Numerical modelling of surface – reinforced rod tube structures

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Abstract

The study treats rod structures consisting of tubes whose external surfaces are reinforced by means of suitable reinforcing material (reinforced composites, special nanomaterials etc.). The technology of reinforcement deposition is assumed to be diffusive (i.e. painting, pulverization, some electrochemical methods of deposition). Thus a transition area of gradual penetration of the reinforcement into the basic material (substrate) emerges. The authors use in the transition area an approach, proposing a quadratic approximation of material elastic modulus, which varies within a range bounded by the elastic modulus of the reinforcement and that of the substrate. Loading is static (axial tension/compression) applied under normal temperature and humidity. The linear axial strains are constant within the core, but following a quadratic parabolic law within the transition area. Strain transition between both areas is also smooth. A rod under tension is separately considered and its strength is calculated. Moreover, a compressed rod is also analysed assuming a possibility of rod stability loss. The authors outline a possibility to homogenize the structural element using the stiffness values and employing FEM codes for complex rod tube structures.

1 Introduction

Consider rod structures composed of tubes fixed by two joints and reinforced by a high-strength material along their external surfaces. The reinforcing material can be a polymer composite reinforced by metal fibers or a special nanomaterial [1], [2]. Consider also some technologies of deposition of the reinforcing layer, which diffusively penetrates the substrate, for instance, painting, pulverization, concrete spraying, electrochemical deposition [3]. Some metal, concrete, steel fiber reinforced concrete etc. can be used as a basic material (substrate). Then, a transition area emerges within the tube, bounded by the tube surface and the basic material, where the concentration of the reinforcement varies from 100% to 0%. Thus, a combined structural element is formed, while the outline of the transition area and the assessment of the mechanical properties of the penetrated material can be performed by solving the diffusion problem. Yet, this turns to be a complex task, since it is not possible to find exactly the specific coefficient of diffusion. To overcome that difficulty, we designed in previous studies an approximate calculation model (ACM), where the distribution of the elasticity modulus and that of the axial linear deformation of the compound material were approximated by means of quadratic relations [4], [5]. The approximation thus attained turned to be of the order of the approximation of FEM codes. Moreover, ACM yields simplified solutions to the problem of determining the stressed and strained state of surface-reinforced, i.e. combined, structural elements. Thus facilitating the calculation of structures composed of those elements. ACM enables also one to perform element homogenization by introducing combined stiffness. Finding the latter, one can use it in various FE commercial codes.

We use ACM in the present study. Rod loading is uniaxial and static, applied under normal temperature and humidity. Displacements and strains are small, and the linear geometrical theory of solid mechanics is assumed to hold [6]. Note also: (i) First-rate stresses and strains are uniformly distributed within the basic material, and they are identical along the rod; (ii) Shear stresses within the basic area are considered to be second-rate quantities, and they are found using known approximate relations of the AMC [6], [7]; (iii) When applying compression, one should check rod strength or find the condition of rod stability loss to be taken as criteria of rod strength calculation [8].

Rod reinforcement makes a structure lighter, since tube reinforcement increases material strength and tubes may be fabricated thinner thus reducing their weight and that of the overall structure. Thus, energy of structural production can be indirectly saved. The same energy-saving effect is attained when using recycled structural materials, which have lost to a certain extent their strength and deformability and are bound to rehabilitation, [5].

Regarding ACM, the distribution of the elasticity moduli follows a quadratic elliptic law in the transition area, while that of the axial strains follows a quadratic parabolic law.

The geometry parameters are as follows: R_1 – internal radius; R_2 – external radius; $h = R_1 - R_2$ – tube thickness; h_R – thickness of the transition area, $h_R = R_2 - z_L$, $h_L = h - h_R$, $R_L = R_1 + h_L$, $F = 2\pi R_M$ – cross section area; $R_M = 1/2(R_2 + R_2)$ – tube mean radius; $r_M = 1/2(R_1 + r_L)$ – core mean radius; $r_R = 1/2(R_2 + r_L)$ – mean radius of the transition area. The following additional geometry parameters are also accounted for:

$$K_F = \frac{F_L}{F}, F_L = 2\pi r_M h_L, K_R = \frac{h_R}{h}, K_M = \frac{r_M}{R_M}, K_R = \frac{r_R}{R_M}, K_L = \frac{h_L}{h} \tag{1}$$

Introduce a cylindrical coordinate system $Oxr\theta$. The cross section symmetry is entirely axial. Following the model, the mechanical characteristics do not depend on θ and x, but only on r.

