Formulation And Experimental Justification Of The Long-Term Strength Criterion

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Abstract

The new results on the effect of thermal embrittlement (embrittlement of structural metallic materials under prolonged action of relatively low stresses and high temperatures) are obtained. The main attention is focused on the formulation of the relations for the damage parameter and the development of the long-term strength criterion. For comparison of the obtained relations with the experimental results observational studies were performed on the determination of damage accumulation under high temperature creep conditions for various metals and alloys: copper, aluminum, Magnox AL80, Nickel and 0.1% palladium alloy, various heat resistant alloys. The experiments were carried out at different temperatures and levels of tensile stresses. Theoretical curves of density change were compared with the experimental results for some of these metals and alloys. At the time interval 30-500 hours the damage function is expressed as a straight line. The theoretical curves have the general character irrespective of the material and the temperature-power effects, which indicates the existence of a common law of damage processes and indirectly confirms the selection of a physical damage parameter as the ratio of the current density of the material to the initial density.

For the description of brittle fractures the conception of continuity (Kachanov [1]) and damage (Rabotnov [2, 3]) was developed. To materialize the damage parameter various definitions were offered [4-6]. In the paper the parameter of continuity is determined by the ratio $\psi = \rho/\rho_0$ (ρ_0 is initial, ρ is current density) and it is an integral measure of the accumulation of structural microdefects during long-term high-temperature loading [7-14]. In the initial conditions $t = 0$, $\rho = \rho_0$, $\psi = 1$, at the fracture time $t = t_f$, $\rho = 0$, $\psi = 0$.

In the brittle model of Kachanov it is supposed that creep deformation doesn't influence to fracture processes, and the kinetic equation of the continuity parameter is taken as a power function of effective stress [1]

$$\frac{d\psi}{dt} = -A\left(\frac{\sigma_{\max}}{\psi}\right)^n, \tag{1}$$

where $A > 0$, $n \geq 0$ are constants, $\sigma_{\max}/\psi$ is effective stress.

The tension problem of specimen under the action of constant load P is solved. It is considered that brittle fracture happens at small deformations therefore it is possible to neglect change of specimen cross section, i.e. the conditions $F = F_0$, $\sigma_{\max} = \sigma = P/F = P/F_0 = \sigma_0 = \text{const}$, ($\sigma$ is true stress, σ_0 is nominal stress, F_0, F are the initial and current cross section area of a specimen) are accepted. At these assumptions the equation (1) can be expressed in the form

$$\frac{d\psi}{dt} = -A\left(\frac{\sigma_0}{\psi}\right)^n. \tag{2}$$

In the Rabotnov's brittle fracture model [3] the damage parameter ω ($0 \le \omega \le 1$) is defined by the following kinetic equation

$$\frac{d\omega}{dt} = A\sigma^n. \tag{3}$$

The damage parameter is introduced as $\omega = F_T/F_0$ (F_T is the total area of pores). From condition $F = F_0 - F_T$, we have $F = F_0(1-\omega)$, $\sigma = P/F = \sigma_0 F_0/F = \sigma_0/(1-\omega)$. Taking into account these relations the kinetic equation (3) can be written as

$$\frac{d\omega}{dt} = A\left(\frac{\sigma_0}{1-\omega}\right)^n, \tag{4}$$

The equations (2) and (4) are identical at $\omega = 1-\psi$, $d\psi = -d\omega$. From the solution of these equations under the initial conditions $t = 0$, $\psi = 1$, $\omega = 0$ we have

$$\psi = 1 - \omega = \left[1 - (n+1)A\sigma_0^{nt}\right]^{\frac{1}{n+1}}. \tag{5}$$

Accepting the fracture conditions $t = t_f^b$, $\psi = 0$, $\omega = 1$ (in the general case, the fracture occurs when $\rho = \rho_*$, $\psi = \psi_*$, $\omega = \omega_*$ where the asterisk indicated the limit values of density and damage parameters), from (5) follows the criterion

$$t_f^b = \frac{1}{(n+1)\cdot A\sigma_0^n}. \tag{6}$$

Such approach can give to the parameter of Kachanov the physical content. However from condition $F = F_0$, which is used in Kachanov's theory, follows $\omega = 0$, i.e. the concept of damage loses meaning. Thus, similar interpretation of Kachanov's continuity parameter isn't represented fully correct. The development of the conception of damage received in work [3], where the system of equations for the creep deformation ε and damage parameter ω was proposed. When the criterion of ductile-brittle fracture is determined using this system of equations, the condition of incompressibility, which is contrary to the damage conception, is accepted.

