac{F}{m} \frac{\partial f}{\partial c_i} = I'$$

$$\Delta^- = dt \, d\vec{x} \, d\vec{\xi} \, f(t, \vec{x}, \vec{\xi}) \int \left[ f_1(t, \vec{x}, \vec{\xi}) + O\left(\Delta t \, \vec{\xi} \frac{\partial f_1}{\partial x}\right) \right] g b \, db \, d\epsilon \, d\vec{\xi}_1$$

$$\Delta^+ = dt \, d\vec{x} \, d\vec{\xi}' \int \left[ f(t, \vec{x}, \vec{\xi}') f(t, \vec{x}, \vec{\xi}'_1) + O\left(\Delta t \, \vec{\xi} \frac{\partial f}{\partial x}\right) \right] g' b' \, d\epsilon' \, d\vec{\xi}'_1$$

$$I = \Delta^- - \Delta^+$$

$$\frac{df}{dt} \leftrightarrow \frac{df}{dt} + \bar{\tau} \frac{\partial^2 f}{\partial t^2}$$

$$f'(t, x, \xi') \leftrightarrow f'(t, x, \xi')$$

$$f(t, x, \xi) \leftrightarrow f(t + \bar{\tau}, x + \bar{\lambda}, \xi) \leftrightarrow f(t, x, \xi) + \bar{\tau} \frac{\partial f}{\partial t} + \bar{\lambda} \frac{\partial f}{\partial x} + \dots$$

$$f_1(t, x, \xi_1) \leftrightarrow f_1(t + \bar{\tau}, x + \bar{\lambda}, \xi_1) \leftrightarrow f_1(t, x, \xi_1) + \bar{\tau} \frac{\partial f_1}{\partial t} + \bar{\lambda} \frac{\partial f_1}{\partial x} + \dots$$

In general, this formula is necessary to write in this form, but for small gradients for simple gas can be limited to one time and one length of path. However, for structural gas, for example, at heights of more than 120 km of the mean for three

Mach numbers the lag time  $10^{-8}$  c. and can be more that can be comparable with the relaxation time. In fact, the expression can be simplified, if given the orders of magnitude. Then

$$ff_1 - f'f'_1 \leftrightarrow ff_1 - f'f'_1 + \tau \frac{\partial f^0}{\partial t} f_1^0 + \tau_1 f^0 \frac{\partial f_1^0}{\partial t} + \lambda \frac{\partial f^0}{\partial x} f_1^0 + \\ + \lambda \frac{\partial f_1^0}{\partial x} f^0 + \dots - \tau' \frac{\partial f'^0}{\partial t} f'^{01} - \tau'_1 f'^0 \frac{\partial f'^0_1}{\partial t} - \dots - \lambda' \frac{\partial f'^0}{\partial x} f'^0_1 - \lambda'_1 \frac{\partial f'^0_1}{\partial x} f' - \dots$$

The integrals can be calculated, and you can find the appropriate kernel of Navier-Stokes equations. Thus, for small and medium gradients mean free time is one and the mean free path for a single-component gas is one. Significant differences will be in the interaction of gases with very different properties. So for some organic molecules the relaxation and time of the delay time with the mean free path is comparable (about  $10^{-9} - 10^{-8}$  c.)

It should be noted that, in general, ergodicity is not observed, which is very important, especially for turbulent flows. This analysis was made by T.G. Elizarova [18]. Averaging is performed for the space but is no for time. This is if we use the integral method for the construction of continuum mechanics equations. Perhaps more appropriate for the theory is the formulation of an integral equations with the average in the space and in the time that should be the average time between collisions of molecules. Otherwise, to record the derivative in a case of finite length of middle-free path of molecules (rarefied gas), we take into account only the molecules at high speed as slow collisions do not have the time to collision. One of the ways using in the kinetic theory, the replacement of discrete distribution on the smooth distribution function. However, even for the uniform distribution of function this way to replace it was not study in the transient case. A new wording of the definition of pressure, temperature and energy are suggested. The conventional formulation [11-16]

$$\frac{3}{2}kT = \frac{1}{n} \sum_k \int \frac{m_k c^{k2}}{2} f_k d\xi_k, \quad \left( n = \sum_{k=1}^N n^k \right),$$

here  $k$  - number of components, the  $T$  is the temperature,  $c^k = \xi^k - u$ , its own rate  $\xi^k$  - molecule speed. Another definition

$$\frac{3}{2}kT = \frac{\int \left( \sum_k \frac{n_k}{n} m_k \right) \left( \sum_k \frac{n_k}{n} c_k \right)^2 f d\xi}{2}.$$

In this case, one term, the traditional second-connected with a second viscosity. For the pressure tensor

$$P_{ij} = \int \left( \sum_k \frac{n_k}{n} m_k c_k \right)_i \left( \sum_k \frac{n_k}{n} m_k c_k \right)_j f d\xi.$$