2 Distribution of the elastic modulus in the transition area

Pursuant to ACM, the elastic modulus E depends on r, only, i.e. $E = E(r), r \in [0,1]$, while the distribution function has a quadratic elliptical form. Introduce a local coordinate $\rho = r - r_L/h_R, \rho \in [0,1]$. Then, the distribution function takes the form

$$E(\rho) = E_B + \Delta E_R \psi(\rho), \Delta E_R = E_R - E_B, \psi(\rho) = 1 - \sqrt{1-\rho^2} \tag{2}$$

The elastic modulus of the core material is constant, i.e. $E = E_B = const$ for $r \in [R_1, r_L]$.

3 Stiffness of an axially tensed combined rod

Consider an axially tensed combined rod $N = N^+$ and find its generalized tension stiffness. If a rod with dimensions (R_1, R_2) is not reinforced, its tension stiffness will be $K_I^0 = E_B J$, while the inertial moment is approximately expressed as $J \approx \pi R_M^3 h$, [8]. For a reinforced tube with a transition area, we express the generalized stiffness as follows:

$$K_I^+ = 2\pi \int_{R_1}^{R_2} E(r) r dr = K_{I1}^* + K_{I2}^* \tag{3}$$

where

$$K_{I1}^* = 2\pi \int_{R_1}^{r_L} E_B r dr = E_B F_L$$

$$K_{I2}^* = 2\pi \int_{r_L}^{R_1} E(r) r dr = 2\pi \int_0^1 E(\rho)(\rho h_R + r_L) h_R d\rho = F_I E_I + F_{II} E_0 \tag{4}$$

where

$$F_I = 2\pi h_R^2, F_{II} = 2\pi h_R r_L$$

We assume the following notation in rel. (4):

$$E_i = \int_0^1 E(\rho)\rho^i d\rho, (i = 0, 1 \cdots) \tag{5}$$

while

$$E_0 = E_B + 0,215\Delta E_R, E_1 = 0,5E_B + 0.167\Delta E_R, E_2 = 0,5E_B + 0,137\Delta E_R, etc.$$

The degree of tube reinforcement is assessed pursuant to the ratio $\chi_I^* = K_I^*/K_I^0$, which reads:

$$\chi_I^* = Q_{I0} + Q_{I1}\Delta e_R, \Delta e_R = \frac{\Delta E_R}{E_B} \tag{6}$$

for

$$Q_{I0} = K_F + K_R(0,5K_M + K_L), Q_{I1} = K_R(0,167K_M + 0,215K_L) \tag{7}$$

Parameters K_F, K_R, K_L, K_M are found from rel. (1). If we introduce $\Delta e_R = (K_E - 1)$, then $K_E = E_R/E_B$ which sets forth the reinforcement. It is seen from expr. (6) that the generalized stiffness K_I^*, expressed by ratio χ_I^*, depends linearly on K_E. Considering the generalized stiffness K_I^* of each rod, we could calculate any combined rod tube structure (truss, frame etc.) via FEM. This procedure is additionally approximate. It can be treated as a homogenization of each combined structure, and it significantly facilitates the calculations.

4 Distribution of the axial linear strains within the cross section of a reinforced rod

We find the axial linear strains $\varepsilon(r)$ within the rod cross section using ACM. They are constant within the core and vary within the transition area following a parabolic quadratic function. Thus, we find:

$$for\ r \in [R_1, r_L], \varepsilon_x = \varepsilon_{x0} = const$$

$$for\ r \in [r_L, R_2], or\ \rho \in [0,1], \varepsilon_x = \varepsilon_x(\rho) = \varepsilon_{x0} + \Delta\varepsilon_{xR}\rho^2, \Delta\varepsilon_{xR} = \varepsilon_{xR} - \varepsilon_{x0} \tag{8}$$

where ε_{x0} is the constant linear strain within the core and ε_{xR} is the edge linear strain for $r = R_2$. Both strains, together with h_R, are to be found.

