Theoretical and Numerical Analysis of the flow separation criterion for hypersonic nonequilibrium flow over

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Abstract

Through combining the triple-deck theory in the analytical treatment of shock wave boundary layer interactions and the numerical simulation of hypersonic nonequilibrium flow over compression corner, the influence factors on flow separation are analyzed, and a criterion parameter to predict whether the separation occurs is proposed. The criterion parameter (S) is the product of the powers of the corner angle, the freestream Mach number and Reynolds number, and the Chapman-Rubesin parameter. It is based on the existing formula of the incipient separation angle for calorically perfect gas flow, but the reference temperature and the corresponding viscosity in the calculation of Chapman-Rubesin parameter are determined by the reference enthalpy with chemical equilibrium assumption, which introduces the high-temperature gas effects. The powers in the criterion parameter (S) and the critical value of S for incipient separation are determined through large number of numerical simulations of hypersonic nonequilibrium compression corner flow for 3 corner angles (15, 18 and 24 degree), where the 12 freestream Mach numbers range from 8 to 35, the 36 gas densities are corresponding to the altitudes from 30 to 65km, the freestream Reynolds numbers range from 104 to5 $\times 10^6$.

Key Words: hypersonic, nonequilibrium flow, compression corner flow, flow separation, numerical simulation

1 Introduction

Shock waveïCboundary-layer interaction is an important flow phenomenon that exists widely in the flow over control surface or in the inlet of the hypersonic vehicle. The flow in these areas can be simplified to a compression corner flow. The occurrence of Shock waveïCboundary-layer interaction will produce local peak pressure and peak heat flux. Moreover, when the shock is strong enough, the separation of the boundary layer occurs, which will make the wave structure of the flow field complicated, the increase the peak value of wall pressure and heat flux further, change the aerodynamic load of the hypersonic vehicle control surface, and increase the

drag. The separation of the boundary layer in the inlet will seriously deteriorate the flowfield quality, and cause the loss of the engine intake. Therefore, it is very important to obtain the separation criterion of hypersonic compression corner flow. Under hypersonic conditions, dissociation and even ionization of the high temperature air take place in the boundary layer. This causes the decrease of the temperature and therefore the decrease of gas viscosity, which results in less loss of the gas kinetic energy, and the increase of the gas ability to overcome the adverse pressure gradient. This means that the chemical reaction of the high temperature gas may delay the flow separation as compared with the calorically perfect gas[1].
The study of flow separation criterion has been done by many researchers through theoretical analysis, experimental research, and numerical simulation. The initial separation criterion proposed is the incipient separation angle. Under a given flow condition, the boundary layer will separate only when the angle of the compression corner exceeds a certain critical value (called the incipient separation angle). The incipient separation angle has many influencing factors, including the flow type of the boundary layer (laminar or turbulent), the freestream Mach number and the Reynolds number, and the wall temperature [2]. In 1967, through correlation analysis of the experimental data, Neeham [3] $\sim$ [5] gave the approximate formula of the incipient separation angle for hypersonic laminar boundary layer

$$Ma_\infty \theta_{is} = K \text{Ø}^{1/2} \tag{1}$$

where \`$_{is}$ is expressed in radians, the coefficient K is taken as 1.13, and the Ø is the viscous interaction parameter

$$\text{Ø} = Ma_\infty^3 \sqrt{C_{\mathrm{REF}}} \Big/ \sqrt{Re_{L\infty}} \tag{2}$$

C_{REF} in the above equation is Chapman-Rubesin parameter

$$C_{\mathrm{REF}} = \frac{\bar{}_{\mathrm{REF}}}{\bar{}_{\infty}} \frac{T_\infty}{T_{\mathrm{REF}}} \tag{3}$$

where T_{REF} is the reference temperature in the boundary layer and $\bar{}_{\mathrm{REF}}$ is the corresponding viscosity of the gas.
As a pioneering work in the theoretical analysis of the shock waveïCboundary-layer interaction, Lighthill [7], Stewartson [8] and Neiland [9] proposed the triple-deck theory and established the governing equation of the disturbance flow field in the late 1970s. The local disturbance field is organized into a vertically layered structure: an outer layer external to the boundary layer consisting of potential disturbance flow associated with the viscous displacement effect of the underlying deckħњa middle layer of negligible shear-stress-perturbation rotational inviscid disturbance flow occupying the outer 90 percent of the incoming boundary layer thickness; a thin inner layer of viscous disturbance flow within the linear portion of the velocity profile that is interactively coupled with the local pressure field. Triple-deck theory provides a basic theoretical explanation for the upstream influence and free interference phenomenon. Inger (in 1994) [6] [10] solved the governing disturbance flow equations

