Specifics of charge accumulation on and transport along the interface between a low-conducting liquid and a solid perfect insulator

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Abstract

Presently, solid insulation is an essential part of various high-voltage systems including the case of electrohydrodynamic ones. The latter use the Coulomb force to put dielectric liquids in motion and thus their characteristics strongly depend on the electric field distribution. When the solid insulation has much smaller electrical conductivity than that of the liquid, its surfaces accumulate electric charge in a thin layer in the liquid near the interface, which changes the electric field configuration and is difficult to account for in computer simulation. Therefore many works use simplified boundary conditions on the insulation surfaces like the absence of the normal component of the electric field. However, the key problem is that the charge takes time to accumulate and remains mobile in the tangential field, thus the simplified condition should be validated. The present study considers an EHD system, employing an insulating barrier, characterizes the layer of the accumulated charge and checks the applicability of the simplified boundary condition by means of computer simulation. The results show when the simplified condition can be invalid resulting in different total current and electric field distribution.

1 Introduction

Electrohydrodynamic (EHD) systems use the strong electric field to charge dielectric liquids and to put them in motion via the Coulomb force [1]. The characteristics of the systems fundamentally depend on both charge formation mechanisms and the electric field distribution, and the latter is even more essential than the former because it controls the intensity of the charge formation.

The electric field configured by metal electrodes is easily accounted for with the use of up-to-date computer simulation techniques. However, recent studies use solid insulation in EHD systems for various purposes (for example, to generate wall jets [2] or to change the electric field distribution [3,4]) with increasing frequency. Introduced in the region near or between the electrodes, solid insulation accumulates electric charge on its surface (the electrical conductivity of the solid insulation is assumed to be much smaller than that of the liquid) and thus affects the electric field in an EHD system. The charge remains in a very thin layer of liquid in immediate proximity to the surface and screens the normal component of the electric field in the liquid outside the layer. The key problem is that the charge takes time to accumulate and remains mobile in the tangential field, which complicates simulation of the EHD systems like that.

The accumulated charge in the tangential electric field can cause both the onset of electroconvection and the migration charge transport. These processes are mostly considered on a micro scale, with the liquid motion referred to as induced-charge electroosmotic flow [5,6]. In the case of low-conducting (dielectric) liquids and macro scales the effect of the accumulated charge transport is yet to be investigated.

The phenomena concerning the electrical double layer (EDL) are similar to those discussed and are extensively being investigated (in [7-8], for instance). In the case of accumulated charge, the total charge density per surface unit can be much greater than that in the case of EDL, therefore, disregarding the effects of the accumulated charge transport could be erroneous.

The aim of the present study is to examine the applicability of the simplified boundary condition used in computer simulation on the insulating walls when the electric field distribution is mainly determined by the accumulated charge [3]. The simplest boundary condition states that the normal component of the electric field is absent ($E_N = 0$). This assumes the charge to be totally accumulated, immobile, and located in infinitely thin layer. The works [3,4] rely on it to study the electric current passage and the EHD flow caused by the field-enhanced dissociation in a low-conducting liquid with raised conductivity. The present paper uses geometry from the work [3] and compares the results of utilizing the simplified and more complete models (the latter disregards field-enhanced dissociation, liquid flow, and assumes the solid insulators to be perfect).

In fact, several studies (e. g. [9, 10]) have already used more complete models of charge accumulation on the surfaces of the solid insulation (side boundaries) but they focused neither on the layer structure nor on the effect of the accumulated charge onto the total current or the electric field distribution.

That is why, just after the description of the mathematical model, the present paper studies an 1D system „HV electrode – slightly conducting liquid – solid perfect insulator – grounded electrode“ to characterize the layer of the accumulated charge and to check the impact of model parameters (the low-voltage conductivity of the liquid, the mobility of ions, and the voltage). Then, the more complicated 2D geometry of the EHD system from [3] is considered.

2 Mathematical model

The present work deals with the current passage through the system that consists of both liquid and solid insulation. Since the electric field distributions and the charge accumulation at the liquid-surface interfaces are of main interest, the mathematical model disregards liquid motion to avoid excessive complicacy. Due to the same reason, a pure conduction model—without the field-enhanced dissociation and the injection—is considered with the two species of ions that are univalent and have equal properties.

