The Role of synchronization in Transition to Two-dimensional and Three-Dimensional Turbulence

Dmitry Zhilenko, Olga Krivonosova

jilenko@imec.msu.ru

Abstract

Turbulent flows of a viscous incompressible fluid in a layer between rotating concentric spheres under the action of the modulation of the velocity of one of the spheres have been studied experimentally and numerically. We used an algorithm of numerical solution based on a conservative finite difference scheme of the discretization of the Navier–Stokes equations in space and semi-implicit Runge–Kutta scheme of the third order integration accuracy in time. Discretization in space was performed on grids nonuniform in radial and meridional directions with concentration near the boundaries and equatorial plane. The experimental setup consisted of two coaxial spheres. The space between the spheres was filled with silicone oil to which aluminum powder was added for visualization of flows. The rotation velocity was periodically varied. Agreement was shown to be between the experimental and calculated results, including the integral properties of turbulent flows. The possibility of the formation of turbulence with spectra qualitatively similar to spectra obtained in measurements in the upper atmosphere is established: with the slope close to -3 at low frequencies and close to $-5/3$ at high frequencies and with the negative longitudinal velocity structure function of the third order. It has been shown that such spectra are formed in the regions of a flow that are strongly synchronized under the action of the modulation of the rotational velocity.

1 Introduction

Large-scale flows in the atmosphere occur in the presence of fast rotation of the Earth, and their properties are usually explained within the concept of вЃЁtwo-dimensionalвЃ№ turbulence [1, 2]. In two-dimensional turbulence, two inertial intervals are usually identified corresponding to energy transfer at low wave numbers and enstrophy transfer at high wave numbers [3]. The inertial interval of energy transfer from high to low wave numbers (inverse cascade) is described by the same Kolmogorov relation as in three-dimensional turbulence [4] for the dependence of the energy spectrum $E(k)$ on the wave number $k : E(k) \sim k^{-5/3}$. In the inertial interval of enstrophy transfer from low to high wave numbers (direct cascade), this dependence has the form $E(k) \sim k^{-3}$. The direction of the cascade is determined by the sign of the third order longitudinal velocity structure function [5], which is defined as $D_{LLL} = \langle [u(l) - u(l')]^{-3} \rangle$, where u is the velocity at the spatially separated points l and l' and angular brackets mean averaging over the ensemble of realizations. The negative and positive signs of D_{LLL} correspond to the direct and inverse cascades, respectively. Conclusions of the theory of two-dimensional turbulence were confirmed in numerous results reviewed in [6, 7]. At the same time, measurements of the horizontal velocity of the wind in the EarthвЂ™s atmosphere revealed an anomalous location of spectral regions that is inconsistent with the theory of two-dimensional turbulence. In particular, spectra of turbulence with a slope of -3 begin at scales larger than $700km$ and are limited by a strong peak at a scale of $10^4 km$. Spectra with a slope $-5/3$ were detected at scales smaller than $500km$ [1]. Analysis of third order structure functions in [2] showed that only one of these regions with the slope of -3, corresponds to two-dimensional turbulence. This indicates the direct energy transfer cascade in both spectral regions under consideration. Despite the existing explanations [1, 8, 9], reasons for the inverse position of spectral regions, as well as the possibility of reproducing this phenomenon under laboratory conditions, are as yet unclear.

Both viscous dissipation [10] and vertical motions, which are components of Large-scale circulation [6], prevent twoBædimensional turbulent flows in the atmosphere. Large-scale circulation also exists in turbulent flows induced by the rotation of the boundaries of the spherical layer, which is responsible for the motion of viscous incompressible fluid between them [11]. It is exactly why the model spherical Couette flow is studied in this work for the qualitative simulation of processes in the atmosphere. By analogy with Baroud [12], we chose the case of the counter rotation of spheres. Under stationary boundary conditions, oppositely directed vortices with an interface between them are formed in the meridional plane of such a flow (see Figure 1, which is similar to Figure 1 in [13]). A similar circulation can be observed in the case of the rotation of only the inner sphere in the presence of altitude-inhomogeneous external heating [14] , typical of the atmosphere. In spherical layers the formation of turbulence with a high correlation dimension occurs by the increase in the rotation velocity of one of the boundaries [11, 15] as well as by their modulation [16]. The spectrum of developed turbulence in the latter case depends on the parameters of force action [17].

