

Four-ion model of an electrohydrodynamic flow in the two-wire electrode system

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Abstract

The paper presents the results of computer simulation of the formation and development of EHD flow in a symmetric electrode-wire system in a closed long channel on the basis of the complete set of EHD equations with four types of ions taken into account. This implements the model of an electrochemical-type EHD converter and allows one to investigate the effect of an external load on its operation. The simulation results are the main characteristics of the through EHD flow of the injection type. The emerging flow was analyzed at different initial ratios of the injection currents, the forced and passive viscous parts of the flow in the closed channel were identified. The extinction of injected ions in the channel is analyzed at different initial ratios of the injection currents at the electrodes.

1 Introduction

An EHD converter is a device that can be used to convert the energy of the electric current into the mechanical energy of a working fluid flow. The structure of an electrohydrodynamic flow of injection type is defined by the electrophysical and electrochemical properties of the working fluid, as well as the electrode-liquid contact parameters. The electrochemical asymmetry of the electrode-liquid contact is required in the symmetrical electrode system to pump fluid through the system, which can be accomplished by means of electrodes of different materials, or various coatings of electrodes, or a liquid with electron-acceptor impurities. In these cases, four types of ions are present in the liquid: those appearing on the electrodes as a result of injection, and the ones dissociated in the volume. These ions, which ensure the intrinsic conductivity of the liquid, can have different properties.

Earlier studies analyzed the structure of the EHD flow in a symmetrical electrode system and identified the effect of the injection intensity on each of the electrodes on the kinematic and dynamic structures of the EHD flow in an open channel [1, 2]. The numerical calculation of computer models of the process of formation and development of an EHD flow, which were implemented within the framework of a two-ionic formulation, has shown that a charge plug can form in the electrode

region under certain conditions. This inhibits the through pumping of the liquid, but can be eliminated by selecting the impurity composition of the liquid so that the injection proceeded on the surfaces of both electrodes [3]. Previously, the EHD flow was calculated numerically in the model of an open short channel, and the effect of the level of low-voltage fluid conductivity [4] and dielectric walls on the flow structure was examined [5].

In a symmetrical electrode system with injection occurring on both electrodes, four types of ions are generated in the liquid: positive and negative ions, which emerge due to either the injection on the electrodes or the dissociation in the volume. In this connection, presented here are the results of computer simulation of the process of formation and development of EHD flow in a symmetric electrode-wire system in a closed long channel on basis of the complete set of EHD equations with four types of ions taken into account. The paper implements the model of an electrochemical-type EHD converter, which allows investigating the effect of an external load on its operation. The simulation yielded the main characteristics of the through-hole EHD flow of the injection type.

A feature of the model with a closed channel is the possibility of analyzing the processes of mutual recombination of injected and dissociated ions. In addition, the model allows taking into account the effects of the uncompensated charge on the cyclic development of EHD flows, as well as the differences in the properties of injected and dissociated ions.

2 Simulation technique

The set of EHD equations contains the Navier-Stokes equation (1), the continuity equation (2), the electrostatic equations (3) and (4), the Nernst-Planck equation for the four ion varieties (5). The considered complete set of EHD equations includes four Nernst-Planck equations - two for injected ions, two for dissociated ions:

$$\gamma \frac{d\vec{v}}{dt} + \gamma (\vec{v}, \nabla) \vec{v} = -\nabla p + \eta \Delta \vec{v} - \rho \nabla \varphi \quad (1)$$

$$\text{div} (\vec{v}) = 0 \quad (2)$$

$$\text{div} (\vec{E}) = \frac{\rho}{\varepsilon \varepsilon_0} \quad (3)$$

$$\vec{E} = -\nabla \varphi \quad (4)$$

$$\frac{dn_i}{dt} + \text{div} (n_i(z_i b_i) \vec{E} - D_i \nabla n_i + n_i \vec{v}) = g_i, i = 1, 2, 3, 4 \quad (5)$$

