Thermoelasticity of micropolar thin plates

Aslanyan N.S., Sargsyan S.H.

asnaira73@mail.ru

Abstract

The problems of the stress state caused by uneven heating are of great importance for the analysis of the strength and correct functioning of the structures of the new technology, including micro and nano techniques, operating under conditions of unevenly distributed temperature fields. Three-dimensional theory of temperature stresses in the micropolar theory of elasticity was developed by Novacki [1] [2]. In papers [3][4] system of asymptotically justified hypotheses is developed and on the basis of them applied theories of micropolar elastic thin plates and shells are constructed. In paper [5] applied theory of thermoelasticity of micropolar thin shells is constructed. Developing this direction in current paper theory of thermal stresses of bending deformation of micropolar elastic thin plates is introduced and on the basis of this theory problems of thermoelastic bending of rectangular and circular plates are studied, which are brought to final numerical results. Effective properties of micropolar material rigidity are revealed compared with classical case.

1 Problem statement

Isotropic elastic plate of constant thickness $2h$ is considered as three-dimensional body. Axes x_1, x_2 are referred to the plate middle plane. We proceed from main equations of three-dimensional asymmetric linear theory of quasistatic thermoelasticity [2]:
Equilibrium equations:

$$\sigma_{ji,j} = 0,\ \mu_{ji,j} + \epsilon_{ijk}\sigma_{jk} = 0 \tag{1.1}$$

Geometric relations:

$$\gamma_{ji} = u_{i,j} - \epsilon_{kji}\omega_k,\ \chi_{ji} = \omega_{i,j} \tag{1.2}$$

Physical relations of elasticity:

$$\begin{aligned}\sigma_{ji} &= (\mu+\alpha)\gamma_{ji} + (\mu-\alpha)\gamma_{ij} + (\lambda\gamma_{kk} - \alpha_T T)\delta_{ij},\\ \mu_{ji} &= (\gamma+\varepsilon)\chi_{ji} + (\gamma-\varepsilon)\chi_{ij} + \beta\chi_{kk}\delta_{ij}.\end{aligned} \tag{1.3}$$

Here σ_{ij} are stresses, μ_{ij}-momental stresses, u_i-displacements, ω_i re body points rotations during the deformation, γ_{ij} are deformations, χ_{ij}-bending-torsions, $\lambda, \mu, \alpha, \gamma, \varepsilon, \beta$ -elastic constants, α_T is the linear coefficient of temperature expansion of body material $(i, j = 1, 2, 3)$.
Boundary conditions should be added to the above mentioned equations. $\sigma_{3i},\ \sigma_{33},\ \mu_{3i},\ \mu_{33} (i = 1, 2)$ are given on the facial planes, stresses or displacements and rotations can be given on the lateral surface, or mixed boundary conditions can be given on different parts of the surface.
Energy balance equation in three-dimensional micropolar thermoelasticity has the following form:

$$\int\int_S \int_{-h}^{h} W dx_1 dx_2 dx_3 = A, \tag{1.4}$$

where W is the density of deformation potential energy:

$$\begin{aligned} W = \frac{1}{2}(&\sigma_{11}\gamma_{11} + \sigma_{22}\gamma_{22} + \sigma_{33}\gamma_{33} + \sigma_{12}\gamma_{12} + \sigma_{21}\gamma_{21} + \sigma_{13}\gamma_{13} + \\ &+\sigma_{23}\gamma_{23} + \sigma_{32}\gamma_{32} + \mu_{11}\chi_{11} + \mu_{22}\chi_{22} + \mu_{33}\chi_{33} + \mu_{12}\chi_{12} + \mu_{13}\chi_{13} + \\ &+\mu_{31}\chi_{31} + \mu_{23}\chi_{23} + \mu_{32}\chi_{32}) - \frac{\alpha_t T}{2}(\sigma_{11} + \sigma_{22} + \sigma_{33}), \end{aligned} \tag{1.5}$$

A is the work of external forces and moments on displacements and rotations of deformation:

$$\begin{aligned} A = \frac{1}{2}\Bigg\{ \Bigg[&\int_{-h}^{h} dx_3 \int_{l_1} \left(\sigma_{21}^0 u_1 + \sigma_{22}^0 u_2 + \sigma_{23}^0 u_3 + \mu_{21}^0 \omega_1 + \mu_{22}^0 \omega_2 + \mu_{23}^0 \omega_3\right) dx_1 + \\ &+ \int_{-h}^{h} dx_3 \int_{l_2} \left(\sigma_{11}^0 u_1 + \sigma_{12}^0 u_2 + \sigma_{13}^0 u_3 + \mu_{11}^0 \omega_1 + \mu_{12}^0 \omega_2 + \mu_{13}^0 \omega_3\right) dx_2 \Bigg] + \\ &+ \Bigg[\iint_{S^+} \left(p_1^+ u_1 + p_2^+ u_2 + p_3^+ u_3 + m_1^+ \omega_1 + m_2^+ \omega_2 + m_3^+ \omega_3\right) dx_1 dx_2 + \\ &+ \iint_{S^-} \left(p_1^- u_1 + p_2^- u_2 + p_3^- u_3 + m_1^- \omega_1 + m_2^- \omega_2 + m_3^- \omega_3\right) dx_1 dx_2 \Bigg] \Bigg\}. \end{aligned} \tag{1.6}$$

On the basis of Hook's law (1.3) density (1.5) of deformation potential energy can be expressed by components of tensors of deformation and bending-torsions, or by components of force and moment stresses. Let's introduce the density W of deformation potential energy by by components of tensors of deformation and bending-torsions:

$$\begin{aligned} W = \frac{1}{2}\{&2\mu\left(\gamma_{11}^2 + \gamma_{22}^2 + \gamma_{33}^2\right) + \lambda\left(\gamma_{11} + \gamma_{22} + \gamma_{33}\right)^2 + \\ &+ (\mu + \alpha)\left(\gamma_{12}^2 + \gamma_{21}^2 + \gamma_{13}^2 + \gamma_{31}^2 + \gamma_{23}^2 + \gamma_{32}^2\right) + \\ &+2(\mu - \alpha)\left(\gamma_{12}\gamma_{21} + \gamma_{13}\gamma_{31} + \gamma_{23}\gamma_{32}\right) + 2\gamma\left(\chi_{11}^2 + \chi_{22}^2 + \chi_{33}^2\right) + \\ &+\beta\left(\chi_{11} + \chi_{22} + \chi_{33}\right)^2 + (\gamma + \varepsilon)\left(\chi_{12}^2 + \chi_{21}^2 + \chi_{13}^2 + \chi_{31}^2 + \chi_{23}^2 + \chi_{32}^2\right) + \\ &+2(\gamma - \varepsilon)\left(\chi_{12}\chi_{21} + \chi_{13}\chi_{31} + \chi_{23}\chi_{32}\right)\} - (3\lambda + 2\mu)\,\alpha_T T\left(\gamma_{11} + \gamma_{22} + \gamma_{33}\right) \end{aligned} \tag{1.7}$$