yielded by this triple-deck theory and derived the incipient separation angle 00, in which the coefficient K is taken as 1.26.
With the arrival of the computer age, the CFD numerical simulation of the flow field based on Navier-Stokes equations becomes an extensively effective method. The numerical study of the high temperature real gas effect on shock waveïCboundary-layer interaction [13, 14, 15, 16, 17] is also being carried out. John [18] assessed the suitability Eq. (1) through numerical simulations for flows with different freestream to wall temperature ratio and at low and high enthalpy conditions. It is pointed out in [19] that Eq. (1) can successfully fit a great deal of experimental data if K=1.4∼1.5 is used in the formula.
Neeham's fitting coefficient applies to the case of small Ma_∞ `, Inger's fitting coefficient applies to the case of larger $Ma_\infty$ `. John included the high temperature gas effects in some of the numerical simulations, however, the detail conditions of the freestream are not taken into account, such as the freestream velocity and gas density. Considering that the velocity and density may influence the flow thermo-chemical state in the boundary layer and the shock intensity, which will further affect the adverse pressure gradient and the corresponding flow separation characteristics, it can be concluded that Ma_∞ ` is not an inclusive parameter to distinguish the flow condition for the determination of appropriate value of coefficient K in Eq. (1).
In the present work, the incipient separation angle formula (1) is transformed into a formula of separation criterion parameter: $S = Ma_\infty^m Re_{L\infty}^{1/4} C_{\mathrm{REF}}^{-1/4} \left(T_\infty / T_s\right)^n \theta_B^l$. The values of the power m, n and l and the critical value of S corresponding the incipient separation are determined by the combination of theoretical analysis and numerical simulation of nonequilibrium compression corner flow. Theoretical analysis is mainly about the influence of chemical reactions on the distribution of temperature and the corresponding viscosity in the boundary layer, on the shock intensity, and on the upstream propagation of the adverse pressure gradient. The nonequilibrium flow simulations are carried out for three compression corners (with angle of 15, 18, and 24 degree) in wide range of hypersonic flight condition, 12 freestream Mach numbers (8, 10, 13, 15, 18, 20, 23, 25, 28, 30, 33, 35) and 36 gas densities (corresponding to the atmosphere at altitude from 30 to 65km) are taken as the case conditions.

2 Theoretical analysis

2.1 Separation Criterion Parameters Based on Triple-Deck Theory

According to Eq. (1), flow separation occurs when the compression corner angle $`_B$ is greater than or equal to $`_B$, namely

$$\theta_B \geq \theta_{is} = \frac{K Ø^{1/2}}{Ma_\infty} \tag{4}$$

In practice, the Mach number and Reynolds number may change during the flight, while the shape of the vehicle does not change generally, that is, $`_B$ keeps constant. For convenience in the prediction of separation, we reform Eq. (4) into

$$\frac{Ma_\infty \theta_B}{\O^{1/2}} \geq \frac{Ma_\infty \theta_{is}}{\O^{1/2}} = K \tag{5}$$

From the above equation, we know that for a given corner angle $`_B$ , the flow separation occurs when $\left(Ma_\infty `_B \big/ \O^{1/2}\right) \geq K$. So a separation criteria parameter can be defined as

$$S_0 = \frac{Ma_\infty \theta_B}{\O^{1/2}} \tag{6}$$

In the above equation, $S_0 = \frac{Ma_\infty \theta_B}{\chi^{1/2}}$ is a hypersonic similarity parameter which reflects the intensity of the corner-generated shock and the corresponding adverse pressure gradient, and the viscous interaction parameter $\O$ reflects the pressure increment caused by hypersonic viscous interaction, which can also reflect the flow ability to resist the adverse pressure gradient,. So the separation criterion parameter S_0 is an index of the adverse pressure gradient relative to the flow ability to withstand it. Thus the greater the separation criterion parameter S_0 , the more prone of the flow to separation.