$$\rho = \sum_{k=1}^{4z_k e n_k} \quad (6)$$

Here \vec{E} is the electric field strength, ρ is the space charge density, φ is the electric potential, n_1 is the concentration of positive injected ions, n_2 is the concentration of negative injected ions, n_3 is the concentration of positive dissociated ions, n_4 is the concentration of negative dissociated ions, g_i is the source function, ε is the relative

electric permittivity, b_i is the ion mobility, D_i is the diffusion coefficient, z_i is the ion valency; ε_0 is the electric constant, e is the elementary electric charge, t is the time; i subscript indicates the ion species, γ is the mass density, \vec{v} is the fluid velocity, p is the pressure, η is the dynamic viscosity. In general, the properties of particles may differ, but the paper assumes them the same. In addition, it should be noted that the ions were univalent, that is, $|z_i|=1$.

In the problem, we consider the injection and dissociation mechanisms of charge formation with allowance for recombination. The injected ions recombine with one another and with dissociated ions of the opposite sign. Dissociated ions are produced in the volume and recombine with one another and with injected ions. The right-hand side of equations (5) is supplemented with a term describing the death of particles in the volume, and that of (5) for $i = 3, 4$ - a term describing the volumetric source of ion generation W :

$$\rho = \alpha_r n_1 (n_2 + n_4) \tag{7}$$

$$\rho = \alpha_r n_2 (n_1 + n_3) \tag{8}$$

$$\rho = W - \alpha_r n_3 (n_2 + n_4) \tag{9}$$

$$\rho = W - \alpha_r n_4 (n_1 + n_3) \tag{10}$$

Here W is the dissociation intensity, α_r is the recombination coefficient. The coefficient of recombination of i and k species is determined by the following formula

$$\alpha_{rik} = \frac{e(b_i + b_k)}{\varepsilon \varepsilon_0}.$$

In a liquid with intrinsic conductivity in the absence of an external electric field, the formation of ions occurs due to the thermal motion of the molecules. The equilibrium concentration of ions, which form due to dissociation, is determined by the condition that the rates of dissociation and recombination are equal. The equilibrium concentration is determined through the low-voltage conductivity and is given in this problem as initial equilibrium value $n_0 = \frac{\sigma_0}{2eb}$.

The source function for positive and negative particles in this problem will be the same. The dissociation coefficient in the absence of an external electric field is determined through the equilibrium concentration and is written as $W_0 = \frac{\sigma_0^2}{2eb\varepsilon\varepsilon_0}$. The Wine effect in the problem is not considered, that is, we assume the dissociation intensity to be constant and equal to the dissociation coefficient in the absence of an external field $W = W_0$. Thus, the source functions in equations (5) for $i = 3, 4$ can be written as follows:

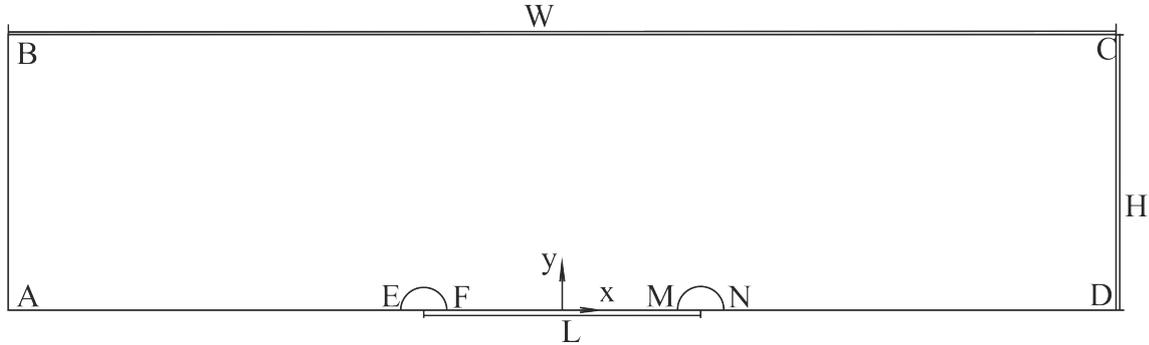
$$\rho = W_0 - \alpha_r n_3 (n_2 + n_4) \tag{11}$$

$$\rho = W_0 - \alpha_r n_4 (n_1 + n_3) \tag{12}$$

The diffusion coefficient was determined by the Einstein relationship $D_i = \frac{k_B T_0}{eb_i}$, where k_B is the Boltzmann's constant, T_0 is the system temperature. The liquid properties are: $b_i = 10^{-8} \frac{m^2}{V \cdot s}$, $D_i = 2.59 \cdot 10^{-10} \frac{m^2}{s}$, $|z_i|=1$, $\gamma = 950 \frac{kg}{m^3}$, $\eta = 4.75 \cdot 10^{-3} Pa \cdot s$, $\sigma_0 = 3 \cdot 10^{-11} \frac{S}{m}$.

