An Analogy of the Equilibrium of a Two-legged Robot on a Cylinder for the Problem of Transfer by a Manipulator With a Two-finger Grasp of a Cylinder

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Abstract

We consider a walking robot with m legs that ensure the desired motion of the robot body. Each of the robot legs contacts the surface in a single foothold. We describe the robot motion within the framework of general dynamics theory, with six differential equations for the robot motion derived from the momentum and angular momentum theorems. In the case of two-legged robot, $n = 2$, we reduce the problem of the existence of the solution to a system of algebraic inequalities. We present the classification of footholds positions for different values of the friction coefficient k.

We show an analogy of the problem of the equilibrium of a two-legged robot on an inclined rough cylinder for the problem of transfer by a manipulator with a two-finger grasp of a rough cylinder.

1 A cylinder grasping problem

In this paper, we discuss the problem of walking robot dynamics on one-side constraint. While the general walking robot motion on a plane was analyzed in detail in Ref. [1] the case of the dynamics on a curved surface is far more complicated. Model dynamics and control problems was considered in [2]. Equilibrium conditions for a solid on a rough plane was considered in [3]. Walking robot parameters optimization for the motion in tubes was considered in [4]. The special case of a robot with eight legs whose up porting points are restricted to the inner surface of a tube was considered in [5]. In the present work, we consider the more general case of a robot with two arbitrary supporting points on a rough cylinder and on a curved surface.

Then we consider the problem of curved object grasping by the fingers of the robot-manipulator. For example we discussed monkey-robot with 10 arms and 10 legs fingers or two legged human-robot with 10 arms fingers. The robot can hold the object by one and grasp by two-fingers. An object grasping problem is equivalent to the problem of the walking robot with n legs [6], [7]. Consider a grasp with m fingers. Each finger contacts an object in one foothold.

Let the point O is an origin fixed in absolute space. Suppose that robot arms fingers accomplish the desired motion with respect to the body of the robot. Using general dynamics theorems to describe the cylinder motion, we obtain six different equations for the cylinder dynamics from the momentum and angular momentum theorems. Among them there are three equations of the body translation with point A and another three describe body rotation about point A. For prescribed motion be realized then reaction in m footholds should satisfy following kinetostatic equations [8], [9]:

$$\sum_{i=1}^{m} \tilde{\mathbf{R}}_i = -\tilde{\mathbf{\Phi}}, \qquad \sum_{i=1}^{m} \tilde{\mathbf{r}}_i \times \tilde{\mathbf{R}}_i = -\tilde{\mathbf{M}}, \tag{1}$$

where $\tilde{\mathbf{R}}_i$ is reaction component, $\tilde{\mathbf{r}}_i$ corresponds to the i-th finger supporting point vector, $\tilde{\mathbf{\Phi}}$ is the sum of the external active forces plus time derivative of desired momentum, and $\tilde{\mathbf{M}}$ is the sum of external active forces momentum and time derivative of desired angular momentum with respect to the point O. In two vector equations in (1), the former corresponds to the momentum of the object (and is equivalent to three scalar equations when projected onto the basis vectors), while the latter defines the desired change of the angular momentum.
Assuming that $\tilde{\mathbf{\Phi}}$ is orthogonal to $\tilde{\mathbf{M}}$, we obtain [10] that the system $\{\tilde{\mathbf{\Phi}}, \tilde{\mathbf{M}}\}$ can be also used at the point C

$$\tilde{\mathbf{r}}_C \times \tilde{\mathbf{\Phi}} = \tilde{\mathbf{M}}, \qquad \tilde{\mathbf{r}}_C = -\frac{\tilde{\mathbf{M}} \times \tilde{\mathbf{\Phi}}}{\tilde{\Phi}^2}, \qquad \tilde{\Phi} = |\tilde{\mathbf{\Phi}}|,$$

where $\tilde{\mathbf{r}}_C$ is the vector $\mathbf{OC}$, and C corresponds to the point at which the resultant of the reactions is acting.
Further problem of reactions distribution $\tilde{\mathbf{R}}_i$ in some fixed point of time is investigated by the proposal that force $\tilde{\mathbf{\Phi}}$ is acting at the point $\tilde{\mathbf{r}}_C$ and force moment there is zero. Motion equations (1) for finding reactions of fingers prescribed motion can be transformed [11]:

$$\sum_{i=1}^{m} \tilde{\mathbf{R}}_i = \tilde{\mathbf{\Phi}}, \quad \sum_{i=1}^{m} \tilde{\mathbf{r}}_i \times \tilde{\mathbf{R}}_i = \tilde{\mathbf{r}}_C \times \tilde{\mathbf{\Phi}}. \tag{2}$$