As it is accepted in the classical theory of elasticity, as well as in the micropolar theory of elasticity, the application of variation methods is effective during the determination of temperature stresses. General variation principle in micropolar theory of thermoelasticity is studied, the functional of which is the following :

$$
\begin{aligned}
I = \iint_S \int_{-h}^{h} \Bigg\langle W - \Bigg\{ \sigma_{11}\left(\gamma_{11} - \frac{\partial u_1}{\partial x_1}\right) + \sigma_{22}\left(\gamma_{22} - \frac{\partial u_2}{\partial x_2}\right) + \sigma_{33}\left(\gamma_{33} - \frac{\partial u_3}{\partial x_3}\right) + \\
+\sigma_{12}\left[\gamma_{12} - \left(\frac{\partial u_2}{\partial x_1} - \omega_3\right)\right] + \sigma_{21}\left[\gamma_{21} - \left(\frac{\partial u_1}{\partial x_2} + \omega_3\right)\right] + \\
+\sigma_{13}\left[\gamma_{13} - \left(\frac{\partial u_3}{\partial x_1} + \omega_2\right)\right] + \sigma_{31}\left[\gamma_{31} - \left(\frac{\partial u_1}{\partial x_3} - \omega_2\right)\right] + \\
+\sigma_{23}\left[\gamma_{23} - \left(\frac{\partial u_3}{\partial x_2} - \omega_1\right)\right] + \sigma_{32}\left[\gamma_{32} - \left(\frac{\partial u_2}{\partial x_3} + \omega_1\right)\right] + \\
+\mu_{11}\left(\chi_{11} - \frac{\partial \omega_1}{\partial x_1}\right) + \mu_{22}\left(\chi_{22} - \frac{\partial \omega_2}{\partial x_2}\right) + \mu_{33}\left(\chi_{33} - \frac{\partial \omega_3}{\partial x_3}\right) + \\
+\mu_{12}\left(\chi_{12} - \frac{\partial \omega_2}{\partial x_1}\right) + \mu_{21}\left(\chi_{21} - \frac{\partial \omega_1}{\partial x_2}\right) + \mu_{13}\left(\chi_{13} - \frac{\partial \omega_3}{\partial x_1}\right) + \\
+\mu_{23}\left(\chi_{23} - \frac{\partial \omega_3}{\partial x_2}\right) + \mu_{31}\left(\chi_{31} - \frac{\partial \omega_1}{\partial x_3}\right) + \mu_{32}\left(\chi_{32} - \frac{\partial \omega_2}{\partial x_3}\right) \Bigg\} \Bigg\rangle dx_1 dx_2 dx_3 - \\
- \iint_{s^+} \left[p_1^+ u_1 + p_2^+ u_2 + p_3^+ u_3 + m_1^+ \omega_1 + m_2^+ \omega_2 + m_3^+ \omega_3\right]_{x_3 = h} dx_1 dx_2 + \\
+ \iint_{s^-} \left[p_1^- u_1 + p_2^- u_2 + p_3^- u_3 + m_1^- \omega_1 + m_2^- \omega_2 + m_3^- \omega_3\right]_{x_3 = -h} dx_1 dx_2 + \\
+ \int_{-h}^{+h} dx_3 \int_{l_1'} \left(\sigma_{21}^0 u_1 + \sigma_{22}^0 u_2 + \sigma_{23}^0 u_3 + \mu_{21}^0 \omega_1 + \mu_{22}^0 \omega_2 + \mu_{23}^0 \omega_3\right) dx_1 + \\
+ \int_{-h}^{+h} dx_3 \int_{l_1''} \left[\sigma_{21}\left(u_1 - u_1^0\right) + \sigma_{22}\left(u_2 - u_2^0\right) + \sigma_{23}\left(u_3 - u_3^0\right) + \right. \\
\left. +\mu_{21}\left(\omega_1 - \omega_1^0\right) + \mu_{22}\left(\omega_2 - \omega_2^0\right) + \mu_{23}\left(\omega_3 - \omega_3^0\right)\right] dx_1 + \\
+ \int_{-h}^{+h} dx_3 \int_{l_2'} \left(\sigma_{11}^0 u_1 + \sigma_{12}^0 u_2 + \sigma_{13}^0 u_3 + \mu_{11}^0 \omega_1 + \mu_{12}^0 \omega_2 + \mu_{13}^0 \omega_3\right) dx_2 + \\
+ \int_{-h}^{+h} dx_3 \int_{l_2''} \left[\sigma_{11}\left(u_1 - u_1^0\right) + \sigma_{12}\left(u_2 - u_2^0\right) + \sigma_{13}\left(u_3 - u_3^0\right) + \right. \\
\left. +\mu_{11}\left(\omega_1 - \omega_1^0\right) + \mu_{12}\left(\omega_2 - \omega_2^0\right) + \mu_{13}\left(\omega_3 - \omega_3^0\right)\right] dx_2
\end{aligned} \tag{1.8}
$$

The functional (1.8) is called full functional of three-dimensional micropolar theory of thermoelasticity. On the basis of it variation equation $\delta I = 0$ can be obtained, accepting that virtual increments $\delta\gamma_{mn}, \delta\chi_{mn}, \delta u_n, \delta\omega_n, \delta\sigma_{mn}, \delta\mu_{mn}$ are mutually independent. Then all main equations (1.1) - (1.3) and natural boundary conditions of three dimensional problem of micropolar thermoelasticity will be obtained.
It is accepted that plate thickness is small compared with its other sizes. We?ll start from the following main concept: in static case general thermoelastic state of thin three-dimensional body consists of internal state, covering the plate, and of boundary layers, localizing near the plate edge Σ. The construction of the general

applied two-dimensional theory of thermoelasticity of micropolar elastic thin plates is closely connected with the construction of the internal problem.
Considering that the hypotheses method rather intensively and easily for engineering practice leads to final results, the model of thermoelasticity of micropolar isotropic thin plates will be constructed on the basis of hypotheses method. The hypotheses are formulated on the basis of the asymptotic analysis result of the stated three-dimensional boundary-value problem of micropolar theory of thermoelasticity in thin three-dimensional domain of the plate [6].