For ease of use, the separation criterion parameter S_0 can also be expressed directly as a function of the freestream Mach number and Reynolds number

$$S_0 = Ma_\infty^{-1/2} Re_{L\infty}^{1/4} C_{\mathrm{REF}}^{-1/4} `_B \tag{7}$$

when , $S_0 > S_{0,is} = K = Ma_\infty \theta_{is} \big/ \chi^{1/2}$ the flow separation occurs.

2.2 Introduction of the Effects of High Mach Number and High Temperature

The separation criterion parameter S_0 in Eq. (7) is defined based on the results of triple-deck theory for calorically perfect gas. Considering the change of temperature distribution by chemical reactions and the limitation of the first-order approximation in the triple-deck theory under very high Mach number conditions, Eq. (7) is modified as follows.

The Chapman-Rubesin parameter is usually determined using the reference temperature method under the assumption of calorically perfect gas. The reference temperature is a function of the freestream Mach number and the wall temperatureħə

$$\frac{T_{\mathrm{REF}}}{T_\infty} \cong 0.50 + 0.039 Ma_\infty^2 + 0.5 \frac{T_w}{T_\infty} \tag{8}$$

The corresponding viscosity is generally determined by Sutherland formula. Considering the chemical reactions in hypersonic flow and the effects on temperature, the reference enthalpy method is used instead of the reference temperature method. The reference enthalpy in the plate boundary layer is

$$h_{\mathrm{REF}} \cong 0.50\,(h_\infty + h_w) + 0.22 \times \sqrt{Pr}\,(H_\infty - h_\infty) \tag{9}$$

where H_∞ is the total enthalpy of the freestream and h_w is the static enthalpy at wall. It can be seen that the influence of wall temperature is taken into account already. To determine the reference pressure in the boundary layer, the viscous interaction should be taken into account. In the strong interaction zone (when $Ø > 3$)

$$p_{\mathrm{REF}} \cong p_\infty \times (0.514Ø + 0.759) \tag{10}$$

In the weak interacting zoneħïwhen $Ø > 3$ ħκ,

$$p_{\mathrm{REF}} \cong p_\infty \times \left(1 + 0.31\chi + 0.05\chi^2\right) \tag{11}$$

With the reference enthalpy and the reference pressure, the reference temperature T_{REF} can be determined with chemical equilibrium assumption, and the corresponding equilibrium chemical composition can be obtained. Then the viscosity of the multicomponent gas mixture can be calculated from the species viscosity by means of mixture rules [20]. Note that the viscous interaction parameter $Ø$ is needed in calculating the reference pressure with Eq. (10) and Eq. (11), however $Ø$ is also related to C_{REF} [see Eq.(2)]. So the above process to determine the reference temperature T_{REF} and the parameter C_{REF} requires the iteration. In general situation, two to four iterations are enough to meet convergence.
It can be seen from Eq. (7) that the increase of Mach number, the decrease of Reynolds number, and the increase of compression angle will promote the flow separation. The reason why the the decrease of Reynolds number promotes separation is that it leads to the thickening of the boundary layer and the increase of wall friction. There are several aspects of the influence of Mach number increase. On the one hand, the increase of Mach number means the increase in shock intensity and the adverse pressure gradient. On the other hand, the Mach number increase means the increase in flow kinetic energy and the ability to withstand the adverse pressure gradient. Moreover, as Mach number increases, the subsonic region in the boundary layer is reduced, so the range for the propagation of adverse gradient decreases. In combination, the Mach number increases will suppress the flow separation.
The influence of Mach number is embodied by $Ma_\infty^{-1/2}$ in Eq. (7), which is based on the triple-deck theory with first-order approximation and the calorically perfect gas assumption. This result is obtained in the case where the streamline expansion effect under very high Mach number conditions is omitted and $Ø$ is not much greater than 1. It is pointed out in [6] that the upstream influence distance and the incipient pressure will decrease if the streamline expansion effect is considered, which means that the extent of suppressing flow separation by the increase Mach number should be stronger than what Eq. (7) reflects. So the power of Ma_∞ in Eq. (7) can be modified to embody the above effects.
The increase in `$_B$ will increase the shock intensity and the adverse pressure gradient, therefore promote flow separation. Considering that the extent of increase in adverse