To overcome these contradictions a system of equations for the creep rate and damage, based on the continuity parameter $\psi = \rho/\rho_0$, is proposed. Let's consider the following system of equations

$$\psi^\beta \frac{d\varepsilon}{dt} = B\sigma^m, \tag{7}$$

$$\psi^\alpha \frac{d\psi}{dt} = -A\sigma^n, \tag{8}$$

where B, A, α, β are constants.
Taking into account the mass conservation law $\rho_0 l_0 F_0 = \rho l F$ the true stress can be expressed as $\sigma = \sigma_0 \psi e^{\varepsilon}$. Taking into account this relation the equations (7)-(8) can be written in the form

$$\frac{d\varepsilon}{dt} = B\sigma_0^m \psi^{m-\beta} e^{m\varepsilon}, \tag{9}$$

$$\frac{d\psi}{dt} = -A\sigma_0^n \psi^{n-\alpha} e^{n\varepsilon}. \tag{10}$$

The system of equations (9)-(10) can be solved approximately, for example, for the case of purely brittle fracture and small deformations, when the approximations $e^{m\varepsilon} \approx 1$, $e^{n\varepsilon} \approx 1$ can be considered. In this case, using the initial conditions $t = 0$, $\psi = 1$, $\omega = 0$ we can receive the following analytical solutions

$$\psi = \left[1 - (\alpha - n + 1)A\sigma_0^{nt}\right]^{\frac{1}{\alpha-n+1}}, \tag{11}$$

$$\varepsilon = \frac{B\sigma_0^{m-n}}{A\gamma}\left\{1 - \left[1 - (\alpha - n + 1)A\sigma_0^{nt}\right]^{\frac{\gamma}{\alpha-n+1}}\right\}, \tag{12}$$

where $\gamma = m - \beta + \alpha - n + 1$.
Consider the approximate and exact solutions for the damage function $\psi(\varepsilon)$. Taking $e^{m\varepsilon} \approx 1$, $e^{n\varepsilon} \approx 1$ from the system of equations (9)-(10) we get

$$\frac{d\psi}{d\varepsilon} = -\frac{A}{B}\sigma_0^{n-m}\psi^{n-\alpha-m+\beta}. \tag{13}$$

The solution of equation (13) with initial conditions $\psi = 1$, $\varepsilon = 0$ has the form

$$\psi(\varepsilon) = \left[1 - \frac{A\sigma_0^{n-m}\left(1 - n + \alpha + m - \beta\right)}{B}\varepsilon\right]^{\frac{1}{1-n+\alpha+m-\beta}}. \tag{14}$$

The exact solution of equations (9)-(10) for function $\psi(\varepsilon)$ can be received. Dividing (10) to (9), we will obtain the following equation

$$\frac{d\psi}{d\varepsilon} = -\frac{A}{B}\sigma_0^{n-m}\psi^{n-\alpha-m+\beta}e^{(n-m)\varepsilon}. \tag{15}$$

Using the initial conditions $\psi = 1$, $\varepsilon = 0$ and solving (15) we receive

$$\psi(\varepsilon) = \left[1 + \frac{A\sigma_0^{n-m}(1 - n + \alpha + m - \beta)}{B(n-m)}(1 - e^{(n-m)\varepsilon})\right]^{\frac{1}{1-n+\alpha+m-\beta}}. \tag{16}$$

On Fig. 1 the curves $\psi(\varepsilon)$ according formulas (14) and (16) for different values of parameter α ($\alpha = 6$ - curves 1, 1', $\alpha = 4$ - curves 2, 2' и $\alpha = 2$ - curves 3, 3') are shown. In the calculations the following values of coefficients were used: $A = 10^{-9}[MPa]^{-2}$, $B = 5 \cdot 10^{-14}[MPa]^{-4}$, $\sigma_0 = 100\,MPa$, $n = 2$, m $= 4$, $\beta = 1$.
As can be seen from Fig. 1 the damage curves for formulas (14) and (16) are identical.
Taking the fracture conditions $t = t_f$, $\psi = 0$, from (11) we obtain the creep fracture criterion