The dependence of the type of the azimuthal velocity spectra on the frequency and amplitude of the modulation of the rotation velocity of one of the spheres is determined in this work experimentally and numerically.

2 Methods of calculation and experiment. Field of study

An isothermal flow of a viscous incompressible fluid is described by the NavierвЂ"Stokes and continuity equations:

$$\frac{\partial U}{\partial t} = U \times rotU - grad\left(\frac{p}{\rho} + \frac{U^2}{2}\right) - \nu rotrotU, divU = 0 \quad (1)$$

where U, p, ν, and ρ, are the velocity, pressure, viscosity, and density of the fluid. These equations are numerically solved in a spherical coordinate system with the radial (r), polar (θ), and azimuthal (φ) directions, in which the impermeability and no-slip boundary conditions have the form $u_\varphi(r = r_k) = \Omega_k(t)r_k \sin(\theta)$, $u_r(r = r_k) = 0$, $u_\theta(r = r_k) = 0$, $k = 1, 2$, where u_φ, u_r, and u_θ are the azimuthal, radial, and polar components of the velocity; and $k = 1$ and 2 correspond to the inner and outer spheres, respectively. We used an algorithm of numerical solution [18] based on a conservative finite difference scheme of the discretization of the NavierвЂ"Stokes equations in space and semi-implicit Runge–Kutta scheme of the third order integration accuracy in time. Discretization in space was performed on grids nonuniform in r and θ directions with concentration near the boundaries and equatorial plane and the total number of nodes $5.76 \cdot 10^5$. This algorithm was used for calculations with both stationary [11] and periodic [19] boundary conditions. Spectra of pulsations of the square of the azimuthal velocity component u_φ (minus the average value determined for the entire sample) were calculated at points $1 - 7$ shown in Figure 1 (θ and φ are constant and only r is varied). To this end, u_φ time series with a length of no less than 72000 points were written with a time step $\triangle t = 0.015 - 0.025s$. D_{LLL} was obtained using the dependence of u_φ on the azimuth angle φ during 16 rotation periods ($0 \leq \varphi \leq 32\pi$). All calculations were performed for the initial and boundary conditions corresponding to the experimental conditions.

The experimental setup consisted of two coaxial spheres. The outer radius of the inner sphere was $r_1 = 0.075m$ and the inner radius of the outer sphere was $r_2 = 0.150m$. The space between the spheres was filled with silicone oil. The rotation velocity was periodically varied by the law $\Omega_k(t) = \Omega_{k0}(1+A_k \sin(2\pi f_k t+\psi_k)$ with an accuracy of no worse than 0.5% (where A_k and f_k are the amplitude and frequency of modulation; Ω_{k0} is average angular velocity of rotation; initial phase ψ_k is arbitrary). The modulation frequencies $f_1 = 0.01 - 0.1Hz$ and $f2 = 0.01 - 0.02Hz$ were no higher than the average rotation frequencies of the spheres ($\Omega_{10}/2\pi = 0.59Hz$, $\Omega_{20}/2\pi = 0.32Hz$). The measurements of u_φ were performed near the outer sphere at a distance of $0.078m$ from the equator plane and at a distance of $0.105m$ from the rotation axis (near point 7 in Figure 1). The experiments were performed at Reynolds numbers $Re_1 = \Omega_{10}r_1^2/\nu = 412.5 \pm 0.5$ and $Re_2 = \Omega_{20}r_2^2/\nu = 900 \pm 1$. At these Reynolds numbers in the absence of modulation, a periodic flow with the frequency $f_0 = 0.0376Hz$ is formed in the layer; this flow referred to as initial is a result of mutual synchronization of individual linear modes [13]. The initial flow has the form of traveling azimuthal waves with the wave number $m = 3$. The modulation of the rotation velocity of one of the boundaries leads to the flow induced synchronization. With an increase in the amplitude of modulation at a