Thus, the conduction is described by the following set of equations:

$$\mathrm{div}\,(E) = \rho/\varepsilon\varepsilon_0 \tag{1}$$

$$\partial n_i/\partial t + \mathrm{div}\left(\vec{j_i}\right) = W_0 - \alpha_r n_1 n_2 \tag{2}$$

where E is the electric field, ρ is the electric charge density, ε is the dielectric permittivity, ε_0 is the vacuum permittivity, n_i is the concentration of positive (negative) ions, t is the time, $\vec{j_i}$ is the flux density of positive (negative) ions, W_0 is the intensity of dissociation, α_r is the recombination coefficient.

The electric field, the space charge density, the ion flux, the intensity of dissociation and the recombination coefficient are defined as follows:

$$\vec{E} = -\nabla\varphi \tag{3}$$

$$\rho = e\,(n_1 - n_2) \tag{4}$$

$$\vec{j_i} = \mathrm{sign}\,(Z_i)\,n_i b\vec{E} - D\nabla n_i \tag{5}$$

$$W_0 = \sigma^2/(2eb\varepsilon\varepsilon_0) \tag{6}$$

$$\alpha_r = 2eb/(\varepsilon\varepsilon_0) \tag{7}$$

Where φ is the electric potential, e is the elementary electric charge, Z_i is the ion charge number, b is the ion mobility, D is the diffusion coefficient, σ is the liquid conductivity.

According to the Einstein relation, the diffusion coefficient is proportional to the ion mobility:

$$D = bk_bT/e \tag{8}$$

where k_b is the Boltzmann constant, T is the temperature.
The present study employs computer simulation to solve (1-2) with regard to (3-8). The computations were carried out using software package COMSOL Multiphysics® based on the finite element method.

3 One-dimensional approach

Charge accumulation time
Consider the system consisting of two infinite layers – that of liquid dielectric (the barrier) and that of perfect solid insulation. If the voltage applied, the electric charge will accumulate in the liquid at the surface of the barrier until the electric field in the liquid becomes zero. Obviously, the symmetry of this system allows us to solve the problem in one-dimensional approach. The geometry and the boundary conditions of the model are shown in Fig. 1. L is the thickness of the liquid layer, H is the thickness of the solid layer.
The dielectric permittivities of the liquid, ε_H, and that of the solid, ε_L, equal 2 and are of no interest for the present study. V_0 is the voltage applied to one of

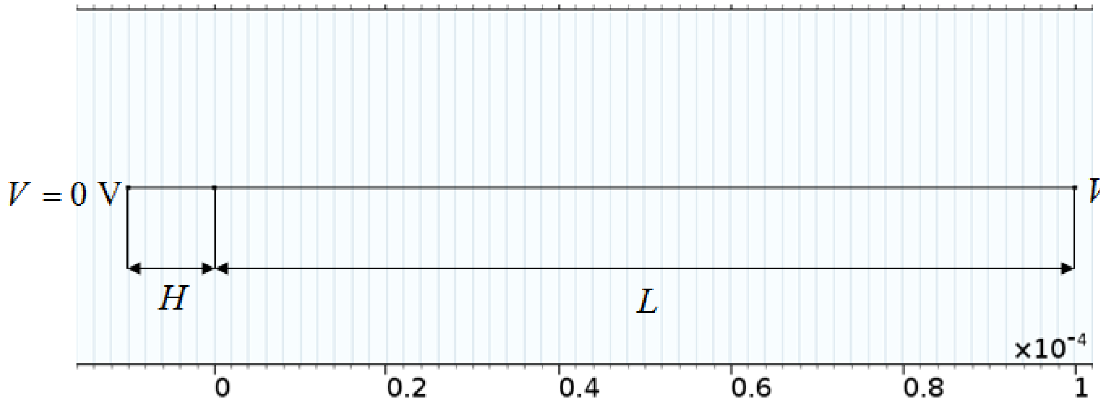

Figure 1: The scheme of the problem.