A symmetrical wire-wire electrode system in a closed channel was considered (Fig. 1). By virtue of the symmetry of the model about the horizontal axis passing through the electrodes, only half of the model was calculated. The two-dimensional problem was

considered since the wire lengths are much larger than the interelectrode distance. The geometric dimensions were as follows: $W = 6 \text{ cm}$, $H = 0.75 \text{ cm}$, $L = 1 \text{ cm}$, $d = 0.1 \text{ cm}$, $r = 0.025 \text{ cm}$ is the radius of electrodes.



Zakirianova 1: Geometry and boundary conditions.

The channel closure was effected with the help of an original boundary condition, which makes it possible to transfer the values of the unknown functions from the right-hand boundary of the channel to the left-hand one. The choice of boundaries is determined by the direction of the through flow.

Electric potentials $\pm U_0 = \pm 10 \text{ kV}$ were specified for the Poisson equation at electrode boundaries EF and MN. The condition of the normal component of the electric field strength being zero is set as $\vec{N} \cdot \vec{D} = 0$ on dielectric wall BC. In this case, it is assumed that the charge on the walls shields the field, and the normal component of the field is zero. The condition for transferring charge flux $\rho \vec{v}$ from boundary CD to boundary BA is used as a condition for the closure of the channel.

Flux of ions of the corresponding sign, $\vec{j}_i = f_i(\vec{E})$, and the extinction of ions of the opposite sign, which was set by equation: $\vec{j}_i \cdot \vec{N} = -(n_i(z_i b_i) \vec{E} - D_i \nabla n_i + n_i \vec{v}) \cdot \vec{N}$, were set for the Nernst-Planck equations for the injected ions at electrode boundaries EF and MN. The extinction of the negative injected and dissociated ions was determined at boundary EF, that of the positive injected and dissociated ions was determined at boundary MN. At channel boundary AB, the fluxes of different sorts of ions were set equal to the corresponding fluxes at boundary CD, which were determined by equation $-\vec{j}_i \cdot \vec{N} = (n_i(z_i b_i) \vec{E} - D_i \nabla n_i + n_i \vec{v}) \cdot \vec{N}$. The isolation condition was specified by equation $-\vec{j}_i \cdot \vec{N} = 0$ at upper boundary BC and lower boundaries AE, FM, and ND. The free passage of ions was set on right-hand boundary CD. The injection current at the electrodes is given in the form of a quadratic polynomial in the local electric field strength according to formula

$$\vec{j}_i = \left(A \cdot |\vec{E}| + B \cdot |\vec{E}|^2 \right) \cdot \vec{N} \quad (13)$$

where A and B depend on the material of the electrodes and impurity additives to the liquid. The injection current on the left-hand electrode was considered in the problem to be greater than on the right-hand one. This determined the direction of the through flow from left to right. The injection functions were chosen in such a way that the initial injection current densities on the left-hand electrode were two or three times higher than on the right-hand electrode.