Heat flux

$$q_j = \frac{\int \left( \sum_k \frac{n_k}{n} m_k \right) \left( \sum_k \frac{n_k}{n} c_k \right)_j \left( \sum_k \frac{n_k}{n} c_k \right)^2 f d\xi}{2}.$$

The question is what is measured in the experiment!

$$f_{\nu}^{(0)} = n^{\nu} \left( \frac{m}{2\pi kT} \right)^{3/2} \exp \left( -\frac{m}{2kT} c^{\nu 2} \right)$$

or temperature

$$f_{\nu}^{(0)} = n^{\nu} \left( \frac{m}{2\pi kT^{\nu}} \right)^{3/2} \exp \left( -\frac{m}{2kT^{\nu}} c^{\nu 2} \right)$$

We may have the wrong result for average temperature.

Old formulas remain for internal energy, but the definition of temperature varies. The results allow one to obtain Maxwell's equation for temperature.

## 4 The S.V. Vallander equations and the Chapman-Enskog method for the Boltzmann equation

A known solution of the Chapman-Enskog obtained using many approximations [15]. On the other hand, the classical laws of conservation, that we study in this part, the normal velocity component [19] is enters.

$$\begin{aligned} \frac{\partial}{\partial t} \int_{\tau} \rho \delta \tau + \int_{\sigma} \rho V_n \delta \sigma &= \int_{\tau} M \delta \tau . \text{ Consequently,} \\ f \left( t + dt, x + (\xi \cdot n) dt, x + (\xi \cdot \tau) dt, \xi_i + \frac{x_{0i}}{m} dt \right) dx d\xi &= \\ &= f(t, x, \xi_i) dx d\xi + (\Delta^+ - \Delta^-) dx d\xi dt . \end{aligned}$$

As a result, we need to get the conservation law in the form of

$$\frac{\partial \rho}{\partial t} + \frac{\partial [(\rho u) \cdot n + (\rho u) \cdot \tau]}{\partial x_i} = 0 .$$

Where  $n, \tau$  - the unit vectors along the normal and tangential to the surface. If the Boltzmann equation is written out in the projections, more properly, all the same in the arbitrariness of the volume should be considered normal and tangential velocities. The velocity projections on the coordinate axes are used in the numerical analysis. Therefore, the error values are of the order of self-diffusion and thermal diffusion, which will be determined by the tangential components. To understand the process of self-diffusion and thermal turn to the equilibrium distribution function and investigate the effect of small additions to the values of macroscopic parameters on its value. The equilibrium function

$$f_0 = n^0 \left( \frac{m}{2\pi kT^0} \right)^{3/2} e^{-\frac{m(\xi-u)^2}{2kT^0}} .$$

Let  $\Delta$  is a small correction. The behavior of the function we are interested in the effect of calculating the Hilbert hypothesis macro parameters through the equilibrium distribution function.



For  $\rho \cdot u$

$$\frac{1}{(n + \Delta n)} \int (n + \Delta n + \dots) \left( \frac{m}{2\pi k(T + \Delta T)} \right)^{\frac{3}{2}} e^{-\frac{m(\xi - u - \Delta u)^2}{2k(T + \Delta T)}} d\xi .$$

$$\int (n + \Delta n + \dots) \xi \left( \frac{m}{2\pi k(T + \Delta T)} \right)^{\frac{3}{2}} e^{-\frac{m(\xi - u - \Delta u)^2}{2k(T + \Delta T)}} d\xi .$$

We are using the formula in the series expansion given that in both cases one of the formulas for  $\rho \cdot u$  and  $(\rho u)$  matches. The difference between the approximations defined with first-degree order and has a structure of the solution of the Chapman-Enskog.

$$\frac{1}{n} \left( 1 - \frac{\Delta n}{n} + \dots \right) \int n \left( 1 + \frac{\Delta n}{n} + \dots \right) \left( \frac{m}{2\pi kT} \right)^{\frac{3}{2}} \left( 1 - \frac{3}{2} \frac{\Delta T}{T} \dots \right) e^{-\frac{m(\xi - u - \Delta u)^2}{2k(T + \Delta T)}} d\xi =$$

$$\frac{1}{n} \left( 1 - \frac{\Delta n}{n} + \frac{\Delta n}{n} + \dots \right) \left( 1 - \frac{3}{2} \frac{\Delta T}{T} \dots \right) \int n \left( \frac{m}{2\pi kT} \right)^{\frac{3}{2}}$$

$$e^{-\frac{m(\xi - u)^2}{2kT^0}} e^{-\frac{m(\xi - u)^2}{2kT^0} (-2(\xi - u)\Delta u - \dots \frac{\Delta T}{T})} d\xi =$$