the electrodes, another one is grounded. σ_0 is the conductivity of the liquid. The voltage, the conductivity and the mobility of ions are the parameters that are varied. This problem can be solved without Nernst-Planck equation (2) in terms of electrostatics (1). The condition of the end of the charge accumulation on the surface of solid insulator:

$$\rho_s = \varepsilon\varepsilon_0 E_H = \varepsilon\varepsilon_0 \frac{V_0}{H}, \tag{9}$$

where ρ_s is the surface charge density on the solid insulator.
Considering the equation (9) and the Ohm's law, we get the expression of ρ_s versus time:

$$\rho_s = \frac{V_0 \varepsilon_H \varepsilon_0}{H}\left(1 - e^{-\alpha t}\right) \tag{10}$$

Where the characteristic time τ of charge accumulation is

$$\tau = \frac{1}{\alpha} = \frac{\varepsilon_0 \left(L\varepsilon_H + H\varepsilon_L\right)}{\sigma H} = \frac{\varepsilon_0 \varepsilon_H L}{\sigma H} + \frac{\varepsilon_0 \varepsilon_L}{\sigma} = \tau_{RC} + \tau_{relaxation} \tag{11}$$

The equation (11) shows that the characteristic time τ is the sum of the two terms. The first term is the RC-time, the characteristic time of the barrier capacitance charging through the resistance of the liquid layer. The second term is the characteristic time of charge relaxation in the liquid dielectric. If the liquid layer is much thicker or much thinner than the barrier, the first or the second term can be neglected. Otherwise, they both are of importance.
When the accumulated charge is located within a thin layer ($<< H$) close to the barrier surface, the characteristic time of the charge accumulation will remain nearly the same. Therefore, for further calculations involving the Nernst-Planck equation (2), it is reasonable to use tenfold characteristic time as the time when charge accumulation reaches stationary value.

Parametric study

To allow for further investigations of more complicated 2D models, the impact of the system parameters should be estimated in the 1D approach now. Fig. 2 shows the electric field distributions in the liquid, and the distributions of the accumulated space charge are very similar. The electric field is as strong as in the barrier in the close proximity to its surface (the left end), and drops down almost to zero within 1 μm. The point where the electric field is twice weaker shows the characteristic layer thickness that also has the interpretation of a distance where the half of the total net charge is located closer to the barrier surface.

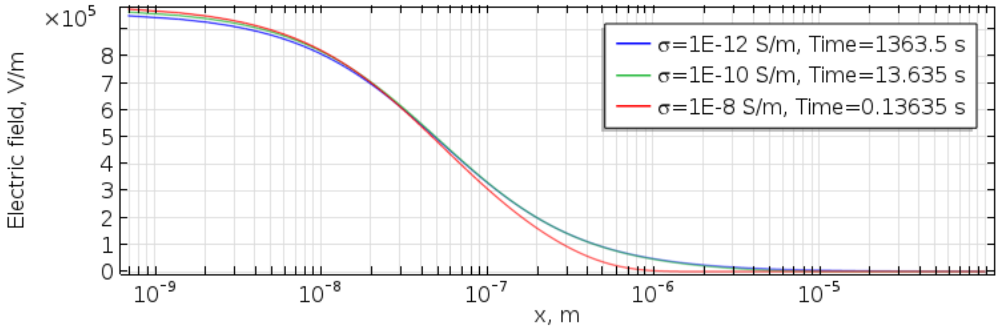

Figure 2: Different conductivity.

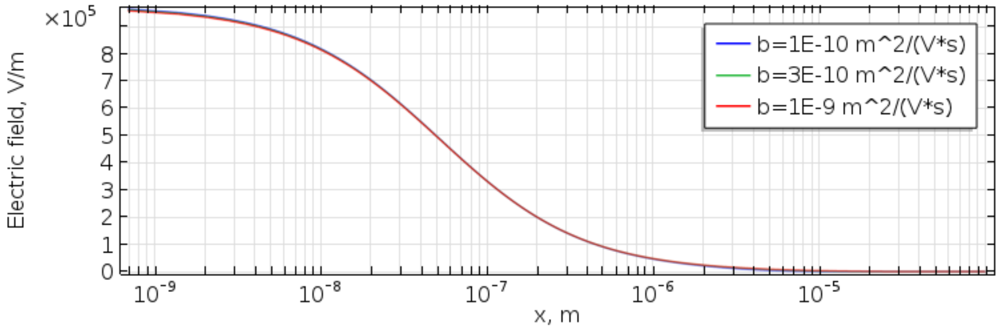

Figure 3: Different mobility of ions.