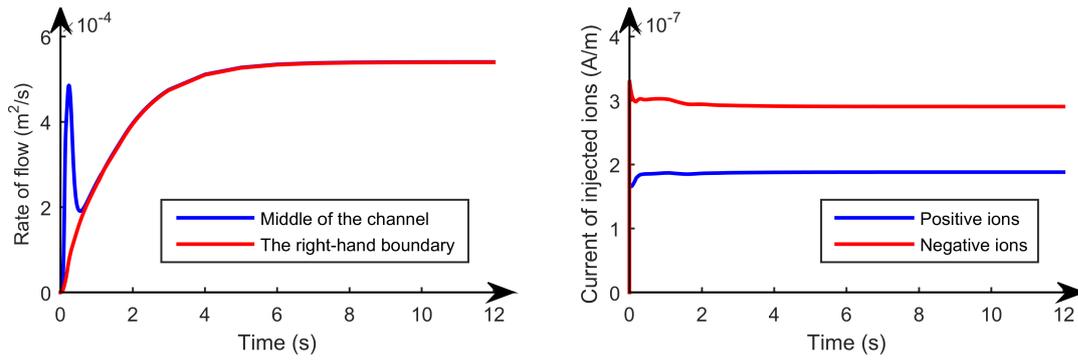
The adhesion condition $\vec{v} = 0$ was specified for the Navier-Stokes equation at the upper and lower boundaries and on electrodes EF and MN. An incoming fluid flow, whose velocity was equal to that of the flow at boundary CD ($\vec{v} = \vec{v}_{right}$), was set at boundary AB. At boundary CD, there is the outflow with the pressure equal to that pressure at boundary AB, without viscous resistance, following the equation
$$\left[\mu \left(\nabla \vec{v} + (\nabla \vec{v})^T \right) \right] \cdot \vec{N} = 0.$$

The non-stationary problem was solved. The initial conditions are voltage switching, fixed liquid with conductivity equal to the equilibrium value.

