

Flow of viscoplastic suspensions in a hydraulic fracture: implications to overflush

Sergei S. Boronin, Andrei A. Osiptsov, and Jean Desroches

S.Boronin@skoltech.ru

Abstract

The study is devoted to modeling of multiphase flows of immiscible viscoplastic (or yield-stress) fluids in a hydraulic fracture. In the framework of the lubrication approximation, three-dimensional Navier-Stokes equations are reduced to hyperbolic transport equations for the fluid tracers and a quasi-linear elliptic equation in terms of the fluid pressure. The governing equations are solved numerically using the finite-difference approach. A parametric study of the displacement of Bingham fluids in a Hele-Shaw cell is carried out. It is found that fingers developed through the pillar of a yield-stress suspension trigger the development of unyielded zones. An increase in the Bingham number leads to an increase in the so-called finger shielding effect, which manifests itself via an increase in the overall finger penetration zone and a decrease in the total number of fingers. The effect of flow parameters on the displacement of hydraulic fracturing proppant-laden suspension by a clean fluid in the vicinity of the perforation zone is carried out. This particular case is considered in application to overflush at the end of a stimulation treatment, when a small portion of a thin clean fluid is injected to wash out the particles from the wellbore into the fracture. It is found that an increase in the yield stress and the viscosity contrast between the fracturing and the overflush fluids typically reduces the area of the cavity thus mitigating the risk of losing the conductive path between the wellbore and the fracture after the fracture closure.

1 Introduction

Proppant transport models incorporated into existing hydraulic fracturing simulators describe the flow of particle-laden suspension inside a hydraulic fracture in the framework of the lubrication approximation using the power-law rheological model (see [7]). Hydraulic fracturing suspensions with large concentration of solids or fibers show a yield-stress behavior in rheological experiments ([8, 5], which is not taken into account in the proppant transport models implemented into commercial simulators of hydraulic fracturing.

A state of the art in the modeling of injection of particle-laden suspensions into hydraulic fractures is the family of 2D width-averaged models based on the lubrication

approximation to Navier-Stokes equations [8, 7, 7, 8]. In the regime of non-inertial settling, the momentum conservation equation for particles is reduced to an algebraic relation for the particle velocity slip in the vertical direction given by the Stokes formula with a correction for hindered-settling effects due to a finite particle volume fraction. In the case of Newtonian suspension rheology, the total momentum conservation equation for the suspension is reduced to the linear expression of the fluid (or mixture) velocity through the pressure gradient (similar to the Darcy law, hence a well-known analogy between filtration and a flow in a Hele-Shaw cell). The dependence of the width-averaged fluid velocity on the pressure gradient is nonlinear, if the rheology of the carrier fluid or the suspension as a whole is non-Newtonian (see, e.g., [11, 9, 10]).

A large number of papers deal with the Saffman-Taylor (S-T) instability accompanying displacement of fluids in a narrow plane channel or annulus, starting from the pioneering study by Muskat [13]. The instability at the interface between the fluids is triggered when a high-viscosity fluid is displaced by a low-viscosity one. Note that in the absence of a certain “cut-off” mechanism (molecular diffusion or surface tension), the growth rate of disturbances at the interface between the fluids increases unboundedly with a decrease in the wavelength. Review of studies on the S-T instability in Newtonian fluids is presented in [12].

In the present study, we continue to analyze the displacement of viscoplastic fluids in a Hele-Shaw cell approximating a hydraulic fracture. This work was started in [1]. A parametric study of different injection scenarios of yield-stress fluids is carried out. In particular, we analyzed the effect of flow parameters on the shape and dimensions of the particle-free zone developed during overflush at the end of a hydraulic fracturing treatment. It is the final stage of a hydraulic fracturing operation, when the proppant-laden slurry is displaced by a low-viscosity particle-free fluid in order to clean the well and perforations.

2 Problem Formulation

We consider the flow of immiscible incompressible fluids with the Bingham rheology in a narrow Hele-Shaw cell approximating a hydraulic fracture. A detailed derivation of 2D width-averaged governing equations is presented in [1], while here we present only the final system of equations in the non-dimensional form:

$$\frac{\partial w C_i}{\partial t} + \nabla \cdot (w C_i \mathbf{v}) = -2C_i v_l, \quad i = 0, 1, 2 \quad (1)$$

$$\nabla \cdot \left(\frac{w^3}{12\mu_m} G(\phi) [\nabla p + \text{Bu} \rho_m \mathbf{e}_y] \right) = \frac{\partial w}{\partial t} + 2v_l, \quad (2)$$