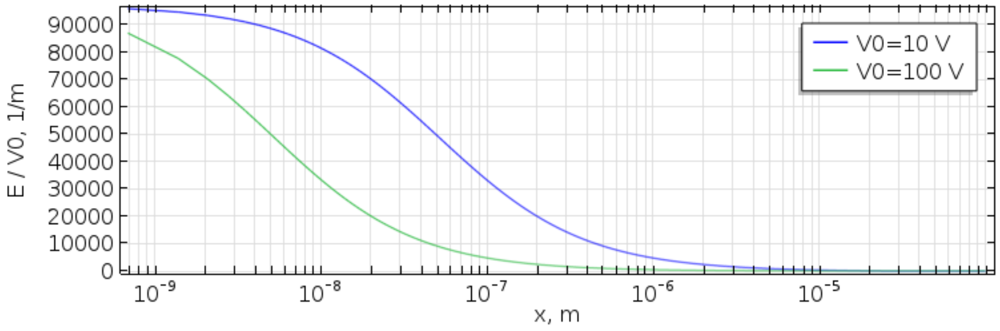

Figure 4: Different voltage.

Figure 5: The results of parametric study.

As it can be seen in Fig. 2a, the value of the liquid conductivity has almost no impact on the layer size and influences the electric field distribution only at the distances on the order of 1μm. However, according to (11), it takes much more time for the charge to be accumulated. The Fig. 2b shows the absence of any effect of the ion mobility at all. This is the outcome of the Einstein relation (8). Nevertheless, if the tangential electric field exists, the impact of the ion mobility can be significant. The absolute value of the electric field to be screened has the strongest influence and can cause the layer to change the thickness in a wide range of values (Fig. 2c); the stronger the field, the thinner the layer.

4 Realistic geometry

Computer model

The next step of the present study is to consider more complicated system from [3] where the charge transport along the interface is possible. The not-to-scale geometry of the system is shown in the Fig. 3. The system consists of two flat parallel electrodes and a dielectric plate (barrier) having a small circular hole. The barrier is placed between the plates and splits the chamber filled with a dielectric liquid into two equal parts, with the hole remaining the only link to connect them. The charge accumulates on the barrier surface, screens the normal component of the electric field, and moves the electric field lines to the hole (the only available way). As a result, a region of the strong electric field emerges inside the hole, which was used to study the field-enhanced dissociation phenomenon both experimentally and numerically. The computer model in [3] used liquids with the low-voltage conductivity ranging from nearly 10^{-9} S/m to 10^{-8} S/m and the simplified boundary condition $E_N = 0$ that needs validation.

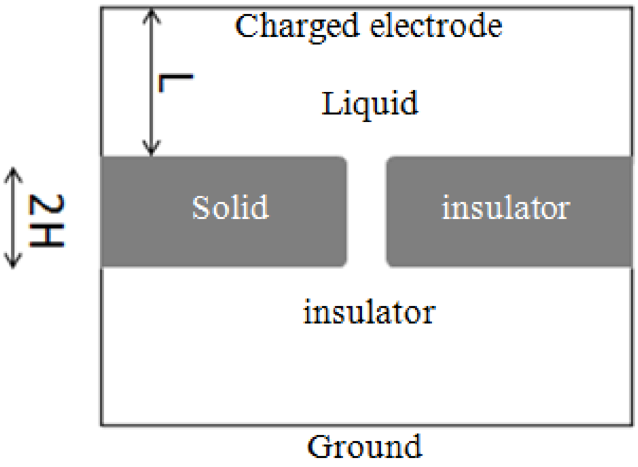

Figure 6: The schematic not-to-scale illustration of the EHD system used in [3].

The system has axial symmetry, thus a 2D axisymmetric model can be implemented. Moreover, there is a horizontal plane of symmetry, so it is reasonable to build geometry only with a quarter of system shown in Fig. 3.