3 Results and discussion

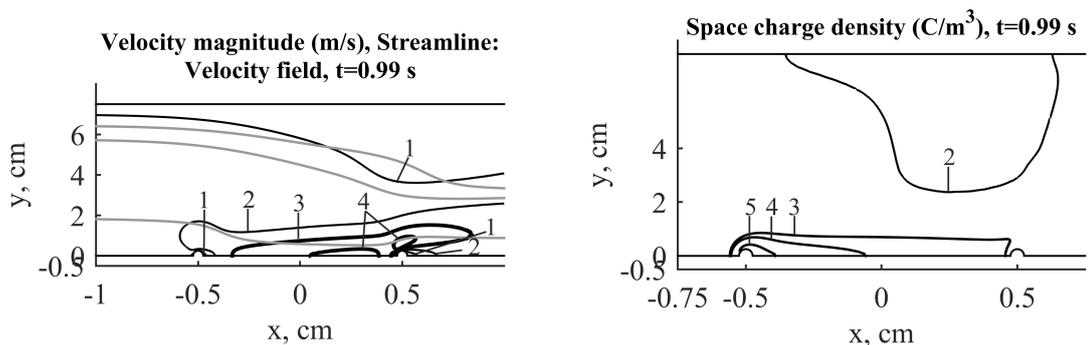
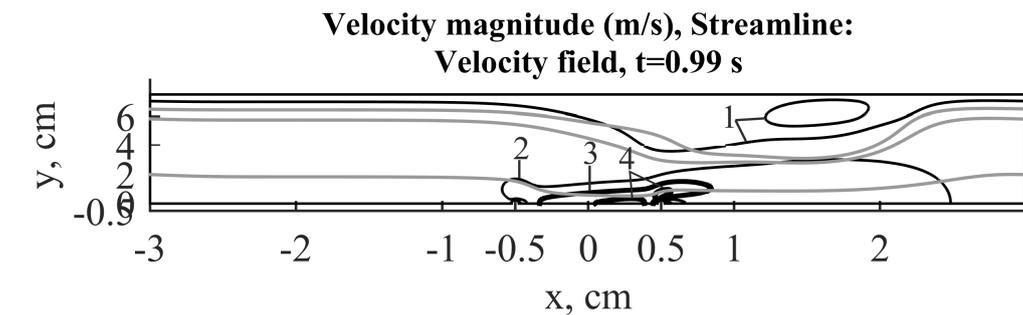
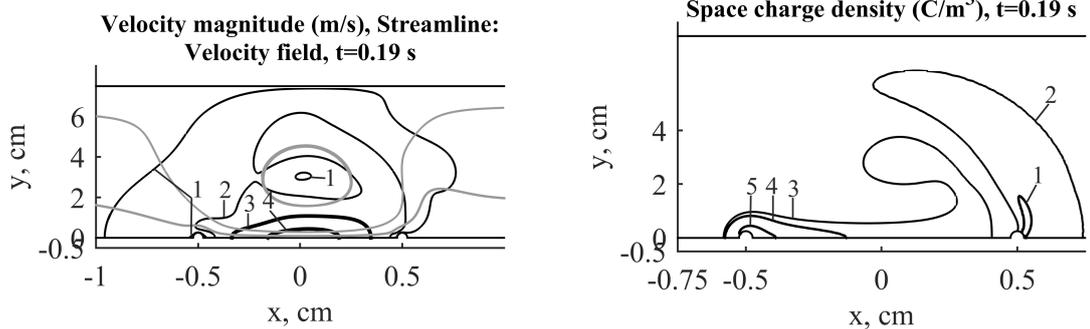
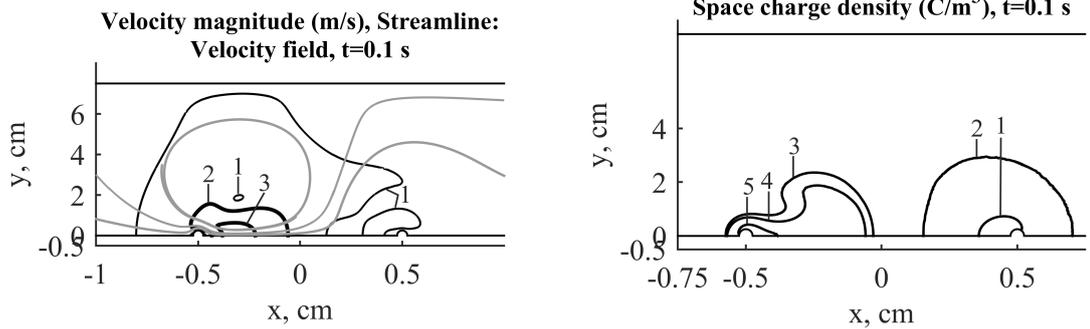
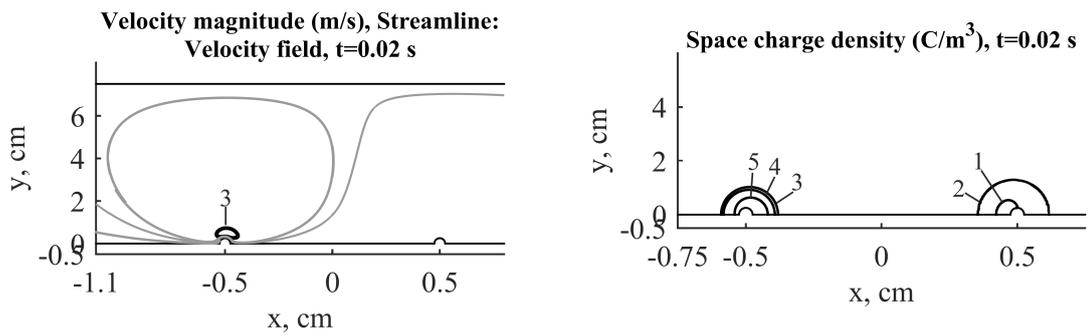
At the initial moment, there is no fluid flow, a voltage of 10 kV is applied to the electrodes. After that, the counter flows from both electrodes form in the interelectrode gap, with the flow velocity from the left-hand electrode higher than from the right-hand one. When jets from different electrodes meet, a more intense jet from the left-hand electrode blows a counter jet into the region behind the electrode. So, the through flow forms in the interelectrode gap and in the rest of the channel; the flow rate of the liquid through the channel cross section is sustained. Fig. 2 shows that the process of balancing the flow in a closed channel lasts about 10 seconds, and then the flow rate remains constant. The time is longer than that of crossing of the interelectrode gap by the charged jet. The time dependence of the injection currents on the electrodes also displays regions of attaining the steady state: the current from the left-hand active electrode decreases, and that from the right-hand passive electrode increases. These processes are associated with the formation of charged structures in the bulk, which affect the surface field strength, and therefore, the injection currents. When homocharges form at the electrodes, the injection current of positive ions decreases, and that of negative ions increases. As the smaller, positively-charged jet propagates to the counter electrode, the injection currents increase slightly. After the flux of positively charged ions closes the interelectrode gap, the injection currents decrease. If the injection currents are balanced, the moving charges of the positive and negative ions will be equal and the jet at the outlet of the channel will be neutral. In our case, the injection current of positive ions is approximately one and a half times larger than that of negative ions.