5 Distribution of the normal tension stress within the cross section of a reinforced rod tube

Knowing $\varepsilon_{x0}, \varepsilon_{xR}$ and h_R, we can find the normal tension stresses $\sigma_x(r)$ within the rod cross section. Following ACM, we consider an operating tube, which deforms linearly elastic. Hence, HookeвЂ™s law links the axial linear strains$\varepsilon_x(r)$ and the normal stresses $\sigma_x(r)$, i.e. $\sigma_x(r) = E(r)\varepsilon_x(r)$ or, accounting for the different areas

$$for\ r \in [R_1, r_L], \sigma_x = \sigma_{x0} = E_B\varepsilon_{x0} = const$$

$$for\ r \in [r_L, R_2], \rho \in [0,1], \sigma_x = \sigma_x(\rho) = E(\rho)\varepsilon_x(\rho) = E(\rho)\varepsilon_{x0} + E(\rho)\rho^2\Delta\varepsilon_{xR} \tag{9}$$

where $E(\rho)$ is given by expr. (2).

6 Determination of the linear axial strain in the basic core

To find the axial linear strain ε_{x0} in the basic core, we use a fictitious mean stress $\sigma_{xM} = N^+/F$ set forth in the problem statement. It is expressed as

$$\sigma_{xM} = \frac{1}{h}\int_{R_1}^{R_2} \sigma(r)dr = \sigma_{xM}(1) + \sigma_{xM}(2) \tag{10}$$

where

$$\sigma_{xM}(1) = \frac{1}{h}\int_{R_1}^{r_L} \sigma_{x0} dr = (1 - K_R)E_B\varepsilon_{x0}, \sigma_{xM}(2) = \frac{1}{h}\int_0^1 E(\rho)\varepsilon_x(\rho)h_R d\rho \quad (11)$$

Put $E(\rho)$ from expr. (2) and $\varepsilon(\rho)$ from expr. (8) in expr. (11) for $\sigma_{xM}(2)$. Then, after certain transformations, we find that

$$\varepsilon_{x0} = e_I - e_{II}\varepsilon_{xR} \quad (12)$$

where

$$e_I = \frac{\sigma_{xM}}{E_1}, E_1 = E_B(1 - K_R) + K_R\Delta E_{02}, \Delta E_{02} = E_0 - E_2, e_{II} = K_R\frac{E_2}{E_1} \quad (13)$$

7 Determination of the model parameters

Assume two methods of finding parameters h_R and ε_{xR}.
First method: Perform nanoindentation and a subsequent FEM numerical simulation [9] over a specific specimen to find h_R , assuming one and the same ratio K_R for the specimen and the tube. Then, we find ε_{xR} from the equilibrium condition

$$N^* = 2\pi\int_{R_1}^{R_2} \sigma_x(r)rdr = N_I^+ + N_{II}^+ \quad (14)$$

where

$$N_I^+ = 2\pi\int_{R_1}^{r_L} \sigma_{x0}(r)rdr = E_B F_L \sigma_{x0}$$

$$N_{II}^+ = 2\pi\int_{r_L}^{R_2} \sigma_x(r)rdr = 2\pi\int_0^1 E(\rho)\varepsilon_x(\rho)(h_R\rho + r_L)h_R d\rho \quad (15)$$

and

$$N_{II}^+ = \Delta P_{I,II}\varepsilon_{x0} + P_{II}\varepsilon_{xR}$$

$$\Delta P_{I,II} = P_I - P_{II}, P_I = F_I\Delta E_{13} + F_{II}\Delta E_{02}, P_{II} = F_I E_3 + F_{II}E_2 \quad (16)$$

where

$$\Delta E_{13} = E_1 - E_3, \Delta E_{02} = E_0 - E_2, E_3 = 0,250E_B + 0,117\Delta E_R \quad (17)$$

Unify expressions (14), (15), (16) and perform a subsequent revision. Then, we get

$$\varepsilon_{xR} = \frac{N^+ - P_{II}e_{II}}{P_{II} - P_{III}e_{II}}, P_{III} = E_B F_L + \Delta P_{I,II} \quad (18)$$

Secondt method: Perform tension of a reinforced rod tube specimen, applying load N^+, and measure ε_{xR} along its surface. Then, h_R can be found from eq. (18).