2 Initial hypotheses

On the basis of qualitative results [6] of asymptotic solution of the system of equations (1.1) - (1.4) with the above mentioned boundary conditions and asymptotic integration process of this boundary-value problem, following general hypotheses are stated for the construction of the model of micropolar thermoelasticity of isotropic thin plates with free fields of displacements and rotations [3] - [5]:
1)Assumption of linear distribution of components of vectors of displacement and free rotation by coordinate x_3 is accepted as kinematic hypothesis:

$$u_i = x_3\psi_i(x_1, x_2), u_3 = w(x_1, x_2) \qquad (i = 1, 2), \tag{2.1}$$

$$\omega_i = \Omega_i(x_1, x_2), \omega_3 = x_3\iota(x_1, x_2) \qquad (i = 1, 2), \tag{2.2}$$

where u_i, w are displacements of middle plane points along the x_i and x_3; ψ_i-full angels of rotation of the normal to the middle plane element around the axis x_i, Ω_i-free rotations of the three-dimensional plate points around the axis x_3. In papers [3] - [5] the kinematic hypothesis (2.1), (2.2) is called Timoshenko's generalized kinematic hypothesis of theory of micropolar plates and shells.
Following hypotheses are accepted as static ones:
2) In formulas for γ_{ii} of generalized Hook's law (1.2) force stress σ_{33} can be neglected in relation to force normal stresses, σ_{ii} and in formulas for $\chi_{i3}, (i = 1, 2)$ moment stresses μ_{3i} can be neglected in relation to moment stresses $\mu_{i3}, (i = 1, 2)$.
3) For determination of deformations, bending-torsions, force and moment stresses first we accept following relations for force stresses σ_{3i} and moment stress μ_{33}:

$$\sigma_{3i} = \sigma_{3i}^0(x_1, x_2), (i = 1, 2), \mu_{33} = \mu_{33}^0(x_1, x_2). \tag{2.3}$$

After determination of mentioned quantities values of σ_{3i} and μ_{33} will be determined as sum of (2.3) and result of integration of the first two and sixth equilibrium equations of (1.1), or which condition will be required that averaged along the plate thickness quantities are equal to zero.
4) Linear change along the plate thickness is accepted for temperature function T [7]:

$$T = \frac{x_3}{2h}T_0(x_1, x_2). \tag{2.4}$$

The accepted kinematic, static hypotheses and hypothesis on linear distribution of the temperature function let us reduce the problem of determination of spatial stress state of micropolar plate to two-dimensional problem.

3 Determination of components of deformation and bending-torsions tensors

On the basis of kinematic hypothesis (2.1), (2.2) following formulas will be obtained for components of deformation and bending-torsion tensors:

$$\begin{aligned}
&\gamma_{11} = x_3 K_{11}(x_1,x_2), \ \gamma_{12} = x_3 K_{12}(x_1,x_2), \ \gamma_{32} = \Gamma_{32}(x_1,x_2), \\
&\gamma_{22} = x_3 K_{22}(x_1,x_2), \ \gamma_{21} = x_3 K_{21}(x_1,x_2), \ \gamma_{23} = \Gamma_{23}(x_1,x_2), \\
&\gamma_{13} = \Gamma_{13}(x_1,x_2), \quad \gamma_{31} = \Gamma_{31}(x_1,x_2), \ \gamma_{33} = 0 \\
&\chi_{11} = k_{11}(x_1,x_2), \quad \chi_{12} = k_{12}(x_1,x_2), \quad \chi_{31} = 0 \\
&\chi_{22} = k_{22}(x_1,x_2), \quad \chi_{21} = k_{21}(x_1,x_2), \quad \chi_{32} = 0 \\
&\chi_{33} = k_{33}(x_1,x_2), \quad \chi_{13} = x_3 l_{13}(x_1,x_2), \quad \chi_{23} = x_3 l_{23}(x_1,x_2)
\end{aligned} \tag{3.1}$$

where following notations are accepted:

$$\begin{aligned}
&K_{11} = \frac{\partial \psi_1}{\partial x_1}, \ K_{22} = \frac{\partial \psi_2}{\partial x_2}, \ K_{12} = \frac{\partial \psi_2}{\partial x_1} - \iota, \ K_{21} = \frac{\partial \psi_1}{\partial x_2} + \iota, \\
&\Gamma_{31} = \psi_1 - \Omega_2, \ \Gamma_{32} = \psi_2 + \Omega_1, \ \Gamma_{13} = \frac{\partial w}{\partial x_1} + \Omega_2, \ \Gamma_{23} = \frac{\partial w}{\partial x_2} - \Omega_1, \\
&k_{11} = \frac{\partial \Omega_1}{\partial x_1}, \ k_{22} = \frac{\partial \Omega_2}{\partial x_2}, \ k_{12} = \frac{\partial \Omega_2}{\partial x_1}, \ k_{21} = \frac{\partial \Omega_1}{\partial x_2}, \\
&k_{33} = \iota, \ l_{13} = \frac{\partial \iota}{\partial x_1}, \ l_{23} = \frac{\partial \iota}{\partial x_2}.
\end{aligned} \tag{3.2}$$