pressure gradient due to the increase of `$_B$ may be stronger under very high Mach number conditions, the power of `$_B$ can also be adjusted.
In addition, considering the wide range of hypersonic flight altitude which means the variation of freestream temperature, and the freestream temperature variation in hypersonic or high enthalpy tunnel, it is necessary to analyze the effects of freestream temperature on the flow separation. The effect of the increase in freestream temperature is twofold. On the one and, the temperature rise causes the Reynolds number to decrease and thus promotes separation. On the other hand, the freesteam temperature rise at unvarying Mach number also means the increase in the flow kinetic energy and an enhancement of the ability to withstand the adverse pressure gradient. The effect to promote separation by temperature rise through the decrease of Reynolds number has been reflected by $Re_{L\infty}^{1/4}$ in Eq. (7) already. Here the effect to suppress separation by temperature rise through the increase in kinetic energy should be added. In the present work, the effects of freestream temperature on separation are introduced into the separation criteria parameter formula by means of the ratio of freestream temperature to the air temperature at standard conditions.
Based on the above analysis, Eq. (7) is modified, adjusting the power of Ma_∞ and `$_B$ in to m and l respectively, and introducing T_∞/T_s , the ratio of the freestream temperature to that at standard conditions, with the power of n. Namely, an improved separation criterion parameter is defined as

$$S = Ma_\infty^m Re_{L\infty}^{1/4} C_{\mathrm{REF}}^{-1/4} \left(T_\infty/T_s\right)^n \;`_B^l \tag{12}$$

The values of m, n and l are determined by numerical simulations of nonequilibrium compression corner flow over wide range of Mach number, Reynolds number with various freestream temperature and corner angles.

3 Numerical Analysis

3.1 Governing Equations and Numerical Methods

The governing equations for the flow field are the two-dimensional Navier-Stokes equations coupled with the vibrational and chemical kinetics, which are solved to obtain the steady state solution of the flow field. A finite difference method is used in the calculation. All inviscid terms are discretized with AUSMPW+ scheme [21]. The viscous terms are discretized with center difference scheme.The inviscid fluxes are discretized implicitly while the viscous terms explicitly. The implicit parts of the differential equations are disposed in two steps with the LU-SGS approach [22]. The details of the flow field governing equations are described in [23].
The total length of the compression corner model is 0.6096m, of which the length of the front plate is 0.3048m. The computational mesh (Figure 1) is 131×81 with 131 points along the surface and 81 points in the flowfield normal to the body, only half of the grid points in both directions are shown for clarity. Refined grids are used near the leading edge, the corner and the position of peak pressure. Exponential stretch is used from the wall. The first normal grid height at the wall is 6.096×10^{-5}m.

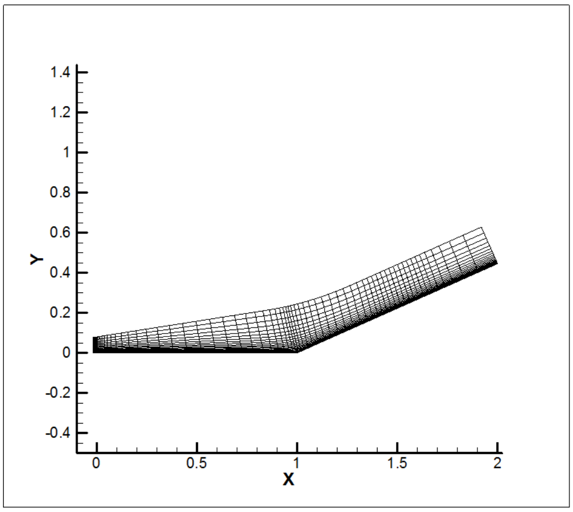

Figure 1: Computational mesh

3.2 Case Conditions

The compression corner angle, freestream Mach number and temperature of the calculation cases are listed in Table 1. For any one case condition (a set of given corner angle, freestream Mach number and temperature) in Table one, flow simulation is carried out with various freestream density. The density values are those of the atmosphere at altitude of 30km to 65km. Through changing the density step by step (the corresponding change of the altitude is 1 km in one step), the density value just corresponding the incipient separation is identified, so is the separation criterion parameter value [by Eq.(7)] at incipient separation, denoted as $S_{0,is}$. With the values of $S_{0,is}$ under each set of case condition, the change of with the freestream Mach number and temperature and the corner angle is analyzed, and the values of the powers in the improved formula of separation criterion parameter [Eq. (12)] are determined.