The geometry and the boundary conditions of the computer model are shown at the Fig. 4. The finite-element model for this geometry accounts for steep gradients of the ion concentrations near the surfaces of the solid insulation.

Results

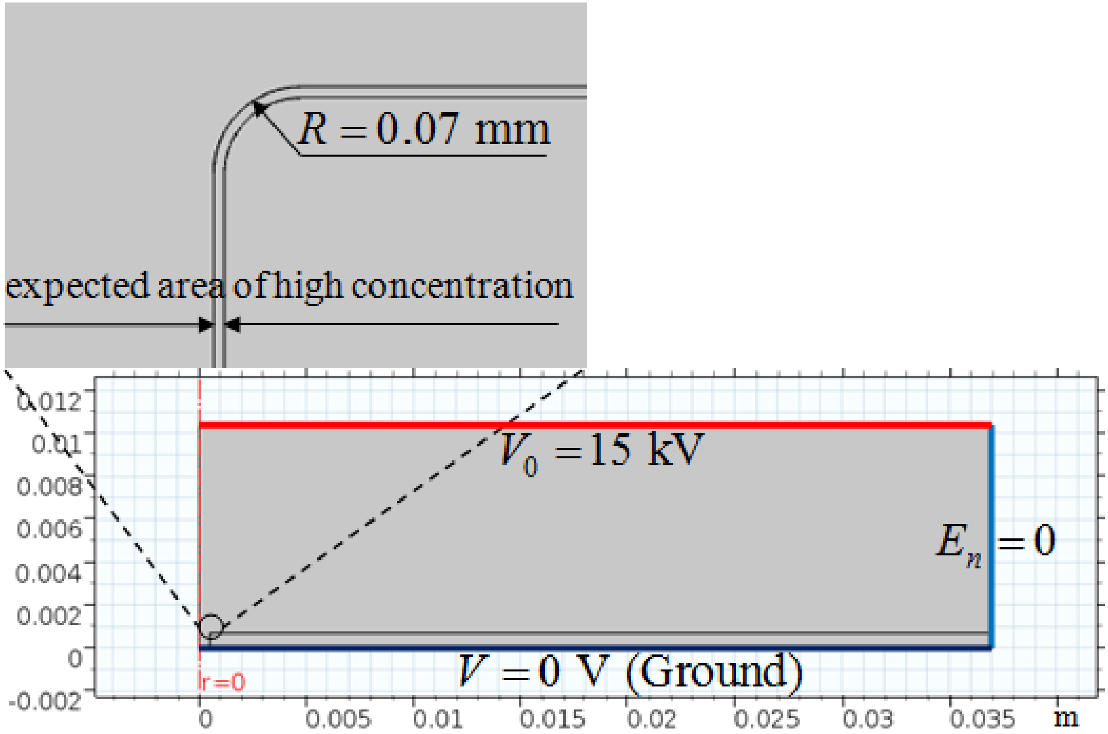

Figure 7: The geometry and the boundary conditions of the computer model.

The most convenient quantities to compare the simplified model and the more complete one are the total current and the electric field distribution. To obtain the results in the case of the simplified boundary condition $E_N = 0$, equation (1) was solved and the total current was calculated using the Ohm's law. In the case of the more complete model, the current is the integral of the current density computed basing on both the ion distributions and that of the electric field.

Figure 5 shows the relative increase of the total current compared to the simplistic model. It is clearly seen that the currents are equal at 10^{-8} S/m, differ slightly at 10^{-9} S/m and dramatically at lower values of the liquid conductivity. At 10^{-12} S/m the current in the more complete model becomes more than a hundred times higher than in the simplified one. Consider the distributions of the electric field normalized to the electric field value in the barrier in case of the absence of the hole (Fig. 6). Figure 6a shows the distribution for the case of the simplistic model where the normal component of the electric field is set to zero on the barrier surface. This model is incapable of computation the electric field strength distribution inside the barrier so it is zero here. In the rest of the model (in the liquid) all electric field lines pass through the hole and there is a region of the strong electric field inside. Comparing Fig. 6a and 6b, one can note that the electric field distributions agree well and just a few electric field lines pass through the barrier instead of the hole. The Fig. 5 shows that this effect is negligible. However, in the case presented in Fig 6c, the difference is apparent: most of the electric field lines pass through the barrier, which means that the normal component of the electric field close to the