4 Determination of components of force and moment stresses tensors

On the basis of physical relations (1.2), formulas for deformations, bending-torsions (3.1)-(3.2) and hypotheses 2)-4) following formulas will be obtained for force and moment stresses:

$$\begin{aligned}
&\sigma_{11} = x_3 \frac{E}{1-\nu^2}\left[K_{11} + \nu K_{22} - (1+\nu)\alpha_t \frac{T_0}{2h}\right], \\
&\sigma_{22} = x_3 \frac{E}{1-\nu^2}\left[\nu K_{11} + K_{22} - (1+\nu)\alpha_t \frac{T_0}{2h}\right], \\
&\sigma_{12} = x_3\left[(\mu+\alpha)K_{12} + (\mu-\alpha)K_{21}\right], \\
&\sigma_{21} = x_3\left[(\mu+\alpha)K_{21} + (\mu-\alpha)K_{12}\right],
\end{aligned} \tag{4.1}$$

$$\begin{aligned}
&\sigma_{13} = (\mu+\alpha)\Gamma_{13} + (\mu-\alpha)\Gamma_{31}, \ \sigma_{23} = (\mu+\alpha)\Gamma_{23} + (\mu-\alpha)\Gamma_{32}, \\
&\sigma_{31}^0 = (\mu+\alpha)\Gamma_{31} + (\mu-\alpha)\Gamma_{13}, \ \sigma_{32}^0 = (\mu+\alpha)\Gamma_{32} + (\mu-\alpha)\Gamma_{23}.
\end{aligned} \tag{4.2}$$

$$\begin{aligned}
&\sigma_{31} = \sigma_{31}^0(x_1;x_2) + \left(\frac{h^2}{6} - \frac{x_3^2}{2}\right)\left(\frac{\partial \sigma_{11}^1}{\partial x_1} + \frac{\partial \sigma_{21}^1}{\partial x_2}\right) \\
&\sigma_{32} = \sigma_{32}^0(x_1;x_2) + \left(\frac{h^2}{6} - \frac{x_3^2}{2}\right)\left(\frac{\partial \sigma_{22}^1}{\partial x_2} + \frac{\partial \sigma_{12}^1}{\partial x_1}\right)
\end{aligned} \tag{4.3}$$

$$\sigma_{33} = -x_3\left(\frac{\partial\sigma_{13}}{\partial x_1} + \frac{\partial\sigma_{23}}{\partial x_2}\right) + \frac{p_3^+ - p_3^-}{2} = x_3\frac{\tilde{p_3}}{2h} + \sigma_{33}^0, \tag{4.4}$$

$$\begin{aligned}
&\mu_{11} = (\beta+2\gamma)k_{11} + \beta(k_{22}+k_{33}),\ \mu_{22} = (\beta+2\gamma)k_{22} + \beta(k_{11}+k_{33}),\\
&\mu_{33}^0 = (\beta+2\gamma)k_{33} + \beta(k_{11}+k_{22}),\\
&\mu_{12} = (\gamma+\varepsilon)k_{12} + (\gamma-\varepsilon)k_{21},\ \mu_{21} = (\gamma+\varepsilon)k_{21} + (\gamma-\varepsilon)k_{12},
\end{aligned} \tag{4.5}$$

$$\mu_{13} = x_3\frac{4\gamma\varepsilon}{\gamma+\varepsilon}l_{13},\ \mu_{23} = x_3\frac{4\gamma\varepsilon}{\gamma+\varepsilon}l_{23}. \tag{4.6}$$

$$\begin{aligned}
&\mu_{31} = -x_3\left(\frac{\partial\mu_{11}}{\partial x_1} + \frac{\partial\mu_{21}}{\partial x_2} + \sigma_{23} - \sigma_{32}\right) + \frac{m_1^+ - m_1^-}{2}\\
&\mu_{32} = -x_3\left(\frac{\partial\mu_{12}}{\partial x_1} + \frac{\partial\mu_{22}}{\partial x_2} + \sigma_{31} - \sigma_{13}\right) + \frac{m_2^+ - m_2^-}{2}.
\end{aligned} \tag{4.7}$$

$$\mu_{33} = \mu_{33}^0\left(x_1; x_2\right) + \left(\frac{h^2}{6} - \frac{x_3^2}{2}\right)\left(\frac{\partial\mu_{13}^1}{\partial x_1} + \frac{\partial\mu_{23}^1}{\partial x_2} + \sigma_{12}^1 - \sigma_{21}^1\right) \tag{4.8}$$

Here $\sigma_{11}^1, \sigma_{22}^1, \sigma_{12}^1, \sigma_{21}^1, \mu_{13}^1, \mu_{23}^1$ are coefficients of coordinate x_3 in relations (4.1) and (4.6).

5 Averaged forces, moments and hypermoments

In order to bring three-dimensional problem of micropolar thermoelasticity for thin plates to two-dimensional one, which is already done for deformations, bending-torsions, force and moment stresses, statically equivalent to them integral characteristics are introduced:

$$N_{13} = \int_{-h}^{h}\sigma_{13}dx_3,\ \ N_{23} = \int_{-h}^{h}\sigma_{23}dx_3,\ \ N_{31} = \int_{-h}^{h}\sigma_{31}dx_3,\ \ N_{32} = \int_{-h}^{h}\sigma_{32}dx_3 \tag{5.1}$$

$$\begin{aligned}
&M_{11} = \int_{-h}^{h}x_3\sigma_{11}dx_3,\ \ M_{22} = \int_{-h}^{h}x_3\sigma_{22}dx_3,\\
&M_{12} = \int_{-h}^{h}x_3\sigma_{12}dx_3,\ \ M_{21} = \int_{-h}^{h}x_3\sigma_{21}dx_3
\end{aligned} \tag{5.2}$$

$$\begin{aligned}
&L_{11} = \int_{-h}^{h}\mu_{11}dx_3,\ \ L_{22} = \int_{-h}^{h}\mu_{22}dx_3,\ \ L_{12} = \int_{-h}^{h}\mu_{12}dx_3,\\
&L_{21} = \int_{-h}^{h}\mu_{21}dx_3,\ \ L_{33} = \int_{-h}^{h}\mu_{33}dx_3,
\end{aligned} \tag{5.3}$$

$$\Lambda_{13} = \int_{-h}^{h}x_3\mu_{13}dx_3,\ \ \Lambda_{23} = \int_{-h}^{h}x_3\mu_{23}dx_3. \tag{5.4}$$

6 Main equations and boundary conditions of the applied theory of thermoelasticity of micropolar isotropic thin plates with free fields of displacements and rotations

Equilibrium equations for two-dimensional case can be obtained from the equations, defining force stresses $\sigma_{31}, \sigma_{32}, \sigma_{33}$ and moment stresses $\mu_{31}, \mu_{32}, \mu_{33}$, if we satisfy boundary conditions on plate planes $x_3 = \pm h$. It should be noted that the system of two-dimensional equations splits into two separate systems for the problems of bending and generalized plane stress state. The problem of bending is studied below.