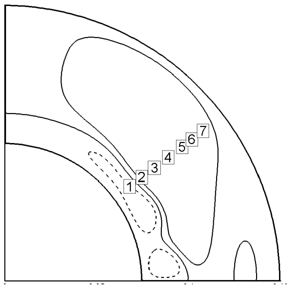

Figure 1: Calculated stream functions Ψ (in $[m^3/s]$)) in the meridional plane of the axisymmetric steady state flow at $Re_2 = -900, Re_1 = 414$: $\Psi_{max} = 6 \cdot 10^{-6}, \Psi_{min} = -6 \cdot 10^{-6}$, and $\triangle\Psi = 6 \cdot 10^{-6}$. Dashed lines are negative value contours. Points $1-7$ are located at the relative distance $l = (r - r_1)/(r_2 - r_1) = 0.135, 0.246, 0.359, 0.484, 0.611, 0.7, 0.803$ from the inner sphere with a deviation of 0.206π from the equatorial plane.

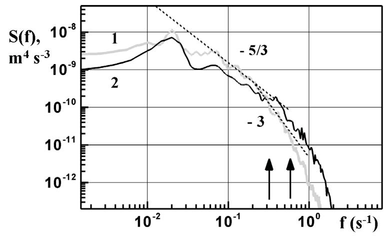

Figure 2: u_φ^2 spectra obtained in the 1 experiment and 2 calculation for point 7 in the case of the modulation of the inner sphere velocity with $f_1 = 0.01Hz$ and $A = 0.163$. The left and right vertical arrows correspond to the average frequencies of rotation of the outer and inner spheres, respectively.

fixed frequency, the initial flow is destroyed. Turbulence appears at the transition from mutual synchronization to induced synchronization [16].

3 Results

With an increase in the amplitude in the case of the modulation of the rotation velocity of the inner sphere, the spectra can be transformed to the form characteristic of two-dimensional turbulence. For example, in the case of the modulation of $\Omega_1(t)$ at $f_1 \leq f_0$ (Figure 2), the spectra obtained both from measurements and numerically exhibit a pronounced segment with a constant slope of $-5/3$ at low frequencies $(0.06-0.27Hz)$ and a segment with a constant slope of -3 at high frequencies $(0.27-0.8Hz)$. At an increase in the amplitude of the modulation of $\Omega_2(t)$, the spectra are modified to the form qualitatively corresponding to the spectra of atmospheric turbulence [1] with a slope -3 at frequencies below $0.1Hz$ and $-5/3$ at higher frequencies $(0.1 - 0.31Hz)$ (Figure 3a). Under the condition $f_k \leq f_0$, the form of the spectrum depends on the position of the point at which the azimuthal velocity is calculated. The most characteristic differences in the form of the spectra at points $1-7$ (Figure 1) are observed in the case of $\Omega_2(t)$ modulation. In particular, near the outer sphere and at a certain distance from it (points $7-3$), the observed spectra are typical to atmospheric turbulence, whereas the spectrum observed near the inner sphere (point 1) has a constant slope of $-5/3$ and is typical to three-dimensional

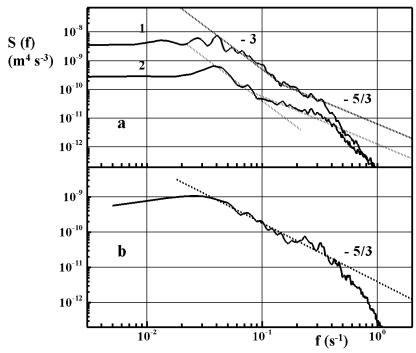

Figure 3: u_φ^2 spectra at points (a) 7 and (b) 1 obtained in the (1) experiment and (2) calculation in the case of the modulation of the velocity of the outer sphere with $f_2 = 0.02Hz$ and $A = 0.2$.