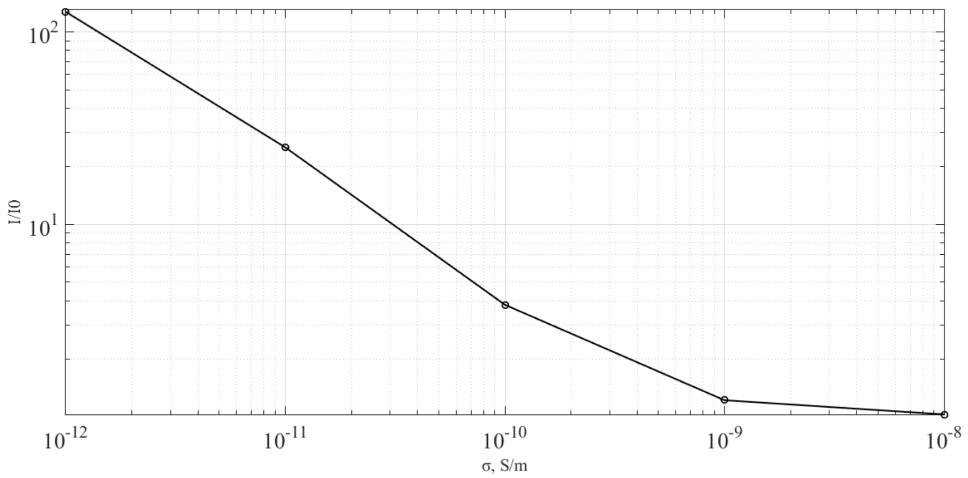

Figure 8: The comparison of models by current.

barrier is not completely screened. Moreover, the electric field strength distribution in the hole differs greatly from the cases presented in Fig 6a and 6b.

These results can be explained by the following: there appears a tangential component of the electric field at the solid-liquid interface, which cause the accumulated charge to migrate along the surface. If the time of the charge accumulation (11) is short, the effect of the charge transport is negligible (the case of Fig. 6b). Otherwise, the charge „slips“ along the barrier to the hole, providing the higher currents and reducing the screening of the normal component of the electric field in the liquid near the interface (the case of Fig. 6c).

Basing on the results, the limit of the applicability of the boundary condition lies in the interval between 10^{-9} S/m and 10^{-8} S/m. However, the limit can shift towards the low-conductivity region in Fig. 5 due to a number of factors. Firstly, if the field-enhanced dissociation is accounted for, it increases the effective value of the conductivity in the region of the strong field. Secondly, the ion mobility that determines the intensity of the charge transport along the barrier may decrease if the ion is very close to the surface.

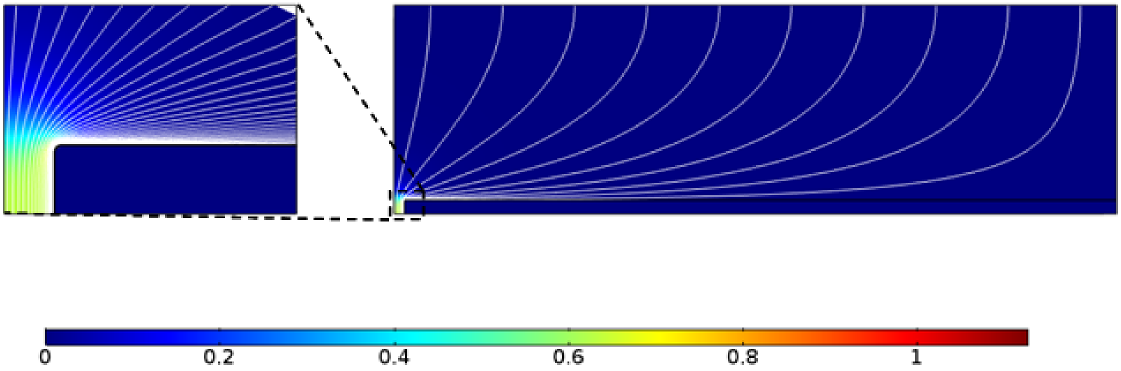

Figure 9: The distribution of the normalized electric field, the case of the full screening of electric field by solid insulator.