Physical relations of thermoelasticity will be obtained on the basis of formulas (5.1) - (5.4) for averaged forces, moments and hypermoments using the corresponding formulas (4.1) - (4.6) or force and moment stresses.

Main system of equations of the problem of thermoelastic bending of micropolar thin plates with fee fields of displacements and rotations will be as follows:

Equilibrium equations:

$$\frac{\partial N_{13}}{\partial x_1} + \frac{\partial N_{23}}{\partial x_2} = -\tilde{p}_3, \quad N_{3i} - \left(\frac{\partial M_{ii}}{\partial x_i} + \frac{\partial M_{ji}}{\partial x_j} \right) = h\tilde{p}_i,$$
$$\frac{\partial L_{ii}}{\partial x_i} + \frac{\partial L_{ji}}{\partial x_j} + (-1)^j (N_{j3} - N_{3j}) = -\tilde{m}_i$$
$$L_{33} - \left[\frac{\partial \Lambda_{13}}{\partial x_1} + \frac{\partial \Lambda_{23}}{\partial x_2} + (M_{12} - M_{21}) \right] = h\tilde{m}_3. \tag{6.1}$$

Physical relations of thermolasticity:

$$N_{13} = 2h(\mu+\alpha)\Gamma_{13} + 2h(\mu-\alpha)\Gamma_{31}, \quad N_{23} = 2h(\mu+\alpha)\Gamma_{23} + 2h(\mu-\alpha)\Gamma_{32},$$
$$N_{31} = 2h(\mu+\alpha)\Gamma_{31} + 2h(\mu-\alpha)\Gamma_{13}, \quad N_{32} = 2h(\mu+\alpha)\Gamma_{32} + 2h(\mu-\alpha)\Gamma_{23},$$
$$M_{11} = \frac{2Eh^3}{3(1-\nu^2)} \left[K_{11} + \nu K_{22} - (1+\nu)\alpha_t \frac{T_0}{2h} \right],$$
$$M_{12} = \frac{2h^3}{3} \left[(\mu+\alpha)K_{12} + (\mu-\alpha)K_{21} \right],$$
$$M_{22} = \frac{2Eh^3}{3(1-\nu^2)} \left[K_{22} + \nu K_{11} - (1+\nu)\alpha_t \frac{T_0}{2h} \right],$$
$$M_{21} = \frac{2h^3}{3} \left[(\mu+\alpha)K_{21} + (\mu-\alpha)K_{12} \right],$$
$$L_{11} = 2h\left[(\beta+2\gamma)k_{11} + \beta(k_{22}+k_{33}) \right], \ L_{22} = 2h\left[(\beta+2\gamma)k_{22} + \beta(k_{11}+k_{33}) \right],$$
$$L_{12} = 2h\left[(\gamma+\varepsilon)k_{12} + (\gamma-\varepsilon)k_{21} \right], \ L_{21} = 2h\left[(\gamma+\varepsilon)k_{21} + (\gamma-\varepsilon)k_{12} \right],$$
$$L_{33} = 2h\left[(\beta+2\gamma)k_{33} + \beta(k_{11}+k_{22}) \right],$$
$$\Lambda_{13} = \frac{2h^3}{3}\frac{4\gamma\varepsilon}{\gamma+\varepsilon} l_{13}, \quad \Lambda_{23} = \frac{2h^3}{3}\frac{4\gamma\varepsilon}{\gamma+\varepsilon} l_{23}. \tag{6.2}$$

Geometric relations ((3.2)):

$$K_{11}=\frac{\partial\psi_1}{\partial x_1},\ K_{22}=\frac{\partial\psi_2}{\partial x_2},\ K_{12}=\frac{\partial\psi_2}{\partial x_1}-\iota,\ K_{21}=\frac{\partial\psi_1}{\partial x_2}+\iota$$
$$\Gamma_{31}=\psi_1-\Omega_2,\ \Gamma_{32}=\psi_2+\Omega_1,\ \Gamma_{13}=\frac{\partial w}{\partial x_1}+\Omega_2,\ \Gamma_{23}=\frac{\partial w}{\partial x_2}-\Omega_1$$
$$k_{11}=\frac{\partial\Omega_1}{\partial x_1},\ k_{22}=\frac{\partial\Omega_2}{\partial x_2},\ k_{12}=\frac{\partial\Omega_2}{\partial x_1},\ k_{21}=\frac{\partial\Omega_1}{\partial x_2},$$
$$k_{33}=\iota,\ l_{13}=\frac{\partial\iota}{\partial x_1},\ l_{23}=\frac{\partial\iota}{\partial x_2}. \tag{6.3}$$

Following boundary conditions should be added to the system of equations (6.1) - (6.3) (on $x_1=const$) [3]:

$$M_{11}=M_{11}^*\ or\ K_{11}=K_{11}^*,\ M_{12}=M_{12}^*\ or\ K_{12}=K_{12}^*,\ N_{13}=N_{13}^*\ or\ w=w^*,$$
$$L_{11}=L_{11}^*\ or\ k_{11}=k_{11}^*,\ L_{12}=L_{12}^*\ or\ k_{12}=k_{12}^*,\ \Lambda_{13}=\Lambda_{13}^*\ or\ l_{13}=l_{13}^*, \tag{6.4}$$

Taking into consideration the formulas for stresses (4.1) - (4.8), formulas for forces, moments and hypermoments (5.1) - (5.4), formulas for deformations and bending-torsions (3.1) - (3.2), energy conservation law (1.4) for the applied theory of thermoelasticity of micropolar thin plates will be as follows:

$$\iint_S W_0 dx_1 dx_2 = A_0,$$

where density W_0 of the deformation potential energy is expressed by the following formula:

$$W_0=\frac{1}{2}(M_{11}K_{11}+M_{22}K_{22}+M_{12}K_{12}+M_{21}K_{21}+N_{13}\Gamma_{13}+N_{31}\Gamma_{31}+N_{23}\Gamma_{23}+$$
$$+N_{32}\Gamma_{32}+L_{11}k_{11}+L_{22}k_{22}+L_{33}k_{33}+L_{12}k_{12}+L_{21}k_{21}+\Lambda_{13}l_{13}+\Lambda_{23}l_{23})-$$
$$-\frac{\alpha_T}{2h}(M_1+M_2)T_0, \tag{6.5}$$