3.3 Analysis of the Numerical Results

The calculation results of all cases show that the decrease of freestream Mach number and temperature, the increase of the freestream density and the corner angle, will promote the flow separation. Take 18deg compression corner at Mach number of 15 and altitude of 45km as an example, the flowfield pressure distribution and the streamline in the recirculation zone for three different freestream temperauture (100K, 300K, and 500K) are given in Figure 2. The separation zone for the case of $T_\infty = 100$K is obviously larger than the cases with higher T_∞.

Table 1 Freestream Mach number and temperature of the calculation cases

Corner angle (deg)	Mach number	Freestream temperature(K)
15	8, 10, 13, 15, 18, 20, 23, 25	100, 200, 300, 500
18	10, 13, 15, 18, 20, 23, 25, 28	100, 200, 300, 500
24	20, 23, 25, 28, 30, 33, 35 100,	200, 300, 500

There are obvious differences in the values of separation criterion parameter [see Eq. (7)] at incipient separation $S_{0,is}$ for the calculation cases. Take the 15deg compresses

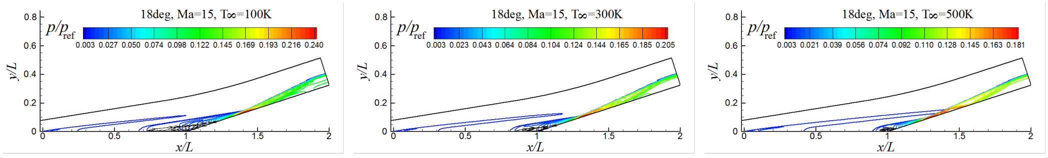

Figure 2: Pressure contour and separation zone streamline at different freestream temperature(18eg corner, Ma=15)

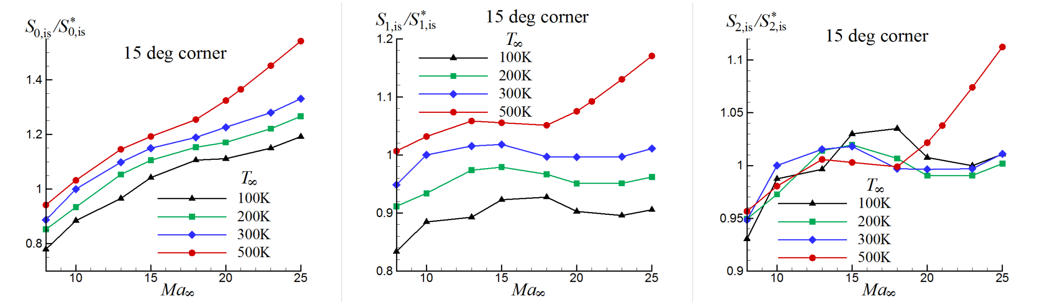

Figure 3: 15deg corner, Variation of separation criterion parameter with freestream Mach number and temperature

corner flow as an example, when the Mach number increases from 8 to 25, increases from 1.13 to 1.74 for the case of T_∞ = 100K, while from 1.01 to 1.94 for the case of T_∞ = 300K, and from 1.05 to 2.25 when T_∞ = 500K. Figure 3(a) gives the variation of the normalized incipient separation parameter (namely $S_{0,is}/S^*_{0,is}$) with Mach number for the 15deg corner flow at different freestream temperature. $S^*_{0,is}$ is the value of incipient separation criterion parameter at $Ma_\infty = 10$ and T_∞ = 300K , which is 1.457.

After the analysis of the change of $S_{0,is}$ with Mach number, the separation criterion parameter is modified to

$$S_1 = Ma_\infty^{-0.8} Re_{L\infty}^{1/4} C_{\text{REF}}^{-1/4} \tag{13}$$

Figure 3(b) shows the change of $S_{1,is}/S^*_{1,is}$) with freestream Mach number and temperature. $S^*_{1,is}$ (= 0.7308) is the value at Ma_∞ =10 and T_∞ = 300K. It can be seen that the change range of $S_{0,is}$ with Ma_∞ is much smaller than that of $S_{0,is}$, especially for the case of T_∞ = 300K and Ma_∞ = 10∼25, the change of $S_{0,is}$ is within 1.5%.