$$\frac{1}{n} (1 + \dots) \left( 1 - \frac{3}{2} \frac{\Delta T}{T} \dots \right) \int n \left( \frac{m}{2\pi kT} \right)^{\frac{3}{2}} e^{-\frac{m(\xi - u)^2}{2kT^0}}$$

$$\left( 1 - \frac{m(\xi - u)^2}{2kT^0} \left( -2(\xi - u)\Delta u - \dots \frac{\Delta T}{T} \right) \right) d\xi$$

$$\frac{m(\xi - u - \Delta u)^2}{2k(T + \Delta T)} = \frac{m(\xi - u)^2}{2kT} (1 + 2(\xi - u)\Delta u + \dots) \left( 1 - \frac{\Delta T}{T} \right) =$$

$$\frac{m(\xi - u)^2}{2kT} \left( 1 + 2(\xi - u)\Delta u + \dots - \frac{\Delta T}{T} \right)$$

For large values of the number of particles of both formulas coincide. In general, we obtain values of various functions. Functionally-Boltzmann equation is invariant with respect to the selection of macro-distribution function. You must compare equilibrium distribution function with macro parameters that taken from the Euler and from the Navier-Stokes equations.

The difference will give us a small increment functions. We find that for the Euler equations (zero approximation of the Chapman-Enskog) the difference is zero. There are differences to the first approximation. The first approximation is responsible for the tangential component ( $p_{ij}$  - tensor of viscous stresses). Euler equations are obtained with the use of locally-equilibrium distribution function. Consequently, they are responsible for the normal component of the velocity values. In case  $p_{ij}$  of receipt of the first order correction of the parameters included in the final decision of the Chapman-Enskog we leave only part of terms after integration over the phase velocity  $\xi$ . The integrals are taken from  $f\xi$  functions, i.e. for  $(\rho u)$ . Consider regardless of macro parameters.

$$\frac{Df_0}{dt} = \frac{1}{n} f_0 \frac{\partial n}{\partial t} + \frac{3}{2} \frac{1}{T} f_0 \frac{\partial T}{\partial t} + \frac{mc^2}{2kT^2} f_0 \frac{\partial T}{\partial t} + f_0 \left( \frac{m}{kT} (\xi - u) \frac{\partial u}{\partial t} \right) +$$

$$\begin{aligned} & \xi \cdot \left\{ \frac{1}{n} f_0 \frac{\partial n}{\partial x} + \left(-\frac{3}{2}\right) \frac{1}{T} f_0 \frac{\partial T}{\partial x} + \frac{m c^2}{2 k T^2} f_0 \frac{\partial T}{\partial x} + f_0 \left( \frac{m}{k T} (\xi - u) \frac{\partial u}{\partial t} \right) \right\} = \\ & = 2J(f_0, f_0 \varphi^k) = \int f_0 f_1^0 \left( \varphi_1^{(k)'} + \varphi^{(k)'} - \varphi_1^{(k)} - \varphi^{(k)} \right) g b db d\epsilon d\xi_1 \xi = 0 . \end{aligned}$$

In classical case

$$\left. \frac{\partial f_0}{\partial t} \right|_{t=0} = f_0 \left\{ \frac{m}{k T} \left( c_i c_j - \frac{1}{3} c^2 \delta_{ij} \right) \frac{\partial u_i}{\partial t} + \frac{1}{2 T} \frac{\partial T}{\partial t} c_i \left[ \left( \frac{m}{k T} \right) c^2 - 5 \right] \right\} .$$

The Boltzmann equation is written relative to the total distribution function and consists of locally-equilibrium functions and additional term.

The tangential component of the velocity, which is obtained due to the arbitrary direction of the velocity relative to the position of the coordinate axes, is equal to

$$\int n \cdot (\tau \cdot f \xi) ds d\xi = \int div (\tau \cdot f \xi) dx d\xi$$

$\tau f$  gives us additional term. In addition to locally equilibrium function has a term

$$f_0 \left[ \frac{p_{ij}}{2p} \left( \frac{m}{2T} \right) c_i c_j - \frac{q_i}{p} \left( \frac{m}{kT} \right) \left( 1 - \frac{c^2}{5} \frac{m}{kT} \right) c_i \right]$$

The main contribution to the integral will give the derivatives of locally equilibrium distribution function, which determines the self-diffusion equations and termo-diffusion S.V. Vallander. The second derivative appears due term  $c_i \cdot \frac{\partial f}{\partial r_i}$  .

## 5 Conclusion

The paper proposes a refinement of the equations of a continuous environment and the Boltzmann equation with allowance for the angular momentum and delay, as well as the position of the center of inertia of the elementary volume. The possibility of describing discrete media in the framework of continuum mechanics is analysed. Set the role of dispersion and delays in physical and chemical processes of relaxation type. Equations by S.V. Vallander theory were obtained from the kinetic results.

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