A_0 is the work of external forces, moments and hypermoments:

$$A_0=\frac{1}{2}\left\{\int_{l_1}(M_{21}^0\psi_1+M_{22}^0\psi_2+N_{23}^0 w+L_{21}^0\Omega_1+L_{22}^0\Omega_2+\Lambda_{23}^0\iota)dx_1+\right.$$
$$+(M_{11}^0\psi_1+M_{12}^0\psi_2+N_{13}^0 w+L_{11}^0\Omega_1+L_{12}^0\Omega_2+\Lambda_{13}^0\iota)dx_2+$$
$$+\iint_S\left[(p_1^+-p_1^-)h\psi_1+(p_2^+-p_2^-)h\psi_2+(p_3^++p_3^-)w+(m_1^++m_1^-)\Omega_1+\right.$$
$$\left.\left.+(m_2^++m_2^-)\Omega_2+(m_3^++m_3^-)h\iota\right]dx_1dx_2\right\}. \tag{6.6}$$

If physical relations of elasticity (6.2) are taken into account in relation (6.5), then following formulas will be obtained for density W_0 of the deformation potential energy:

$$W_0 = \frac{1}{2}\left\langle \frac{2Eh^3}{3(1-\nu^2)}(K_{11}^2 + K_{22}^2 + 2\nu K_{11}K_{22}) + \frac{2h^3}{3}\left[(\mu+\alpha)(K_{12}^2 + K_{21}^2)+\right.\right.$$
$$+2(\mu-\alpha)K_{12}K_{31}\big] + 2h\left[(\mu+\alpha)(\Gamma_{13}^2 + \Gamma_{31}^2 + \Gamma_{23}^2 + \Gamma_{32}^2)+\right.$$
$$+2(\mu-\alpha)(\Gamma_{13}\Gamma_{31} + \Gamma_{23}\Gamma_{32})\big] + 2h\left[(\beta+2\gamma)(k_{11}^2 + k_{22}^2 + k_{33}^2)^2+\right.$$
$$+2\beta(k_{11}k_{22} + k_{11}k_{33} + k_{22}k_{33})\big] + 2h\left[(\gamma+\varepsilon)(k_{12}^2 + k_{21}^2) + 2(\gamma-\varepsilon)k_{12}k_{21}\right] +$$
$$\left.+\frac{2h^3}{3}\frac{4\gamma\varepsilon}{\gamma+\varepsilon}(l_{13}^2 + l_{23}^2)\right\rangle - \frac{Eh^2}{3(1-\nu)}(K_{11} + K_{22})\alpha_T T_0. \qquad (6.7)$$

Analoggicaly general variation functional of the applied theory of thermoelasticity of micropolar thin plates will be obtained from the variation functional (1.8) of the three-dimensional theory:

$$I_0 = \iint_S \left\langle W_0 - \left\{ M_{11}\left(K_{11} - \frac{\partial\psi_1}{\partial x_1}\right) + M_{22}\left(K_{22} - \frac{\partial\psi_2}{\partial x_2}\right) + \right.\right.$$
$$+N_{32}\left[\Gamma_{32} - (\psi_2 + \Omega_1)\right] + N_{31}\left[\Gamma_{31} - (\psi_1 + \Omega_2)\right] + M_{21}\left[K_{21} - \left(\frac{\partial\psi_1}{\partial x_2} + \iota\right)\right] +$$
$$+N_{13}\left[\Gamma_{13} - \left(\frac{\partial w}{\partial x_1} + \Omega_2\right)\right] + N_{23}\left[\Gamma_{23} - \left(\frac{\partial w}{\partial x_2} - \Omega_1\right)\right] + L_{11}\left(k_{11} - \frac{\partial\Omega_1}{\partial x_1}\right) +$$
$$+L_{22}\left(k_{22} - \frac{\partial\Omega_2}{\partial x_2}\right) + L_{33}(k_{33} - \iota) + L_{12}\left(k_{12} - \frac{\partial\Omega_2}{\partial x_1}\right) + L_{21}\left(k_{21} - \frac{\partial\Omega_1}{\partial x_2}\right) +$$
$$\left.\left.+\Lambda_{13}\left(l_{13} - \frac{\partial\iota}{\partial x_1}\right) + \Lambda_{23}\left(l_{23} - \frac{\partial\iota}{\partial x_1}\right) + M_{12}\left[K_{12} - \left(\frac{\partial\psi_2}{\partial x_1} - \iota\right)\right]\right\}\right\rangle ds-$$
$$-\iint_S \left[(p_1^+ h\psi_1 + p_2^+ h\psi_2 + p_3^+ w) + (m_1^+\Omega_1 + m_2^+\Omega_2 + m_3 h\iota)\right] ds+$$
$$+\iint_S \left[-p_1^- h\psi_1 - p_2^- h\psi_2 + p_3^- w + m_1^-\Omega_1 + m_2^-\Omega_2 + m_3^- h\iota)\right] ds+$$
$$+\int_{l_1'} (M_{21}^0\psi_1 + M_{22}^0\psi_2 + N_{23}^0 w + L_{21}^0\Omega_1 + L_{22}^0\Omega_2 + \Lambda_{23}^0\iota)dx_1+$$
$$+\int_{l_1''} \left[M_{21}(\psi_1 - \psi_1^0) + M_{22}(\psi_2 - \psi_2^0) + N_{23}(w - w^0) + L_{21}(\Omega_1 - \Omega_1^0)+\right.$$
$$\left.+L_{22}(\Omega_2 - \Omega_2^0) + \Lambda_{23}(\iota - \iota^0)\right] dx_1 + \int_{l_2'} (M_{11}^0\psi_1 + M_{12}^0\psi_2 + N_{13}^0 w+$$
$$+L_{11}^0\Omega_1 + L_{12}^0\Omega_2 + \Lambda_{13}^0\iota)dx_2 + \int_{l_2''} \left[M_{11}(\psi_1 - \psi_1^0) + M_{12}(\psi_2 - \psi_2^0)+\right.$$
$$\left.+N_{13}(w - w^0) + L_{11}(\Omega_1 - \Omega_1^0) + L_{12}(\Omega_2 - \Omega_2^0) + \Lambda_{13}(\iota - \iota^0)\right] dx_2 : \qquad (6.8)$$

If we study variation equation $\delta I_0 = 0$, all main equations (6.1) - (6.3) of the applied theory of thermoelasticity of micropolar thin plates and boundary conditions will be obtained, when forces, moments and hypermoments are given or boundary conditions in displacements, rotations and hyperrotations or boundary conditions in mixed form.