$$\mathbf{v} = -\frac{w^2}{12\mu_m} G(\phi) \nabla p, \quad G(\phi) = 1 - 3\phi + 4\phi^3, \quad \phi = \frac{\text{Bn} \tau_m}{w |\nabla p|}, \quad (3)$$

$$\rho_m = C_0 + \zeta_1 C_1 + \zeta_2 C_2, \quad \mu_m = C_0 + \xi_1 C_1 + \xi_2 C_2,$$

$$\zeta_i = \frac{\rho_i}{\rho_0}, \quad \xi_i = \frac{\mu_i}{\mu_0}, \quad \text{Bu} = \frac{\rho_0 g d^2}{\mu_0 U}, \quad \text{Bn} = \frac{\tau_0 d}{U \mu_0}$$

Here, Cartesian coordinate system Oxy is introduced in the cell plane, so that y -axis (with the basis vector \mathbf{e}_y) is vertical and origin O is located in the bottom left corner of the computational domain; C_i are the fluid tracer concentrations (with i being the number of fluid, so that $i = 0$ corresponds to the yield-stress fluid filling the slot initially); $w(x, y, t)$ is the width of the Hele-Shaw cell (currently it is a prescribed function of coordinates and time, while in the model describing the hydraulic fracture propagation, w is obtained via coupling the hydrodynamic equations describing the flow inside a hydraulic fracture with geomechanics equations describing the fracture growth [7]); \mathbf{v} is the width-averaged fluid velocity; v_l is the velocity of fluid leak-off through the porous walls; G is the correction to fluid mobility due to the yield-stress rheology ($G = 1$ for Newtonian fluid); differential operator ‘ ∇ ’ acts in the (x, y) plane as we applied the averaging procedure along the cell width. The flow scales are as follows: L is the cell length, U is the scale of the injection velocity, d is the cell width scale, ρ_0 is the fracturing fluid density, μ_0 and τ_0 are the fracturing fluid plastic viscosity and yield stress, respectively; g is the gravity acceleration. Non-dimensional parameters are as follows: Bu is the Buoyancy number, Bn is the Bingham number; ζ_i is the fluid density ratio; ξ_i is the fluid viscosity ratio. The flow domain according to the scaling introduced above is $(x, y) \in [0, 1] \times [0, H/L]$, where H is the height of the Hele-Shaw cell.

For hyperbolic equations (1), we impose initial distribution of fluid tracers and boundary conditions for fluid tracers and velocity at the inlet:

$$x = 0 : C_i^{in} = C_i(y, t), i = 0, 1, 2; \quad (4)$$

$$t = 0 : C = C_{i0}(x, y), i = 0, 1, 2 \quad (5)$$

The quasi-elliptic equation for pressure (2) requires either Neumann or Dirichlet boundary conditions to be specified at the boundaries of the flow domain. At the top and bottom boundaries, we impose the no-flow condition. There is a specified velocity at the inlet segment of the left vertical boundary. We assume that the flux at the right vertical boundary is horizontal (which is the variant of a “soft” non-reflecting outlet boundary condition). The corresponding boundary conditions for pressure are formulated as follows:

$$x = 0 : \frac{\partial p}{\partial x} = -\frac{12\mu_m}{G(\phi)w^2}, \quad (6)$$

$$x = 1 : \frac{\partial p}{\partial y} = -\text{Bu}\rho_m(1, y) \Leftrightarrow p(1, y) = -\text{Bu} \int_0^y \rho_m(1, s) ds \quad (7)$$

$$y = 0 \text{ and } y = h : \frac{\partial p}{\partial y} = 0. \quad (8)$$

Here, $h = H/L$ is the channel height-to-length ratio, and we assumed that the velocity at the inlet is constant, so that in a dimensionless form it is unity. Note that for a shorter notation, the boundary conditions (4–8) are formulated for the flow configuration when the inlet zone occupies the entire height of the fracture.

3 Numerical Implementation and Validation

The governing equations (1–3) coupled with the boundary and initial conditions (4–8) are solved numerically using the finite-difference method on a rectangular staggered grid. The advection equations are solved using the second-order TVD flux-limiting scheme, while the quasi-linear elliptic pressure equation is solved using the iterative process with the multigrid solver applied to the solution of a linearized equation. The details of numerical algorithm and thorough validation of the model (1–3) are presented in [1]. In particular, the model was validated against the following experiments made in Hele-Shaw cells: (i) gravitational slumping of a heavy oil in a confined cell; (ii) Saffman-Taylor instability during the displacement of a water-glycerin solution by water; (iii) set of experiments with the channeling of Newtonian and power-law fluids through the cell filled initially with a yield-stress fluid. The simulations presented below are carried out using the 513×513 mesh.