8 Strength check of a combined rod

Having found all characteristics of the stressed and strained state under a given load N^+, we can perform a strength check of the carrying capacity of a tensed rod, $N^+ \leq N_{per}$. N_{per} is found using an adopted strength criterion. If the reinforced combined rod undergoes compression $N = N^-$ we use formulas valid for tension considering the subsequent load signs and find the characteristics of the stressed and strained state. We perform a check of the rod carrying capacity, employing two criteria: *Strength check of a compressed combined rod*: We perform it similar to the strength check of a tensed rod presented above.*Check of the stability of a compressed combined rod fixed by joints*: We compare the compression load to the admissible value of the axial load avoiding stability loss N^-_{per}. It is found on the basis of the critical compression load N_{cr} for the reinforced rod, pursuant to Euler [8]. The basis of rod strength calculation is reliable. Pursuant to Euler, the critical force of a two-joint centrally compressed and reinforced rod reads [9].

$$N_{cr} = \frac{\pi_2 K^*_{II}}{L^2} \tag{19}$$

where K^*_{II} is the generalized bending stiffness of the combined rod and L is rod length. We find approximately the generalized bending stiffness of the reinforced rod, treating the tube as a two-layer body, consisting of a basic core with an elastic modulus E_B and a transition area with mean elastic modulus $E^* = 1/h_R \int_0^1 E(\rho) h_R d\rho$ for $r \in [r_L, R_2]$ or $\rho \in [0, 1]$. We assume that h_R is one and the same under tension and compression, since we consider a process of diffusion identical in both cases.

9 Test example. Parametric analysis

Consider a small frame structure shown in Fig. 1. It is subjected to a static vertical load P. The structural scheme is symmetrical. Its two-joint rods are aluminum tubes, being reinforced along their surfaces [10]. The static analysis yields the following loads acting on the rods: on Π_{01}, $N^-_{01} = -0,94P$; on Π_{02}, $N^+_{02} = 0,78P$; on Π_{12}, $N_{12} = 0$. For $P = 1,55x10^4[N]$ loads are $N^-_{01} = -1,46x10^4[N]$, $N^-_{02} = 1,21x10^4[N]$. The elastic modulus of the basic material is $E_B = 7x10^6[N/cm^2]$. The reinforcing material is a composite containing nanoparticles and its elastic modulus varies within limits $40 - 80[N/cm^2]$, [1]. Tube dimensions are $\Phi R_1 x1$ according to manufacturer data [10], with thickness $h = 1cm$. Consider data from nanoindentation [9] and assume $h = 0,1cm$. If the elasticity modulus of the reinforcing material is $E_R = 40x10^6[N/cm]$, we compare the calculation of the rods of the non-reinforced structure with that of the rods of the reinforced structure. The dimension of all rods of the non-reinforced structure is $\Phi 10x1$. The reinforced structure yields Π_{01} dimension $\Phi 18x1$, while the rest of the rods have a manufacturer dimension $\Phi 10x1$. This yield reduction of structure weight for reinforcement is 17%. A parametric analysis of the link between ε_{xR} and $K_E = E_R/E_B$ is performed for rod Π_{02}. The results are plotted in Fig. 2. The test example and the parametric analysis prove the efficiency of the reinforcement, thus contributing to the reduction of the structure weight and to the increase of its carrying capacity.