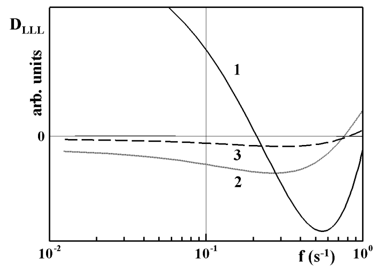

Figure 4: Approximation of the thirdBæorder longitudinal velocity structure function for (1) $f_1 = 0.01Hz, A = 0.163$ and $(2,3)$ $f_2 = 0.02Hz, A = 0.2$ at points 7 $(1,3)$ and 1 (2).

turbulence (Figure 3b). We tried to determine the direction of the energy cascade in the cases corresponding to the spectra, shown in Figures 2 and 3, from the sign of the third order longitudinal velocity structure function D_{LLL}. Sign of D_{LLL} alternates with a period of $2\pi/3$, because large-scale coherent structures [11] characteristic of the initial flow are held in the turbulent flow. Similar large-scale coherent structures in the upper layers of the atmospheres of planets (e.g., Venus) were assumingly interpreted as Rossby waves [20]. For this reason, to determine the sign of D_{LLL}, the results of the calculation were approximated by sixth order polynomials.

Figure 4 shows the dependence of D_{LLL} on the frequency f given by the expression [21] $f = \langle u_\varphi \rangle / l$, where $0 < l < 32\pi$ and $\langle u_\varphi \rangle$ is the average velocity at a distance of $rsin\theta$ from the axis. We first consider flows for which the observed spectra were typical to two-dimensional (Figure 2) and three-dimensional (Figure 3b) turbulence. In the former case (Figure 4, line 1), transition from positive D_{LLL} values to negative is observed at $f = 0.2Hz$. At the same frequency, transition from a slope of -3 to a slope of $-5/3$ is observed in the experiment (Fig. 2, line 1). In the latter case (Fig. 4, line 2) $D_{LLL} < 0$. Both of these cases confirm the correctness of the estimate of the sign of D_{LLL}. In the case of atmospheric turbulence (Figure 4, line 3), $D_{LLL} < 0$ in the frequency range corresponding to the segments of the spectrum with both slopes of -3 and $-5/3$.

The level of turbulence of the entire flow as a whole is determined by the behavior of its kinetic energy, which is represented in the form of the sum of the azimuthal (E_φ) and meridional (E_ψ) components taken as integrals over the entire volume of the spherical layer: $E_\varphi = \int u_\varphi^2$ and $E_\psi = \int (u_r^2 + u_\theta^2)$. Thus, the modulation of the rotation velocity of one of the spheres results in the suppression of turbulence in the azimuthal direction of motion. Synchronizations between $\Omega_k(t)$ and E_φ, E_ψ (treated as frequency and/or phase locking) are significantly different for inner and outer

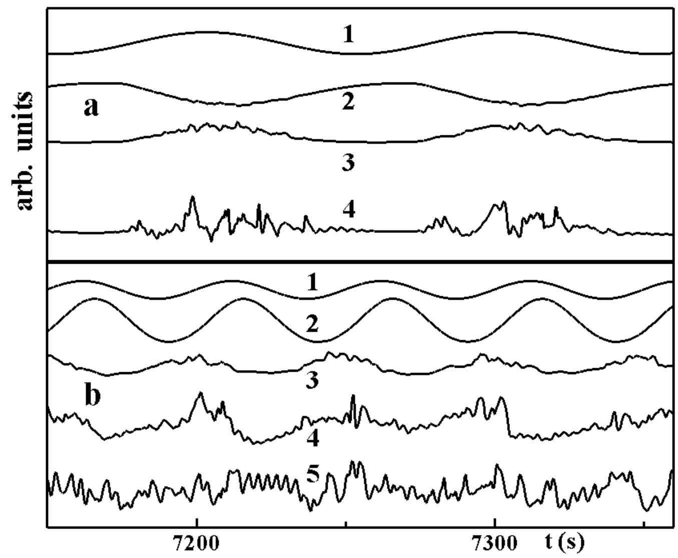

Figure 5: Fragments of the time dependence of various calculated quantities (in arbitrary units) for (a) $f_1 = 0.01Hz, A = 0.163$ and (b) $f_2 = 0.02Hz, A = 0.2$: (1) Ω_i, (2) E_φ, (3) E_ψ, (4) u_φ at point 7, (5) u_φ at point 1, $rms(df)/\Omega_1 = 0.256, 0.130, 0.274$ for cases corresponding to Figs. 3a, 3b, and 4b, respectively.