Both Figure 3(a) and Figure 3(b) show that $S_{1,is}$ increases with the freestream temperature. So the separation criterion parameter is further modified to

$$S_2 = Ma_\infty^{-0.8} Re_{\mathbf{L}\infty}^{1/4} C_{\text{REF}}^{-1/4} \left(T_\infty/T_s\right)^{-0.1} \tag{14}$$

where = 288.15K, the temperature of atmosphere at standard conditions. Figure 3(c) shows the change of $S_{2,is}/S^*_{2,is}$ with freestream Mach number and temperature, where $S^*_{2,is}$ (= 0.7278) is the value at Ma_∞ =10 and T_∞ = 300K. It can be concluded from Figure 3(c) that the freestream temperature effects on flow separation are reflected by introducing (T_∞/T_s) into Eq.(14).

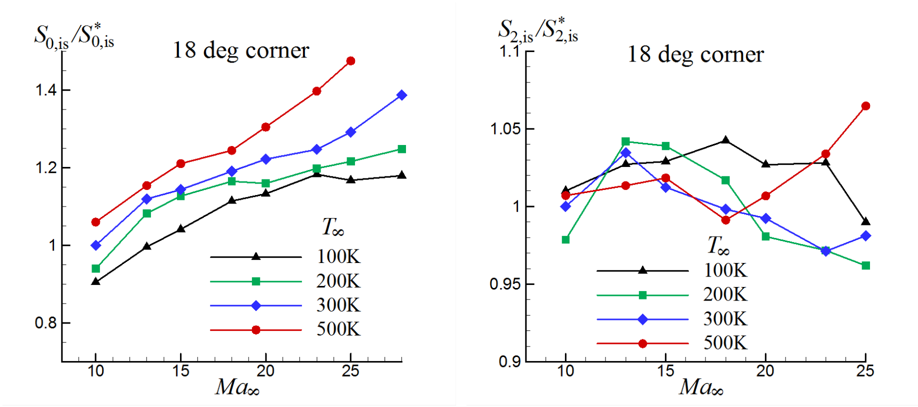

Figure 4: Figure 4 18eg corner, Variation of separation criterion parameter with freestream Mach number and temperature

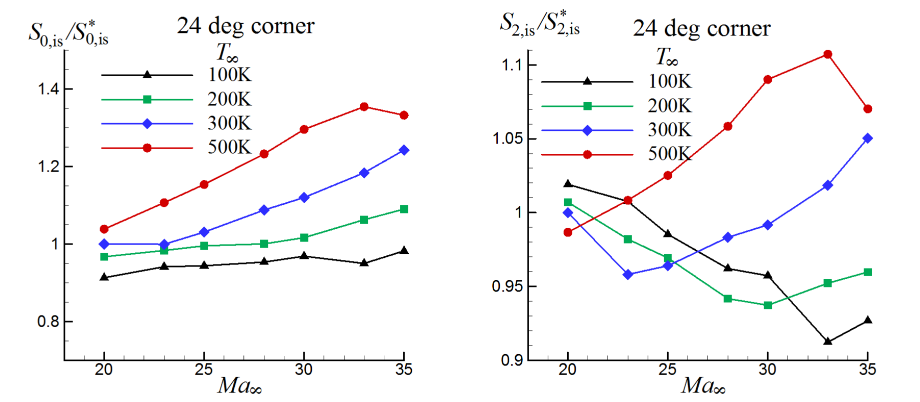

Figure 5: Figure 4 24deg corner, Variation of separation criterion parameter with freestream Mach number and temperature

