Mechanics of penetration and structural protection in the frame of optimization game theory

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Abstract

The questions of the shape optimization of an axisymmetric rigid impactor and structure optimization of layered perforated plates are studied on the base of the Nash game approach [1], [2] for layered plates made on the given set of materials. As a criterion of the multipurpose optimization problem it is chosen the ballistic limit velocity under additional constraint on the layered shield mass. The process of penetration of the rigid body into an elastic-plastic medium is modelled by the application of the two parts representation for the resistance force [3]. It is proposed and realized the solution algorithm of the conflict game problem, namely, the optimal shape impactor against the optimal structure layered shield. It is considered the case when the impactor mass is given and does not depend on its volume. With the application of an evolution numerical method (genetic algorithm) the optimal shapes of penetrating bodies and corresponding optimal shield structures are found and analyzed for all cases.

1 Introduction

The study of processes of high-speed penetration of rigid strikers into deformed media and perforation of shield structures is actual and of theoretical and practical interest. Scientific investigations in this domain are very wide and include many experimental, analytical and numerical components. Also the optimal structural design plays the important role in this aspect. Many studies were devoted to the problem of optimal shape determination of rigid bodies penetrating with high speed into deformed (elastic-plastic, concrete, brittle) media. Also problems of shield structure optimization were investigated by many authors.

Now we propose the game approach to solve the problem of high-speed perforation of the layered slab by the axisymmetric striker (optimal shape striker against optimal shield structure).

2 Ballistic limit velocity (BLV) determiniming

The questions of shape optimization of rigid strikers perforating the layered slab are studied in game statement. As a criterion of multipurpose optimization problem it was taken the ballistic limit velocity, which is a very important characteristic of striker-medium interaction.
Dynamics of high-speed penetration of rigid axisymmetric striker (with velocities up to 10^3m/s) along the axis Ox is studied with application of relation connecting the resistance force $D(x)$ with strength characteristics $A_0(x)$, inertial property $A_2(x)$, striker shape $y(\eta)$ $(0 \leq \eta \leq L)$, its length L, and velocity $v(x)$ as

$$\begin{aligned} &D(x) = D_{nose}(x) + D_{lat}(x) = B_0(x) + B_2(x)v^2(x), \\ &B_0(x) = \pi r^2 A_0(x) - 2\pi \int_{x_*}^{x_{**}} A_0(\eta) y y_\eta d\eta, \\ &B_2(x) = \pi r^2 A_2(x) - 2\pi \int_{x_*}^{x_{**}} \left[A_2(\eta) y_\eta^3\right] \left(1 + y_\eta^2\right)^{-1} d\eta. \end{aligned} \tag{1}$$

Here $D_{nose}(x)$ is the resistance force applied to the truncated head part of striker, $D_{lat}(x)$ is the resistance force applied to the lateral area, r is the radius of truncation, $y_\eta = dy/d\eta$, x is the coordinate of striker nose, x_*, x_{**} are the values characterizing different stages of striker penetration into the medium (boundaries of contact region). If the entry (impact) velocity $v_0 = v_{imp}$ of striker penetration into the slab of thickness H is such that $v(x) > 0$ for $0 \leq x < H + L$ and $v = 0$ for $x = H + L$, then, the impact velocity is called the ballistic limit velocity (BLV), i.e. $v_0 = v_{imp} = v_{BLV}$.
Perforation of layered slab by the axisymmetric striker having ballistic limit velocity is shown in Fig.1.

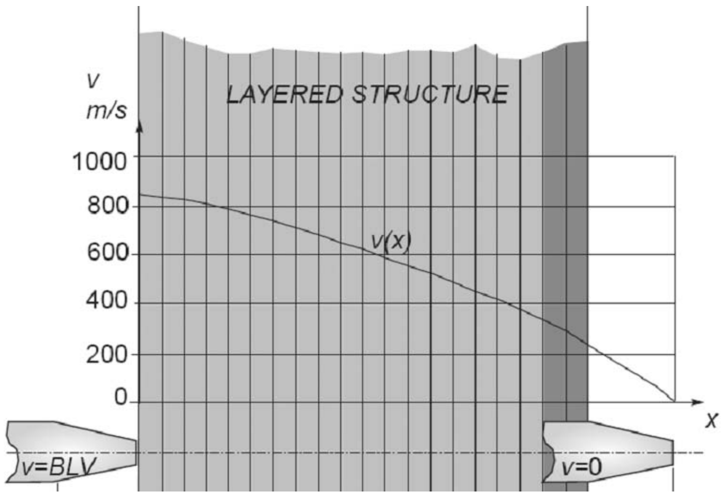

Figure 1: Perforating layered slab

We introduce for convenience the new independent variable

$$\xi = H + L - x, \quad d\xi = -dx$$

and formulate the problem of determining the velocity of striker in the medium as following Cauchy problem:

$$\frac{dv^2}{d\xi} = \beta\left(\alpha + v^2\right), \quad \left(v^2\right)_{\xi=0} = 0, \tag{2}$$