4 Results and Discussion

A parametric study of the interaction between viscoplastic and viscous fluids in a Hele-Shaw cell is conducted using the research code based on the model and its implementation described above. It is found that when a yield-stress fluid is displaced by a high-viscosity Newtonian fluid (so that there is no Saffman-Taylor instability at the interface), the viscoplastic fluid behaves very similar to the viscous fluid (Fig. 1*a,b*). The fingers of a low-viscosity fluid penetrating through the viscoplastic fluid trigger the development of unyielded zones (Fig. 1*c,d*).

The effect of Bingham number on the Saffman-Taylor instability between the yield-stress fluid and the viscous fluid is studied (see Fig. 2). It is found that an increase in the Bingham number intensifies the finger shadowing effect: the growth rate of small fingers is damped, while the longer fingers grows faster and the total number of fingers is decreased. As a result, the finger penetration length is increased with an increase in the Bingham number.

A multistage hydraulic fracturing job in shales (typically in the U.S. Land) is usually followed by an injection of a small portion of particle-free low-viscosity fluid to clean up the well from the proppant. This stage is called an overflush. During this process, a portion of the clean fluid enters the hydraulic fracture and displaces the particle-laden suspension away from the perforations. Therefore, there is a risk of fracture closure in this proppant-free unsupported cavity, which would result in a dramatic decrease in the fracture conductivity. We carried out a sensitivity study of the shape and the area of the particle-free cavity developed during the displacement of a particle-laden suspension by a clean fluid in the vicinity of fracture inlet zone. We found that there are three qualitatively different displacement regimes, namely: (i) the slumping-dominated scenario Fig. 3*a* ; (ii) an intermediate scenario Fig. 3*b*; and (iii) the fingering-dominated scenario Fig. 3*c*. The slumping-dominated regime provokes the development of a large particle-free zone at the top of the hydraulic fracture, which would result in a significant decrease in the productive fracture area after closure. Among the other two regimes, the smallest area of the particle-free cavity is found in the case of a finger-dominated regime, which is achieved by an

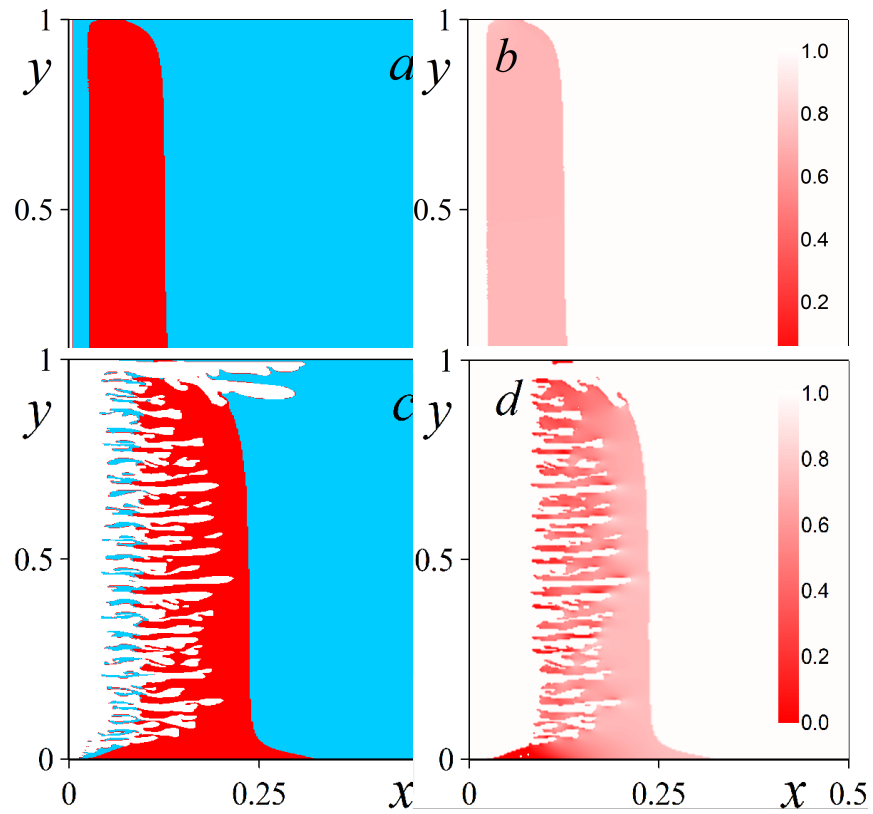


Figure 1: Distribution of fluids (*a, c*) and parameter G (3) describing the unyielding of the viscoplastic Fluid 0 (*b, d*) in a plane channel during a certain injection sequence. Viscoplastic Fluid 0 is red, high-viscosity Fluid 1 is blue and low-viscosity Fluid 2 is white. The inlet velocity is $4.76 \cdot 10^{-2} m/s$, $\xi_1 = 1.43$, $\xi_2 = 8.72 \cdot 10^{-4}$, $\zeta_1 = \zeta_2 = 0.5$, $Bu = 12.9$, $Bn = 1.43$, $t = 0.122$ (*a, b*) and $t = 0.235$ (*c, d*).