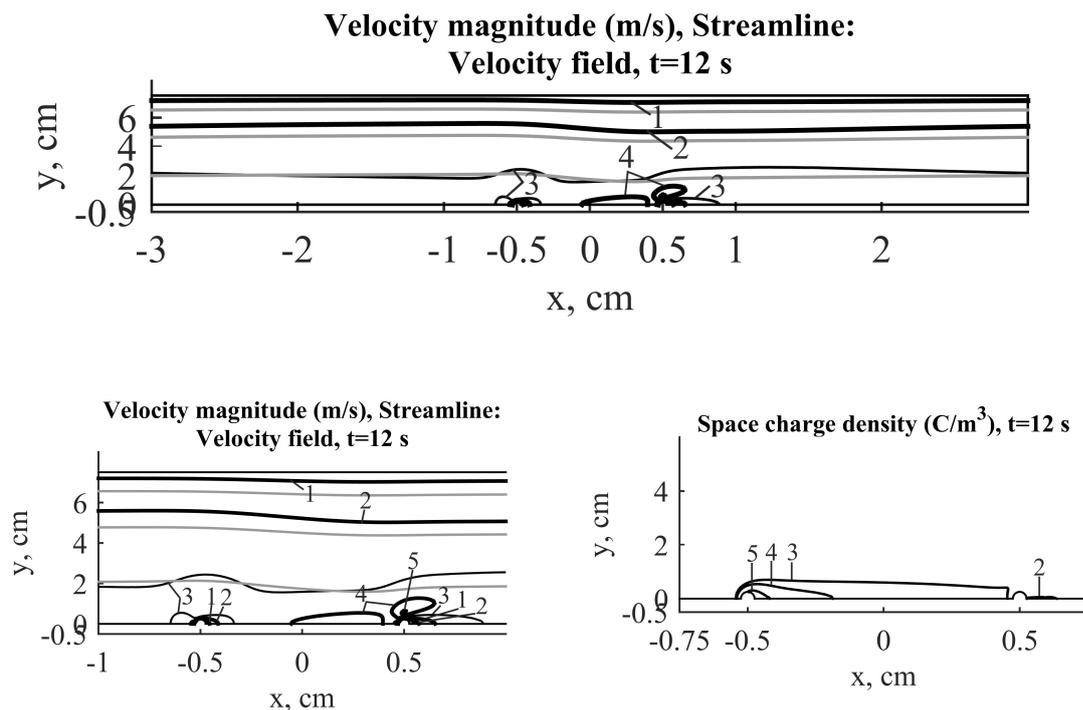
Fig. 3 represents the successive stages of the formation of the EHD flow in a closed channel. The positive space charge is seen to propagate from the right-hand electrode, dominant in the interelectrode gap, into the region behind the electrode. After the stream of the positive charge reaches the right-hand boundary, the smaller charged jet passes to the left side of the cell and the flow attains a steady state. The so-called through EHD flow of injection type forms in the steady state. It is characterized by a thin charged jet, which flows from the active electrode, crosses the interelectrode gap, and becomes a wafer-shaped bipolar charged structure in the region behind the electrode. This prevents the formation of charge plugs and provides some acceleration of the liquid into the region behind the electrode. In general, the flow in the channel can be divided into two parts: the forced flow in the region of the interelectrode gap, where the current lines crowd to the central plane



Zakirianova 2: Time dependences of fluid flow rate in different parts of the channel (on the left) and injection currents with a ratio of injection currents of 2:1 (on the right).