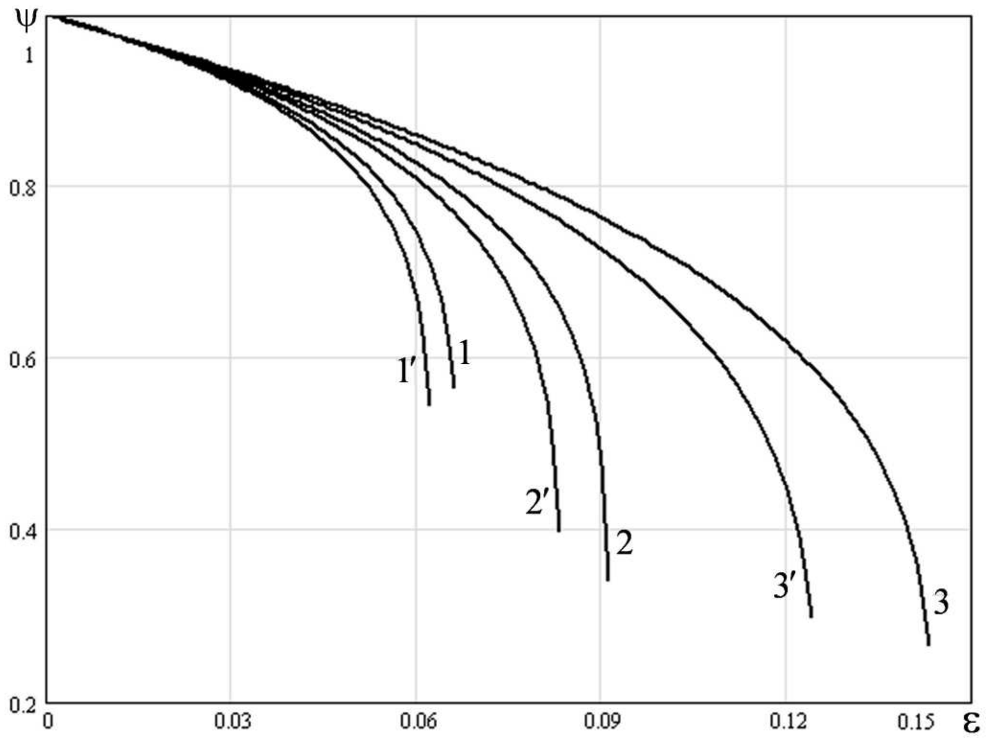

Figure 1: The curves $\psi(\varepsilon)$ according formulas (14) and (16) for different values of parameter α: $\alpha = 6$ - curves 1, 1', $\alpha = 4$ - curves 2, 2' и $\alpha = 2$ - curves 3, 3'.

$$t_f^b = \frac{1}{(\alpha - n + 1) \cdot A\sigma_0^n}. \tag{17}$$

When $\alpha = 2n$ the criterion (17) coincides with the Kachanov-Rabotnov criterion.
In Fig. 2 are shown the theoretical creep deformation curves according to the relation (12) for different values of the coefficient α ($\alpha = 6$ - curve 1, $\alpha = 4$ - curve 2 и $\alpha = 2$ - curve 3). As can be seen from this figure, the system of equations (9)-(10) is able to describe the third phase of creep curves, which is determined by the processes of damage accumulation. In the calculations the following values of coefficients were used: $A = 10^{-9}[MPa]^{-2}$, $B = 5 \cdot 10^{-17}[MPa]^{-4}$, $\sigma_0 = 100\,MPa$, $n = 2$, m = 4, $\beta = 1$.
For comparison of the obtained relations with the experimental results observational studies were performed on the determination of damage accumulation under high temperature creep conditions for various metals and alloys: copper, aluminum, Magnox AL80, Nickel and 0.1% palladium alloy, various heat resistant alloys [8-14]. The experiments were carried out at different temperatures and levels of tensile stresses. Dwell times under load to failure were within 30-500 hours. Theoretical curves of density change were compared with the experimental results for some of these metals and alloys. On Fig. 3 theoretical curves $\psi(\varepsilon)$ (solid line) and experimental points of density changes of pure copper during creep under 500° C [8] (circle points) and 250° C [10] (cross points) are shown.
On Fig. 4 theoretical curves $\psi(t)$ (solid line) and experimental points of density