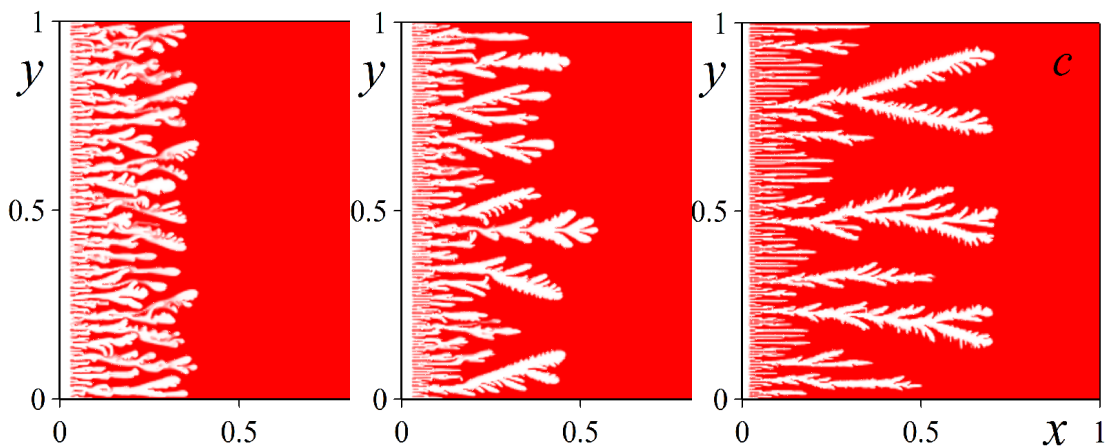


Figure 2: Effect of Bingham number on the fingering of a viscous fluid (white) through the viscoplastic fluid (red) for $Bn = 0.36$ (*a*), $Bn = 1.43$ (*b*) and $Bn = 5.72$ (*c*). The inlet velocity is $4.76 \cdot 10^{-2} m/s$, $\xi = 8.72 \cdot 10^{-4}$, $\zeta = 1$ and $t = 0.203$.

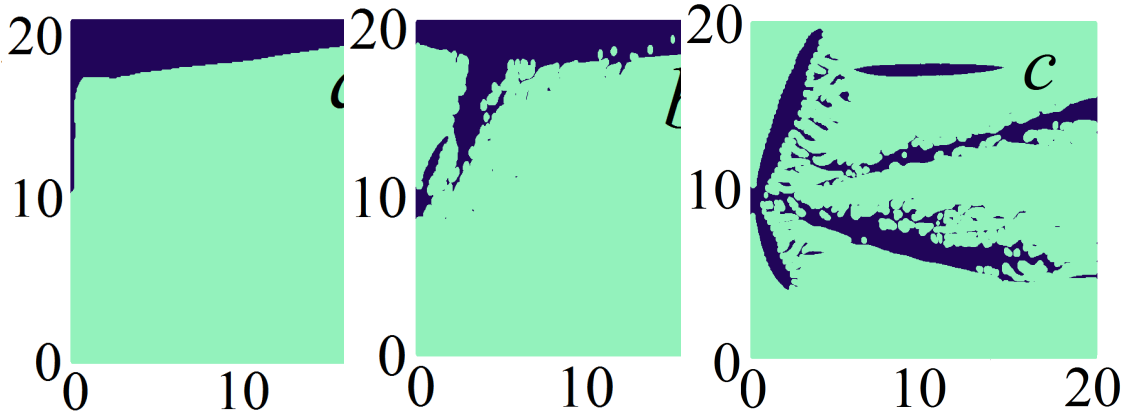


Figure 3: Typical regimes of displacement of a high-viscosity suspension by a low-viscosity fluid in the vicinity of perforations during hydraulic fracture overflush.

increase in the fluid viscosity ratio, injection rate or suspension yield stress.