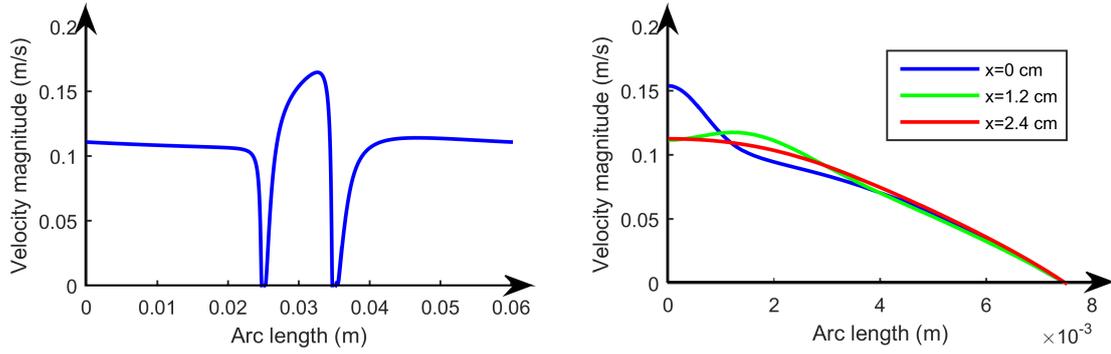
and the profile is of Gaussian shape, and the passive viscous flow elsewhere. Intense liquid acceleration occurs in the forced region(see Fig. 3 and Fig. 4).





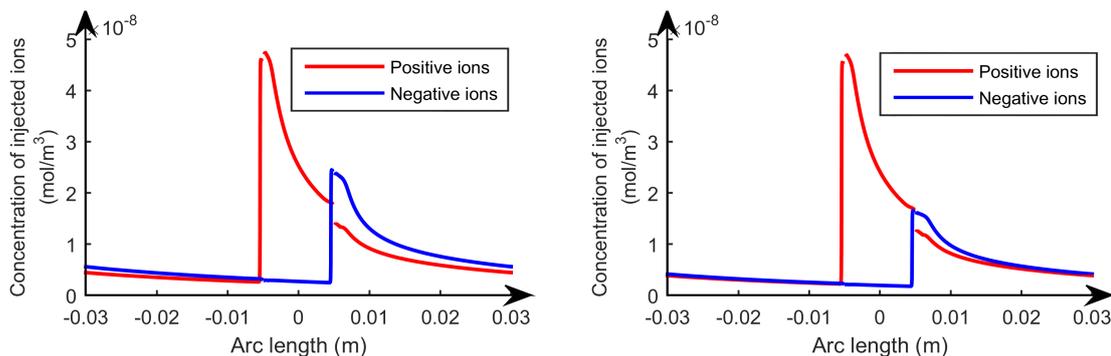
Zakirianova 3: Isolines of the flow velocity distribution and the liquid flow line, the distribution of the space charge at successive instants of time (ratio of injection currents 2: 1). On the velocity distribution plots: 1 - 0.01 m/s, 2 - 0.05 m/s, 3 - 0.1 m/s, 4 - 0.15 m/s, 5 - 0.19 m/s. On the diagrams of the distribution of the density of the space charge: 1 - -0.002 mC/m³, 2 - -0.2 mC/m³, 3 - 1 mC/m³, 4 - 2 mC/m³, 5 - 3 mC/m³.

Fig. 4 shows longitudinal velocity distributions along the central plane of the channel and velocity profiles at different flow levels at the last instant of time. The streamlines are parallel to the walls of the channel in the region of passive viscous flow, the flow velocity decreases slightly along the channel and the velocity profile has a typical parabolic shape. Between these areas, there is the transition area, within which the flow passes from the forced state to the passive one. This region is characterized by a discharge of liquid into the region behind the electrode. Judging from the longitudinal velocity distributions, the length of the transition region corresponds approximately to the size of the interelectrode gap.



Zakirianova 4: Linear longitudinal (on the left) and transverse (on the right) velocity distributions at the last instant of time (ratio of injection currents 2:1).