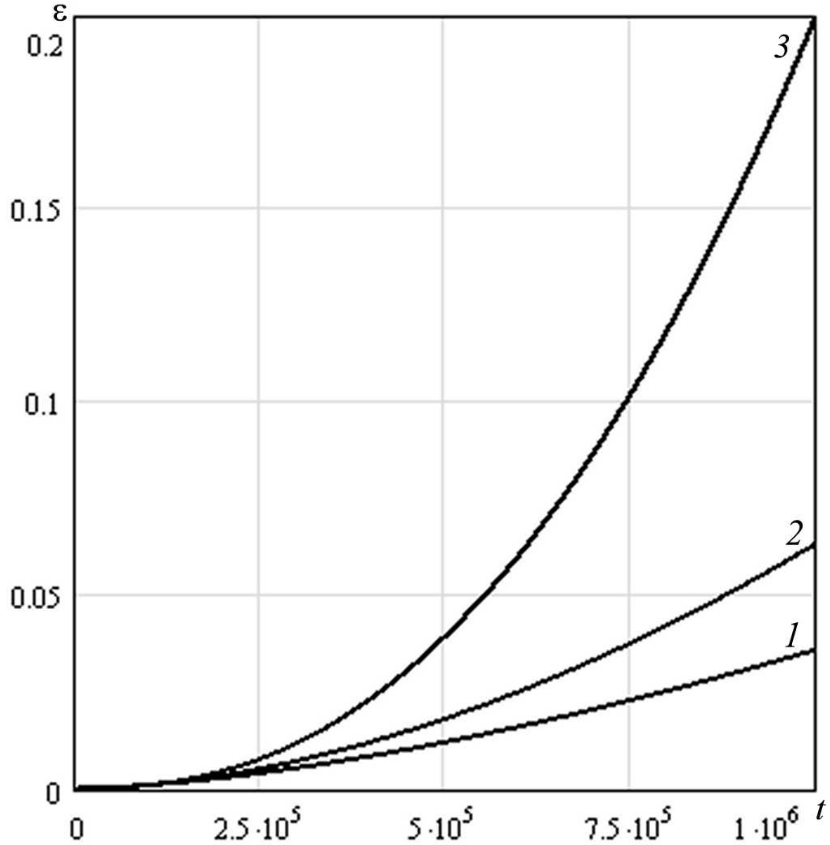

Figure 2: The theoretical creep deformation curves according to the relation (12) for different values of the coefficient α: ($\alpha = 6$ - curve 1, $\alpha = 4$ - curve 2 и $\alpha = 2$ - curve 3.

changes of pure aluminum during creep under 250° C [9] (circle points) and nickel alloy under 503° C [13] (cross points) are shown.
From Fig. 3-4 it follows that the experimental points are described well by straight lines and have the general character for different metals tested under various temperature and force conditions. These results allow us to consider the damage parameter $\psi = \rho/\rho_0$ as universal characteristic of porosity accumulation in the creep process.