7 Thermoelastic bending of micropolar rectangular plate

Problem is studied when the rectangular micropolar plate is hinged supported and is under the temperature field

$$T = x_3 \frac{T_0}{2h}, \quad where \ \ T_0 = const \tag{7.1}$$

Following boundary conditions take place in case of hinged support:

$$\begin{aligned} &w = 0, \psi_2 = 0, M_{11} = 0, L_{12} = 0, \Omega_1 = 0, \Lambda_{13} = 0, \ when \ x_1 = 0, x_1 = a \\ &w = 0, \psi_1 = 0, M_{22} = 0, L_{21} = 0, \Omega_2 = 0, \Lambda_{23} = 0, \ when \ x_1 = 0, x_1 = b \end{aligned} \tag{7.2}$$

System of equations for $w, \psi_1, \psi_2, \Omega_1, \Omega_2, \iota$ will be obtained if formulas (6.3) are substituted into (6.2), then into equilibrium equations (6.1).
If we take into consideration formulas for M_{11} and M_{12} from (6.2), it is easy to show, that temperature summands in these formulas give inhomogeneity in boundary conditions $M_{11} = 0$ when $x_1 = 0, x_1 = a$ and $M_{22} = 0$ when $x_2 = 0, x_2 = b$. In order to obtain homogeneous boundary conditions functions ψ_1 and ψ_2 must be replaced by the following formulas:

$$\begin{aligned} &\psi_1 = \tilde{\psi_1} + \frac{(1+\nu)\alpha_T T_0}{2\delta}\bar{x_1}, \quad \psi_2 = \tilde{\psi_2} + \frac{(1+\nu)\alpha_T T_0}{2\delta}\bar{x_2}, \\ &\bar{x_1} = \frac{x_1}{a}, \quad \bar{x_2} = \frac{x_2}{a}, \quad \bar{w} = \frac{w}{a}, \quad \bar{\iota} = \frac{\iota}{a}. \end{aligned} \tag{7.3}$$

As a result boundary conditions (7.2) will be homogeneous, and in this case method of separation of variables can be used for the solution of the problem (6.1) - (6.3) of rectangular plate. The solution of the mentioned system of equations will be introduced in double trigonometric Fourier series:

$$\begin{aligned} &\bar{w} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn} \sin m\pi\bar{x_1} \sin n\pi\bar{x_1}, \quad \bar{\iota} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} F_{mn} \cos m\pi\bar{x_1} \cos n\pi\bar{x_1}, \\ &\Omega_1 = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} B_{mn} \sin m\pi\bar{x_1} \cos n\pi\bar{x_1}, \quad \Omega_2 = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} C_{mn} \cos m\pi\bar{x_1} \sin n\pi\bar{x_1}, \\ &\tilde{\psi_1} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} D_{mn} \cos m\pi\bar{x_1} \sin n\pi\bar{x_1}, \quad \tilde{\psi_2} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} K_{mn} \sin m\pi\bar{x_1} \cos n\pi\bar{x_1}, \end{aligned} \tag{7.4}$$

which satisfy homogeneous boundary conditions (7.2). Functions $T_0 = const$, $T_0\bar{x_1}$ and $T_0\bar{x_2}$ should be also introduced in double trigonometric Fourier series:

$$\begin{aligned} &T_0 = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} T_{mn} \sin m\pi\bar{x_1} \sin n\pi\bar{x_1}, \quad T_0 x_1 = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} G_{mn} \cos m\pi\bar{x_1} \sin n\pi\bar{x_1}, \\ &T_0\bar{x_2} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} M_{mn} \sin m\pi\bar{x_1} \cos n\pi\bar{x_1} \end{aligned} \tag{7.5}$$

where

$$T_{mn} = 4\int_0^1\int_0^1 T_0 \sin m\pi\bar{x}_1 \sin n\pi\bar{x}_1 d\bar{x}_1 d\bar{x}_2,$$
$$G_{mn} = 4\int_0^1\int_0^1 T_0 \cos m\pi\bar{x}_1 \sin n\pi\bar{x}_1 d\bar{x}_1 d\bar{x}_2,$$
$$M_{mn} = 4\int_0^1\int_0^1 T_0 \sin m\pi\bar{x}_1 \cos n\pi\bar{x}_1 d\bar{x}_1 d\bar{x}_2,$$

Further substituting (7.4) and (7.5) into the system of equations for coefficients $A_{mn}, B_{mn}, C_{mn}, D_{mn}, K_{mn}, F_{mn}$ system of inhomogeneous algebraic linear equations will be obtained. Solving this system and substituting the solution into (7.4) , the solution of the stated problem will be obtained. The result of the numerical calculations is introduced:

$$\delta = \frac{h}{a} = \frac{1}{40}, \nu = 0.33, \bar{\gamma} = \bar{\varepsilon} = 22*10^{-4}, \bar{\beta} = 11*10^{-2}, T_0 = 60^0C, \alpha_T = 125*10^{-7}\ 1/g$$

Table 1: Rectangular micropolar and classical plate bending under the temperature influence, depending on $\frac{\alpha}{\mu}$.

$\frac{\alpha}{\mu}$	micropolar model $\bar{w}_{max}*10^{-3}$	classical model $\bar{w}_{max}*10^{-3}$	$\frac{w^{mik.}_{max}}{w^{cl.}_{max}}$
10^{-5}	3.266	3.277	0.99
10^{-4}	3.173	3.277	0.97
10^{-3}	2.494	3.277	0.76
10^{-2}	1.005	3.277	0.30
$4.2*10^{-2}$	0.569	3.277	0.17
10^{-1}	0.471	3.277	0.14

Numerical results, introduced in Table 1, state that the plate rigidity increases compared with the classical case, when the dimensionless quantity $\frac{\alpha}{\mu}$ increases.