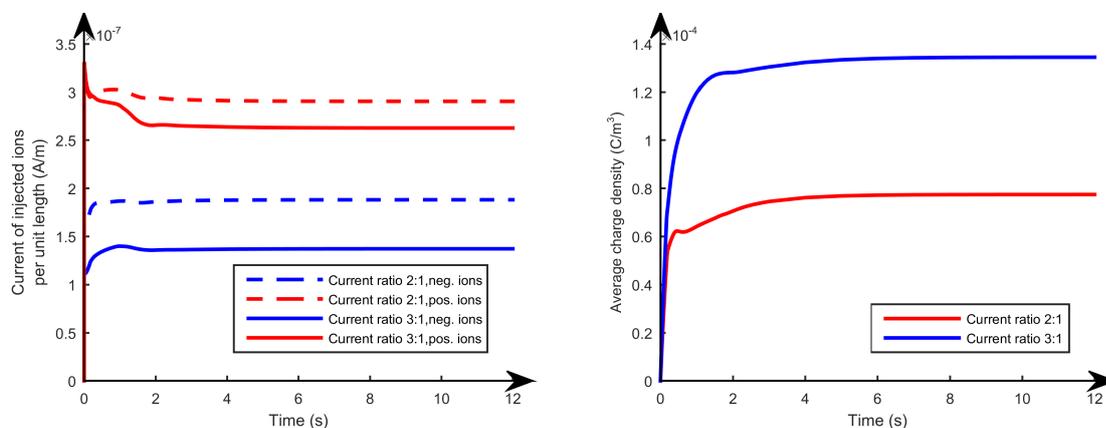
Let us analyze the mechanisms of liquid discharge. Fig. 5 shows the longitudinal distributions of the concentration of injected ions at the initial ratio of the injection currents on the left-hand and right-hand electrodes of 2:1 and 3:1, respectively. The positive injected ions of type 1 are seen to be produced on the active electrode and to propagate across the interelectrode gap. There is an intense decrease in the concentration of positive ions within the interelectrode gap due to recombination with negative type 3 conduction ions. Negative ions of type 2 are produced on the passive electrode, therefore the recombination processes go faster. The recombination processes can be seen to occur throughout the entire channel. However, the injected ions do not have enough time to completely recombine in the gap behind the negative electrode and the injected ions are transferred from the right-hand boundary to the left-hand one. However, the concentration decreases by a factor of e at a distance of 1 cm from the right-hand electrode. With the initial ratio of currents of 3:1, the concentration of injected ions that are transferred to the left boundary of the channel is somewhat smaller than the initial ratio of injection currents of 2:1. Also, the concentrations of injected ions of different signs that reach the boundary of the channel are equal at initial ratio of injection currents of 3:1.



Zakirianova 5: Longitudinal distributions of the concentration of injected positive and negative ions at different initial injection levels (2:1 on the left and 3:1 on the right).

Fig. 6 presents the time dependences of the injection currents on the active and passive electrodes at different initial ratios of the injection currents. The injection

currents in the process of balancing the flow are seen to tend to 1.9:1 at the initial ratio of injection currents of 3:1, and to 1.6:1 at 2:1. At the same time, the average space charge (Fig. 6) circulating through the channel at the initial ratio of 2:1 is half that at the initial ratio of 3:1. Therefore, it is preferable to select the nearest injection currents, with the average flow rate practically unchanged over the channel.



Zakirianova 6: Time dependences of injection currents at the initial ratio of injection currents of 2:1 and 3:1 and the average charge density.

Conclusions

The computer simulation of EHD-flows of injection type from the electrodes of wire-wire type in a closed channel, which is substantially longer than the interelectrode gap, is carried out. A liquid, where four types of ions are present, is considered. The advantage of the model is the possibility of the direct analysis of recombination processes and the study of effect of load and uncompensated space charge on the development of through EHD flows in a closed channel. The structure of the EHD flow is analyzed inside and outside the interelectrode gap.

The model allows one to precisely select the optimum injection function on the electrodes for implementation of the through flow. The results are compared for the initial ratio of injection currents of 2:1 and 3:1; in the first case, the average density of the space charge circulating through the channel decreased almost twofold and the flow rate did not change. Therefore, for the practical application of such a system, it is preferable to choose the nearest initial injection currents that ensure the through flow regime.

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