Acknowledgements

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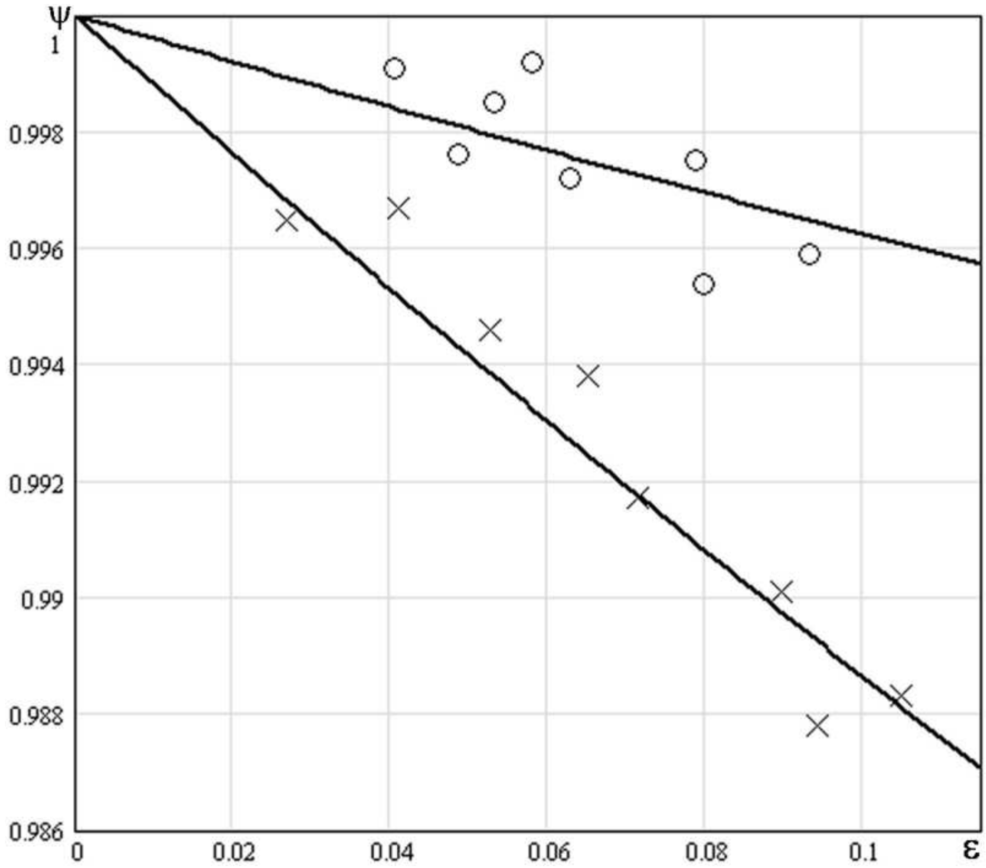

Figure 3: Theoretical curves $\psi(\varepsilon)$ (solid line) and experimental points of density changes of pure copper during creep under 500° C [8] (circle points) and 250° C [10] (cross points).

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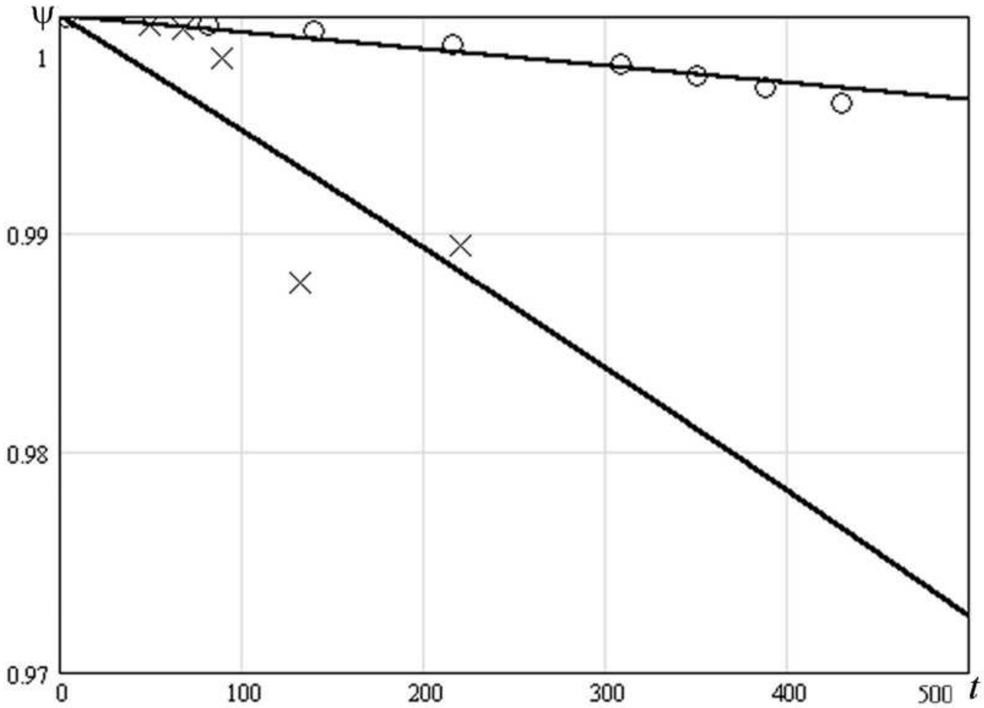

Figure 4: Theoretical curves $\psi(t)$ (solid line) and experimental points of density changes of pure aluminum during creep under 250° C [9] (circle points) and nickel alloy under 503° C [13] (cross points).

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