where $\alpha = B_0/B_2$, $\beta = 2B_2/m$ (m is the mass of striker).
For convenience, we assume that the perforated obstacle includes the slab and rear air region of length L, i.e. has total thickness $H + L$ and consists of n thin layers of equal thickness $h = (H + L)/n$. Each layer contains material with fixed properties. For such piece-wise constant structure, the problem (2) can be represented as

$$\begin{array}{l} \frac{dv^2}{d\xi} = \beta_{j+1}\left(\alpha_{j+1} + v^2\right), \\ v_0^2 = \left(v^2\right)_{\xi=0} = 0, \\ v_j = v(\xi_j), \quad \xi_{j+1} = \xi_j + h, \quad j = 0, 1, 2, ..., n-1, \\ \alpha_{j+1} = \left(\frac{B_0}{B_2}\right)_{j+1}, \quad \beta_{j+1} = \frac{2}{m}\left(B_2\right)_{j+1}, \\ v_n^2 = \left(v^2\right)_{\xi=H+L} = v_{BLV}^2. \end{array} \tag{3}$$

Layer-wise integration of the problem (3) results in algebraic relations

$$\begin{array}{l} \ln\left(\frac{\alpha_{j+1}+v_{j+1}^2}{\alpha_{j+1}+v_j^2}\right) = \mu_{j+1}, \quad \xi_j \le \xi \le \xi_{j+1}, \\ \mu_{j+1} = \beta_{j+1}h, \quad j = 0, 1, 2, ..., n-1, \end{array} \tag{4}$$

which determine the solution, i.e. the velocity distribution for each layer and, in particular, the value of ballistic limit velocity. We have

$$\begin{array}{l} \frac{v_{j+1}^2}{\alpha_{j+1}} = \exp\left(\mu_{j+1}\right) - 1 + \frac{v_j^2}{\alpha_{j+1}}\exp\left(\mu_{j+1}\right), \\ v_0 = 0, \quad v_n = v_{BLV}. \end{array} \tag{5}$$

3 Multipurpose optimization problem

Let us consider the ballistic limit velocity (BLV) as a quality criterion for multipurpose optimization problem according to game approach (with two gamers).
We formulate the problem A (for gamer 1) that consists in minimization of BLV by finding the optimal striker shape for given shield structure (given layered slab), i.e.

$$\begin{array}{l} J_1 = v_{BLV}(y(\eta), t(x)) \to \min_{y\in\Lambda_y}, \\ \Lambda_y = \{y: \ y_{con}(\eta) \le y(\eta) \le R, \ 0 < \eta \le L, \ y(0) = y_{con}(0) = R\}. \end{array} \tag{6}$$

Here $y_{con}(\eta)$ defines the conical shape of the striker, R is given base radius (midel) of the striker and we assume that the mass m is independent on the striker volume. The function $t(x)$, $x \in [0, H + L]$ is piece-wise constant and characterizes the layer-wise distribution of material characteristics. The number r_m of materials is assumed to be given, $s = 1, 2, ..., r_m$, where s is the material number.
The problem B (for gamer 2) consists in maximization of BLV by defining the optimal layered slab (plate) under the constraint on its mass M (on $1m^2$) as

$$\begin{array}{l} \left\{ \begin{array}{l} v_{BLV}(y(\eta), t(x)) \to \max_{t\in\Lambda_t}, \\ M(t(x)) \le M_0, \end{array} \right. \\ \Lambda_t = \left\{ \begin{array}{l} t: \ t = t(x), \quad x \in [0, H+L], \quad t \in \{t_i = s\}, \\ i = 0, 1, 2, ..., n-1, \quad s = 1, 2, ..., r_m, \\ A_0(t(x)) = A_0^{i+1}, \quad A_2(t(x)) = A_2^{i+1}, \quad x \in [x_i, x_{i+1}), \\ \left(A_0^{i+1}, A_2^{i+1}\right) \in \left\{(A_0)_s, (A_2)_s\right\}. \end{array} \right\}. \end{array} \tag{7}$$

Here the properties of material with number s filling the layer $(x_i \le x \le x_{i+1})$ with number $i+1$ are characterized by constants $\left(A_0^{i+1}\right)$ and $\left(A_2^{i+1}\right)$, M_0 is given constant. The problem B (7) can be rewritten by introducing the augmented Lagrange functional J^a as

$$J_2 = J^a(y(\eta), t(x)) \to \max_{t\in\Lambda_t}, \tag{8}$$

$$J^a = v_{BLV}(y(\eta), t(x)) - \lambda \left[M(t(x)) - M_0\right] \to \max_{t\in\Lambda_t},$$
$$\lambda = \begin{cases} 0, & if \quad M - M_0 \le 0, \\ \lambda_* > 0, & if \quad M - M_0 > 0. \end{cases}$$

The multipurpose optimization problem consists in defining the optimal (in the sense of the problem A) striker shape for optimal (in the sense of the problem B) layered structure. As a quality criterion, it is chosen the ballistic limit velocity (BLV).
Solving this multipurpose optimization problem is realized on the base of game iteration approach (Nash approach [1]) by performing the following steps.
Step 1. The initial distribution $t = t_1^*(x)$ of materials is realized for the layered structure.
Step 2. The problem A is solved (by the gamer 1) for given distribution $t_1^*(x)$ and the shape distribution $y_1^*(\eta)$ is defined as

$$y_1^*(\eta) = \arg\min_{y\in\Lambda_y} J_1\left(y(\eta), t_1^*(x)\right). \tag{9}$$