5 Conclusions

In the framework of a lubrication approximation, the displacement of a viscoplastic fluid by viscous fluids in a Hele-Shaw cell is studied. It is found that the yield-stress rheology of the fluid results in a modification to the fluid mobility, which is the coefficient of proportionality between the pressure gradient and the width-averaged fluid velocity. In contrast to the flow of Newtonian fluids in narrow channels, the pressure equation describing the flow of viscoplastic fluid is strongly non-linear.

Based on the numerical simulations, we carried out the parametric study of interactions between viscous and viscoplastic fluids in a Hele-Shaw cell. It is found that when the yield-stress fluid is displaced by a viscous one and the interface is flat, the viscoplastic fluid behaves very similar to a viscous fluid. The instability triggered at the interface between the fluids leads to the development of unyielded zones in the viscoplastic fluid. During the displacement of a yield-stress fluid by a low-viscosity fluid, an increase in the Bingham number leads to an increase in the finger penetration length and a decrease in the total number of fingers.

The overflush stage at the end of a hydraulic fracturing treatment is studied with the aim to minimize the area of a particle-free zone in the vicinity of perforations inside a hydraulic fracture, which mitigates the risk of losing well-to-fracture hydraulic connection. Numerical simulations of the displacement of a viscous or a viscoplastic fracturing fluid by a low-viscosity overflush fluid in the hydraulic fracture demonstrated that there are three qualitatively different overflush scenarios: (i) slumping-dominated; (ii) intermediate; (iii) fingering-dominated. The smallest area of particle-free zone in the vicinity of perforations is achieved in the fingering scenario, which occurs when either there is a large viscosity contrast between the fracturing and the overflush fluids, or the fracturing fluid shows a strong yield-stress behavior.

Acknowledgements

Startup funds of Skolkovo Institute of Science and Technology are gratefully acknowledged. The authors are thankful to Schlumberger for the permission to publish.

References

- [1] Boronin, S. A., Osiptsov, A. A., Desroches, J. 2015 Displacement of yield-stress fluids in a fracture, *Intl. J. Multiphase Flow* **76**, 47–63.
- [2] De Zeeuw, P. M. 1990 Matrix-dependent prolongation and restrictions in black-box multigrid solver, *J. Comput. Appl. Math.* **33**, 1–27.
- [3] Adachi, J., Siebrits, E., Peirce, A., & J. Desroches, 2007 Computer simulation of hydraulic fractures. *Int. J. of Rock Mech. and Mining Sci.* **44**(1), 739–757.
- [4] Economides, M.J., Nolte, K.G. 2000 *Reservoir Stimulation*. Third Edition. Wiley.
- [5] Lecampion, B., Garagash, D. I. 2014 Confined flow of suspensions modelled by a frictional rheology. *Journal of Fluid Mechanics* **759**, 197–235.
- [6] Pearson, J. R. A. 1994 On suspension transport in a fracture: framework for a global model. *Non-Newtonian Fluid Mech.* **54**, 503–513.
- [7] Hammond, P. S. 1995 Settling and slumping in a Newtonian slurry, and implications for proppant placement during hydraulic fracturing of gas wells, *Chem. Eng. Sci.* **50**(20), 3247.
- [8] Boronin, S. A., Osiptsov, A. A. 2010 Two Continua Model of Suspension Flow in a Hydraulic Fracture. *Doklady Physics.* **55**(4), 192–202.
- [9] Frigaard, I. A., Howison, S. D., Sobey, I. J. 2010 On the stability of Poiseuille flow of a Bingham fluid. *J. Fluid Mech.* **263**, 133–150.
- [10] Gorodtsov, V. A., Yentov, V. M. 1997 Instability of the displacement fronts of non-Newtonian fluids in a Hele-Shaw cell. *J. Appl. Maths Mechs.* **61**(1), 111–126.
- [11] Bittleston, S. H., Hassager, O. 1992 Flow of viscoplastic fluids in a rotating concentric annulus. *J. Non-Newtonian Fluid Mech.* **42**, 19–36.
- [12] Homsy, G. M. 1987 Viscous fingering in porous media. *J. Non-Newtonian Fluid Mech.* **19**, 271–311.
- [13] Muskat, M. 1934 *wo Fluid Systems in Porous Media. The Encroachment of Water into an Oil Sand* Morris Muskat. *Journal of Applied Physics.* **5**, 250–264.

Sergei S. Boronin, Skolkovo Institute of Science and Technology, Nobel Street 3, 143026 Moscow, Russia

Andrei A. Osiptsov, Skolkovo Institute of Science and Technology, Nobel Street 3, 143026 Moscow, Russia

REFERENCES

Jean Desroches, Services Petroliers Schlumberger, Le Palatin 1, 1 cours du Triangle, 92936 Paris La Défense Cedex, France