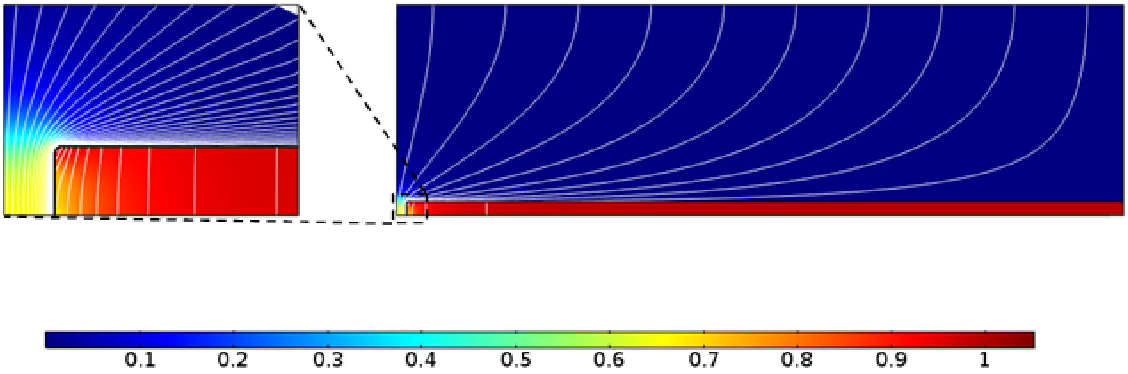

Figure 10: The distribution of the normalized electric field, $\sigma = 10^{-8}$ S/m.

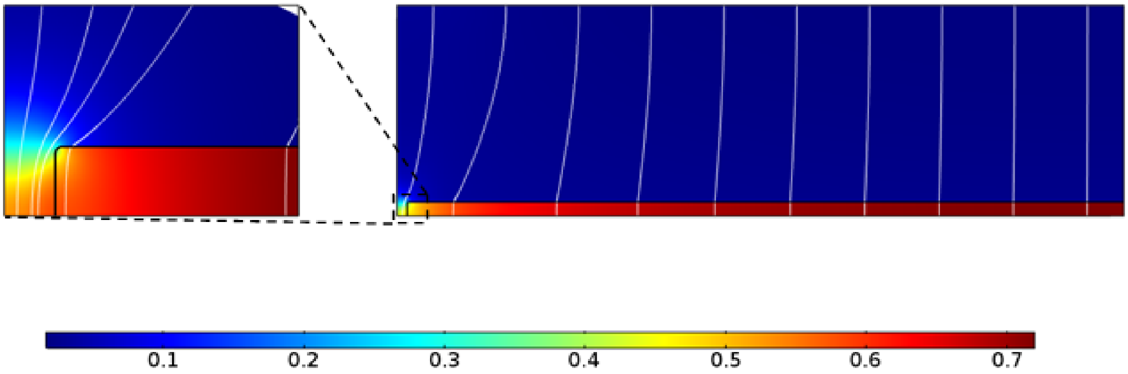

Figure 11: The distribution of the normalized electric field, $\sigma = 10^{-12}$ S/m.

Figure 12: The results of parametric study.

5 Conclusions

The problem of the electric field screening by the accumulated charge on the interface between the dielectric liquid and solid perfect insulator was studied by means of the computer simulation. The results of the calculations allow concluding the following. Considering liquid dielectrics and strong electric fields (on the order of $10^7 V/m$), the

thickness of the charged layer depends on the electric field strength, the conductivity affects only the time of the charge accumulation, and the ion mobility has no effect at all in the case of the 1D approach.
However, when there is the charge transport along the surface (2D model) the steady-state electric field distribution depends on the conductivity. The result shows that the simplified model of the considered EHD system employing the boundary condition $E_N = 0$ can be used if the liquid conductivity is equal to or higher than 10^{-8} S/m. Otherwise, the electric field distribution and the total current do not correspond to the more complete model.
Accounting for the field-enhanced dissociation should extend the applicability limits of the simplified model. Also, the impact of the ion mobility is yet to be investigated.

6 Acknowledgements

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