Step 3. The problem B is solved (by the gamer 2) for given distribution $y_1^*(\eta)$ and the improved distribution $t_2^*(x)$ of materials is found, i.e.

$$t_2^*(x) = \arg\max_{t\in\Lambda_t} J_2\left(y_1^*(\eta), t(x)\right). \tag{10}$$

Step 4. The iteration is completed and we will go to the Step 2 or terminate the optimization process.
For solving the optimization problems A and B, the numerical evolutionary method based on the genetic algorithm was realized. The considered set of admissible shapes (population) consists of thirty solutions (individuals) for each generation. Each individual of population consists of ten (the problem A) or fifty (the problem B) elements. The search of optimal shape was begun (initialization) using initial population consisted of arbitrary distributions $y(\eta)$ with given fixed R and performed up to 500 generations. The parameters of computational process included the probability of crossover $p_{CO} = 0.5$ and the mutation probability $p_m = 0.05$.
The computations were performed for the following values of the problem parameters, namely, $L = 0.02m$, $R = 0.005m$, $H = 0.1m$. Admissible values of material properties for slab layers were taken as [3]

$$\begin{aligned}
&s = 1 \quad (air) \quad (A_0)_1 = 0, \quad (A_2)_1 = 0; \\
&s = 2 \quad (soft\ steel) \quad (A_0)_2 = 1850 \cdot 10^6 M/m^2, \quad (A_2)_2 = 7830 kg/m^3; \\
&s = 3 \quad (copper) \quad (A_0)_3 = 910 \cdot 10^6 M/m^2, \quad (A_2)_3 = 8920 kg/m^3; \\
&s = 4 \quad (duraluminum) \quad (A_0)_4 = 1330 \cdot 10^6 M/m^2, \quad (A_2)_4 = 2765 kg/m^3.
\end{aligned}$$

All optimal strikers have the shape with blunted (a little) nose part. The analysis of numerical results for all considered cases permits to make a conclusion (within the framework of using model of high-speed interaction) that the optimal shape of striker is determined by its geometrical and inertial characteristics and do not depend on the mass of the slab, its thickness and layers ordering. Two optimal shape distribution are shown in Fig.2 by the curves with numbers 1 (thin solid line) and 2 (thick solid line). The curve 1 corresponds to optimal duraluminum slab of the thickness $0.056m$ and the values $m = 0.009kg$, $M_0 = 156.6kg/m^2$. The curve 2 corresponsd to optimal slab that consists of the frontal steel part (of thickness $0.006m$) and the rear duraluminum part (of thickness $0.094m$), and the values $m = 0.05kg$, $M_0 = 313.2kg/m^2$. Note that the optimal shapes shown in Fig.2 are practically the same, although they corresponds to different optimal layered structures and different striker mass values. This fact was noted for all considered cases including also the case where the mass of the striker was not fixed but satisfied the constraint imposed on its value.

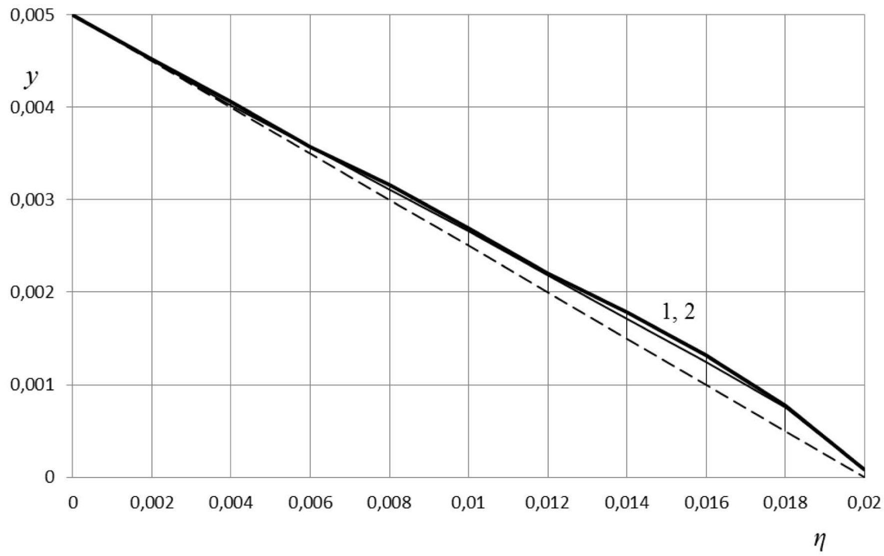

Figure 2: Optimal shapes of striker

Thus, it is sufficient to determine the optimal shape of the striker (minimizing the ballistic limit velocity) for some given layered structure and then use this solution for the shield optimization according to maximum of the ballistic limit velocity.

Acknowledgements

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