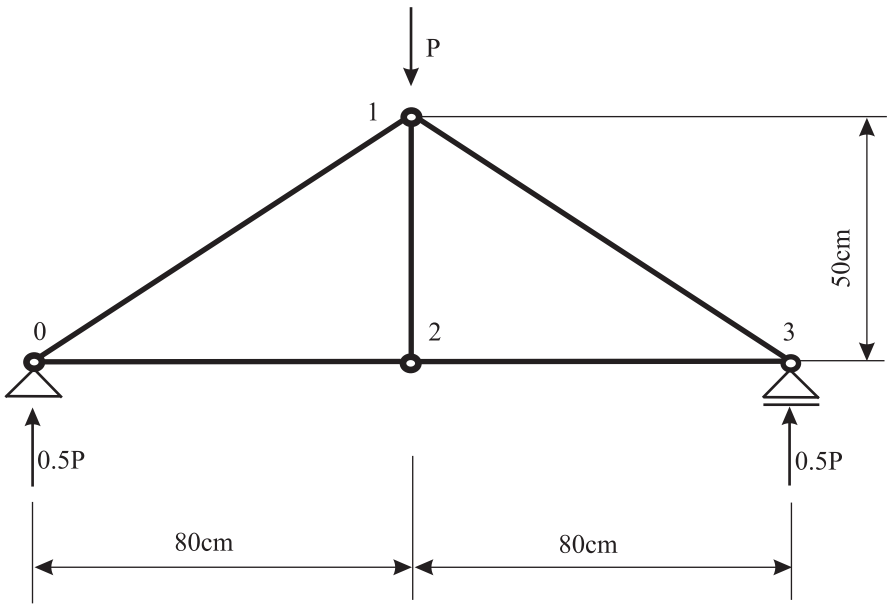

Figure 1: A test frame composed of reinforced aluminum rod tubes

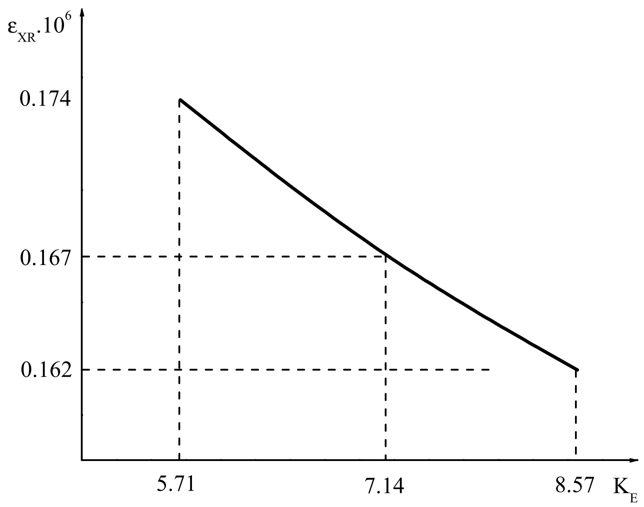

Figure 2: Link between ε_{xR} and K_E

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References

[1] Mechanics of Nanomaterials and Nanotechnology (2012): Thematic Collection "Series in Applied Mathematics and Mechanics", Vol. 3. Publisher Institute of Mechanics, Bulgarian Academy of Sciences, Sofia.

[2] Hui-Shen Shen, J. G. Teng and J. Yang (2001): Interfacial Stresses in Beams and Slabs Bounded with Thin Plate. J. Eng. Mech. 127, pp. 399-406.

[3] Bakis, C. E., L. C. Bank, V. L. Brown, E. Cosenza, J. F. Davalos, J. J. Lesko, A. Machida, S. H. Rizkalla, and T. C. Triantafillou (2002): Fiber-Reinforced Polymer Composites for Construction, State of the Art-Review. J. of Comp. for Construction, 6, pp. 73-87.

[4] Baltov A., A. Yanakieva (2015): Local Deplanation of Double Reinforced Beam Cross Section Under Bending. J. of Theor. and Appl. Mech., 45(4), pp. 31-40.

[5] Baltov A., A. Yanakieva, G. Nikolova (2016) Modelling of the Bending Behaviour of a Double-Reinforced Beam from Recycled Materials for Application in NZEBs. Springer Proceedings in Energy (SPE), pp. 167-173.

[6] Timoshenko Stephen P. James M. Gere (1972): Mechanics of Material., Publisher: Van Nostrand Reinhold Company.

[7] Kazakov K (2012) Theory of Elastisity, Stability and Dynamics of Structures Common Problems. Trafford Publishing.

[8] Birger, I.A., Ya. G. Panovko (1969): Strength, Stability, Vibrations. Handbook 1. Mashinostroene, (in Russian).

[9] Datcheva M., S. Cherneva, M. Stoycheva, R. Iankov, D. Stoychev (2011): Determination of Anodized Aluminum Material Characteristics by Means of Nanoindentation Measurements. Materials Sciences and Applications. Scientific Research Publishing, USA, 2, pp.1452-1464.

[10] Internet information: Aluminium tubes.

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