However, for the case of $T_\infty = 500$K, the incipient separation criterion parameter increases obviously when the Mach number is greater than 23 (corresponding to the flow velocity greater than 10 km /s). This can be seen in Figure 3 (a) to Figure 3(c), with the three parameters, $S_{0,is}$, $S_{1,is}$, and $S_{2,is}$. Such phenomenon may be related to the start of some chemical reaction mechanism at very high speed.
Figure 4 and Figure 5 show the variation of $S_{0,is}/S^*_{0,is}$ and $S_{2,is}/S^*_{2,is}$ with freestream Mach number and temperature for 18deg and 24deg compression corner respectively. In Figure 4, $S^*_{0,is}$ (= 1.4215), $S^*_{2,is}$ (= 0.7125) are the values at Ma_∞ =10 and T_∞ = 300K. In Figure 5, $S^*_{0,is}$ (= 1.6299) and $S^*_{2,is}$ (= 0.6635) are the values at Ma_∞ =10 and T_∞ = 300K. The effects of the modification of the separation criterion parameter [in Eq .(14) and (14)] are also shown in 18deg and 24deg compression corner flows. However, for the case of $T_\infty = 500$K, the incipient separation criterion parameter increases significantly when the Mach number is greater than 23 for 18deg compression corner or greater than 30 for 24deg compression corner flow, which is similar to the 15deg compression corner case.
The comparison between the values of incipient separation criterion parameter for the three compression corner flows show that the incipient separation criterion parameter decreases as the corner angle increases. This indicates the necessity to

introduce the angle into the formula separation criterion parameter. Based on the existing numerical data, the definition of separation criterion parameter is further revised to

$$S = Ma_{\infty}^{-0.8} Re_{L\infty}^{1/4} C_{\text{REF}}^{-1/4} \left(T_{\infty}/T_s\right)^{-0.1} \theta_B^{1.2} \tag{15}$$

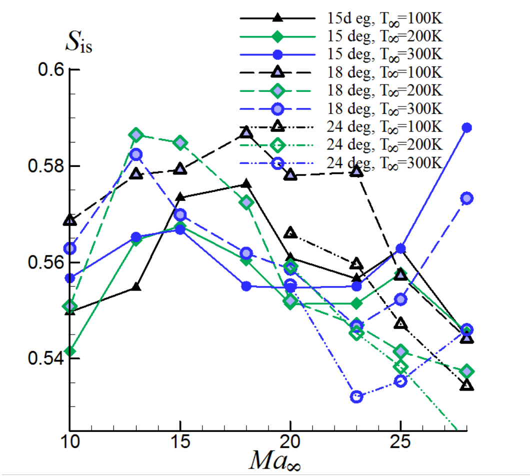

Figure 6: Variation of separation criterion parameter with corner angle, freestream Mach number, and temperature

Figure 6 shows the variation of the value of the incipient separation criterion parameter with the Mach number (10 to 28), corner angle (15deg, 18deg, and 24deg), and the freestream temperature (100K, 200K and 300K). The variation range of S_{is} is 0.525∼0.565 , within 8%. For the case of $T_{\infty} = 500$K, the change of S_{is} is basically in the range, except for the very high Mach number (greater than 23 for 15deg and 18deg corner, and greater than 30 for 24deg corner).

4 Conclusion

(1) Based on the incipient separation angle formula yielded by the triple-deck theory with calorically perfect gas assumption, a separation criterion parameter (S_0) is proposed. S_0 is an index of the adverse pressure gradient relative to the flow ability to withstand it. So the larger the parameter is, the more prone of the flow to separate.
(2) The high temperature real gas effects on flow separation are introduced through the use of reference enthalpy and chemical equilibrium assumption to determine the reference temperature, and the modification of separation criterion parameter formula to $S = Ma_{\infty}^{m} Re_{L\infty}^{1/4} C_{\text{REF}}^{-1/4} \left(T_{\infty}/T_s\right)^{n} \theta_B^{l}$. The values of the power in the formula and the incipient separation criterion parameter value are determined through a large number of nonequilibrium compression corner flow simulations. The present results for the power values are m=-0.8, n=-0.1, and l=1.2. For most of the case, the value range of the incipient separation criterion parameter is $S_{is} = 0.540 \sim 0.585$.

(3) Considering that only three different corner angles are taken in the simulation and analysis, more work is needed to determine the power value of the angle in the separation criterion parameter. For the case with even larger corner angle, the corresponding slope of the flow deflection angle (tan `$_B$ ) can be used instead of `$_B$. The determination of the power values of the freestream Mach number and temperature also need further in-depth work.

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