sphere modulation. In the former case, E_ψ varies almost in phase with the variation of $\Omega_1(t)$ (Figure 4a), whereas in the latter case, the smallest phase shift is observed between $\Omega_2(t)$ and E_φ (Figure 5b). Since E_φ values are two orders of magnitude higher than E_ψ, the above results indicate that synchronization of the flow induced by the modulation of $\Omega_2(t)$ is stronger. As is seen, the level of synchronization between $\Omega_2(t)$ and u_φ is nonuniform in the thickness of the layer of the fluid (Figure 6b): a correlation between the velocity of the sphere and the velocity of the flow is observed at points far from the inner sphere (line 4) and is not observed near the inner sphere (line 5). We calculated the instantaneous frequency differences df between $\Omega_k(t)$ and u_φ. According to [22], the instantaneous frequency and phase are defined as $\chi(t) = \partial\Psi(t)/\partial t$ and $\Psi(t) = \arctan(y(t)/x(t))$, respectively. Here, $x(t)$ is the velocity signal and $y(t)$ is the orthogonal complement to $x(t)$, which is calculated as the Hilbert transform of the series $x(t)$. The minimal root mean square frequency difference $rms(df)$ between $\Omega_k(t)$ and u_φ(the strongest synchronization) is observed near the outer sphere at its modulation (Figure 5). Thus, the strongest synchronization corresponds to spectra of turbulence with the form typical to the upper layers of the atmosphere. The largest frequency difference is observed near the inner sphere at the modulation of the outer sphere. This means that the weakest synchronization corresponds to three-dimensional turbulence. The comparison of the two cases considered above indicates that the effect of the modulation of the outer sphere on the flow is limited by the region of circulation induced by its rotation (points $3-7$ in Figure 1). All presented facts indicate that the appearance of the spectrum of turbulence with slopes of -3 and $-5/3$ at low and high frequencies, respectively, is possible only in the region with strongest synchronization between the rotation velocity and the velocity of the flow.

The results obtained for the model flow under consideration imply that the form of spectra of turbulence in the upper layers of the atmosphere is explained by the induced synchronization of the periodic part of atmospheric flows (e.g., Rossby waves) by an external periodic action with a longer period. Since the main source of the energy for all atmospheric processes is solar heat, seasonal variations of this quantity can be considered as such a periodic external action on the atmosphere.

4 Conclusions

The results of the performed experimental and numerical studies have shown that a decrease in the modulation frequency is accompanied by an increase in differences in the behaviors of the azimuthal and meridional components of the kinetic energy of the flow. The former component remains periodic, whereas the latter component changes the periodic behavior to chaotic. The suppression of turbulence of the azimuthal kinetic energy of the flow promotes the formation of quasi-two-dimensional turbulence. Spectra characteristic of two-Bædimensional turbulence with a constant slope of $-5/3$ and an inverse cascade ($D_{LLL} > 0$) at low frequencies and with a slope -3 and a direct cascade ($D_{LLL} < 0$) at high frequencies have been observed in the case of the modulation of the inner sphere velocity. At a modulation frequency below the frequency of the initial periodic flow, the form of the spectra is spatially nonuniform. In the case of the modulation of the outer sphere velocity, spectra with the qualitative form characteristic to turbulence in the upper layers of the atmoBæsphere with a constant slopes of -3 and $-5/3$ at low and high frequencies, respectively, are observed in the region of circulation induced by the outer sphere. For both segments of the inertial interval $D_{LLL} < 0$. The form of the spectrum near the inner sphere is characteristic of threeBæ-dimensional turbulence: the segment with a constant slope of $-5/3$ presents and $D_{LLL} < 0$. It has been found that the level of synchronization between the rotation velocity of the boundary and the velocity of the flow is different in all flows considered above. The lowest and highest levels of synchronization are observed where spectra are similar to spectra of three-Bædimensional and atmospheric turbulence, respectively.

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Dmitry Y. Zhilenko, Krgiganovski str. 2/21, 51, Moscow, Russia
Olga E. Krivonosova, Krgiganovski str. 2/21,51, Moscow, Russia