8 Temperature bending of micropolar circular plate

Problem of temperature bending of micropolar circular plate is studied when the asymmetric stress state takes place and the temperature field is expressed by the formula (7.1).
It should be noted that the applied theory of thermoelasticity, introduced in the previous paragraphs, is related to the Cartesian coordinate system. Main equations of this theory can also be obtained in curvilinear orthogonal system of coordinates. Particularly, main equations of the applied theory of thermoelasticity of micropolar thin plates can be obtained in the polar system of coordinates. In axisymmetric case, when the bending deformation takes place, on the basis of the main system of equations of the applied theory of thermoelasticity of micropolar circular plates

and with the help of exception method the studied problem can be reduced to the solution of the following equation:

$$\tilde{\nabla}^2\tilde{\nabla}^2\tilde{\psi}_1 - k^2\tilde{\nabla}^2\tilde{\psi}_1 = -\frac{6\left(\frac{\alpha}{\mu}\right)^2(1-\nu)\alpha_T T_0}{\delta^3(\bar{\gamma}+\bar{\varepsilon})\left(1+\frac{\alpha}{\mu}\right)}\bar{r}, \tag{8.1}$$

where

$$\tilde{\psi}_1 = \psi_1 - \frac{\alpha_T T_0}{2\delta}\bar{r}, \quad \tilde{\nabla}^2\tilde{\psi}_1 = \frac{d^2\tilde{\psi}_1}{d\bar{r}^2} + \frac{1}{r}\frac{d\tilde{\psi}_1}{d\bar{r}} - \frac{1}{\bar{r}^2}\tilde{\psi}_1,$$

$$k^2 = \left(\bar{\gamma}+\bar{\varepsilon}+\frac{2\delta^2}{3(1-\nu)}\right)\frac{6\frac{\alpha}{\mu}(1-\nu)}{\delta^2(\bar{\gamma}+\bar{\varepsilon})\left(1+\frac{\alpha}{\mu}\right)}, \quad \bar{r} = \frac{r}{a},$$

a is the plate middle plane radii.
General solution of the inhomogeneous equation (8.1) can be obtained as follows:

$$\tilde{\psi}_1 = -\frac{C_1}{k^2}\bar{r} + C_2 I_1(k\bar{r}) + \frac{3\left(\frac{\alpha}{\mu}\right)^2(1-\nu)\alpha_T T_0}{4k^2\delta^3(\bar{\gamma}+\bar{\varepsilon})\left(1+\frac{\alpha}{\mu}\right)}\bar{r}^3 \tag{8.2}$$

where C_1 and C_2 are constants, $I_1(k\bar{r})$ is Bessel function.
Determining $\tilde{\psi}_1$, bending $\bar{w}$ and free rotation Ω_2 are determined by formulas:

$$\bar{w} = \frac{C_1}{2k^2}\bar{r}^2 + C_2 I_0(k\bar{r})\left[\frac{\delta^2 k}{3(1-\nu)} - \frac{1}{k}\right] - \frac{3\left(\frac{\alpha}{\mu}\right)^2(1-\nu)\alpha_T T_0}{16k^2\delta^3(\bar{\gamma}+\bar{\varepsilon})\left(1+\frac{\alpha}{\mu}\right)}\bar{r}^4 +$$

$$+\frac{\left(\frac{\alpha}{\mu}\right)^2\alpha_T T_0}{k^2\delta(\bar{\gamma}+\bar{\varepsilon})\left(1+\frac{\alpha}{\mu}\right)}\bar{r}^2 - \frac{\alpha_T T_0}{4\delta}\bar{r}^2 + C^*,$$

$$\Omega_2 = C_2 I_1(k\bar{r})\left[1 - \frac{\delta^2 k^2\left(1+\frac{\alpha}{\mu}\right)}{6\frac{\alpha}{\mu}(1-\nu)}\right] + \frac{3\left(\frac{\alpha}{\mu}\right)^2(1-\nu)\alpha_T T_0}{4k^2\delta^3(\bar{\gamma}+\bar{\varepsilon})\left(1+\frac{\alpha}{\mu}\right)}\bar{r}^3 +$$

$$+\left[-\frac{C_1}{k^2} - \frac{\frac{\alpha}{\mu}\alpha_T T_0}{k^2\delta(\bar{\gamma}+\bar{\varepsilon})} + \frac{\alpha_T T_0}{2\delta}\right]\bar{r}. \tag{8.3}$$

The case is studied, when the plate middle plane contour is hinged supported, i.e. following boundary conditions take place:

$$\bar{w} = 0, \ \frac{d\tilde{\psi}_1}{d\bar{r}} + \frac{\nu}{\bar{r}}\tilde{\psi}_1 = 0, \ \bar{L}_{12} = 0 \ \ when \ \ \bar{r} = 1. \tag{8.4}$$

Determining integral constants and substituting them into the corresponding formulas, the solution of the stated problem will be obtained, i.e. functions $\bar{w}(\bar{r}), \psi(\bar{r}), \Omega_2(\bar{r})$.
Results of numerical calculations are introduced below.

$$\delta = \frac{h}{a} = \frac{1}{40}, \nu = 0.33, \bar{\gamma} = \bar{\varepsilon} = 22 * 10^{-4}, T_0 = 60^0 C, \alpha_T = 125 * 10^{-7}\ 1/g$$

Table 2: Circular micropolar and classical plate bending under the temperature influence, depending on $\frac{\alpha}{\mu}$.

$\frac{\alpha}{\mu}$	micropolar model $\bar{w}_{max} * 10^{-3}$	classical model $\bar{w}_{max} * 10^{-3}$	$\frac{w^{mik.}_{max}}{w^{cl.}_{max}}$
$1.2 * 10^{-3}$	7.184	7.5	0.96
$1.4 * 10^{-3}$	6.451	7.5	0.86
$1.8 * 10^{-3}$	5.299	7.5	0.71
$2 * 10^{-3}$	4.825	7.5	0.64
$3 * 10^{-3}$	2.962	7.5	0.40
$4 * 10^{-3}$	1.521	7.5	0.20

As in case of rectangular plate, in case of circular plate it is stated that the plate rigidity increases compared with the classical case, when the dimensionless quantity $\frac{\alpha}{\mu}$ increases.

Conclusion

In the present paper applied theory and variation principle of thermoelasticity of bending deformation of micropolar thin plates are constructed. On the basis of the constructed applied theory of thermoelasticity of thin plates problems of thermoelastic bending of rectangular and circular plates are studied. Numerical results show that the plate rigidity increases in case of micropolar material.

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Aslanyan N.S., Shirak State University after M. Nalbandyan, Armenia
Sargsyan S.H., Shirak State University after M. Nalbandyan, Armenia