For example point C can be the grasping object center of mass.
Assuming that the robot footholds are on the surface of a rough cylinder of radius ρ with a friction coefficient k, we introduce the coordinate system $Oxyz$ such that the axis Ox is directed along the cylinder axis (so that the projection of $\tilde{\mathbf{\Phi}}$ on the axis Ox is negative – see Fig. 1.), the axis Oz is parallel to the vector $\tilde{\mathbf{\Phi}}$, and the angle between the cylinder axis and the vector $\tilde{\mathbf{\Phi}}$ is α.
The problem of finding the reaction forces (2) is similar to the foothold reactions distribution problem for walking robot, when the footholds are on the external surface of a rough inclined cylinder where the axis has an angle α with respect to the vector $\tilde{\mathbf{\Phi}}$. It has been considered in Ref. [9] the problem of searching of the reactions components along the cylinder axis when $\alpha = 0$.
In the coordinates $Oxyz$ we define $\tilde{\mathbf{R}}_i = (\tilde{R}_i^x, \tilde{R}_i^y, \tilde{R}_i^z)$, $\tilde{\mathbf{r}}_C = (\tilde{x}_C, \tilde{y}_C, \tilde{z}_C)$, and $\tilde{\mathbf{\Phi}} = (-\tilde{\Phi}\sin\alpha,\, 0,\, -\tilde{\Phi}\cos\alpha)$, $i = 1, \cdots, m$. In case of a one-sided surface, and the grasp inside the cylinder, we have additional restrictions on normal reactions $\tilde{N}_i$ [13]:

$$\tilde{N}_i = \tilde{\mathbf{R}}_i \cdot \mathbf{e}_\nu^i \geq 0, \tag{3}$$

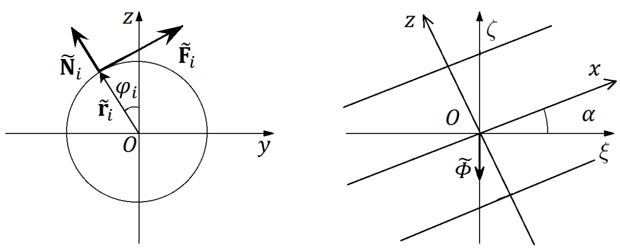

Figure 1: Cylinder.

where $\mathbf{e}_\nu^i$ is an external normal to i-th supporting point on the cylinder, while the tangential components are given by $\tilde{\mathbf{F}}_i = \tilde{\mathbf{R}}_i - \tilde{N}_i \mathbf{e}_\nu^i$.
For the reactions to be in the friction cones (2), we have following inequalities:

$$|\tilde{\mathbf{F}}_i| \le k\tilde{N}_i, \tag{4}$$

i.e. the tangential reactions $\tilde{\mathbf{F}}_i$ are restricted by Coulomb limiting friction value. When $\tilde{\mathbf{F}}_i$ exceeds this limiting value, the robot legs and arms begin to slide along a surface.
The reaction distribution problem then reduces to the solution of equations (2), and inequalities (3), (4), for reactions limited to the friction cones. The restricted motion can only be realized if the solution of Eqns. (2)-(4) does exist.
The same inequalities are for walking robot on the cylinder [9]. If the grasp is out the cylinder this inequalities (3) have opposite sign.
For example if m is even. And one of each par of the supporting points is on and another is in the thin surface such that we consider them like one geometrical point. Then we need only inequalities (4).
For $\mathbf{r}_i = \tilde{\mathbf{r}}_i/\rho = (x_i, y_i, z_i)$, in the cylinder coordinate: $\mathbf{r}_i = (x_i, -\sin\varphi_i, \cos\varphi_i)$, $\mathbf{e}_i^\nu = (0, -\sin\varphi_i, \cos\varphi_i)$, $N_i = \tilde{\mathbf{N}}_i/\tilde{\Phi} = (0, -N_i \sin\varphi_i, N_i\cos\varphi_i)$, where φ_i is the angles between axis Oz and cylinder normal $\mathbf{e}_i^\nu$. We define $\mathbf{e}_x$ as the unitary vector in the Ox axis, while $\mathbf{e}_i^\tau = (0, \cos\varphi_i, \sin\varphi_i)$ as the tangential to the cylinder. Then the tangential reaction: $\mathbf{F}_i = (F_i^x, F_i^{yz}\cos\varphi_i, F_i^{yz}\sin\varphi_i)$, where $F_i^x = \mathbf{F}_i \cdot \mathbf{e}_x$, $F_i^{yz} = \mathbf{F}_i \cdot \mathbf{e}_i^\tau$, $\mathbf{R}_i = \tilde{\mathbf{R}}_i/\tilde{\Phi} = (R_i^x, R_i^y, R_i^z)$, $\mathbf{r}_C = \tilde{\mathbf{r}}_C/\rho = (x_C, y_C, z_C)$.
Let $p = R_1^x - R_2^x$. We further define the coordinate differences, and the supporting points difference of angles of axis Oz are $\Delta x = x_2 - x_1$, $\Delta y = y_2 - y_1$, $\Delta z = z_2 - z_1$, $\Delta\varphi = \varphi_2 - \varphi_1$, and $s_{21} = \sin\varphi_2 - \sin\varphi_1$, $c_{21} = \cos\varphi_2 - \cos\varphi_1$. We then project system (2) onto the axes $Oxyz$. For arbitrary surface we find that the second equation of (2) (corresponding to the moment) has the skew-symmetric matrix with respect to the component R_i^x [9]. These are 2 independent equation, while the third equation corresponds to the restriction of the point C to the plane containing the two footholds. As a result, the system (2) yields 5 independent equation and a restriction.

2 A Two-finger Grasp

During the robot motion one-supporting and two-supporting points phases are changed. First, we consider the one-supporting phase of the grasp. Let $m = 1$,

then the motion existing condition is reaction is equal to force $\mathbf{\Phi}$ and supporting point and the point C are on the line along $\mathbf{\Phi}$, while the angle between $\mathbf{\Phi}$ and the normal do not exceed the friction angle.
If the grasp inside the surface then point C is under the surface. In opposite case the grasp is under the surface. Then point C is inside the surface.
If m is even. And one of each par of the supporting points is on and another is in the thin surface such that we consider them like one geometrical point. Then it does not matter where the point C is on the line.
Let $n = 2$, and $x_1 \neq x_2$. Then $p = F_2^x - F_1^x$, and from (2):

$$\begin{aligned}
F_1^x &= (\sin\alpha + p)/2, \qquad F_2^x = (\sin\alpha - p)/2,\\
N_1 &= \frac{-p\sin^2\dfrac{\Delta\varphi}{2} + (x_2 - x_C)\cos\varphi_1\cos\alpha}{\Delta x} + N_1^\alpha,\\
N_2 &= \frac{-p\sin^2\dfrac{\Delta\varphi}{2} + (x_C - x_1)\cos\varphi_2\cos\alpha}{\Delta x} + N_2^\alpha,\\
F_1^{yz} &= \frac{-p\sin\Delta\varphi + 2(x_2 - x_C)\sin\varphi_1\cos\alpha}{2\Delta x} + F_1^{(yz)\alpha},\\
F_2^{yz} &= \frac{p\sin\Delta\varphi + 2(x_C - x_1)\sin\varphi_2\cos\alpha}{2\Delta x} + F_2^{(yz)\alpha},\\
\tan\alpha &= \frac{\Delta x(\sin\varphi_2 + y_c) + (x_C - x_2)s_{21}}{y_C c_{21} + z_C s_{21} - \sin\Delta\varphi},
\end{aligned} \tag{5}$$

where N_i^α and F_i^{yz} are the functions of x_i, φ_i, y_C and z_C.
From the conditions (3)

$$p \le \frac{(x_2 - x_C)\cos\varphi_1\cos\alpha + N_1^\alpha\Delta x}{\sin^2\Delta\varphi/2}, \qquad p \le \frac{(x_C - x_1)\cos\varphi_2\cos\alpha + N_2^\alpha\Delta x}{\sin^2\Delta\varphi/2}. \tag{6}$$

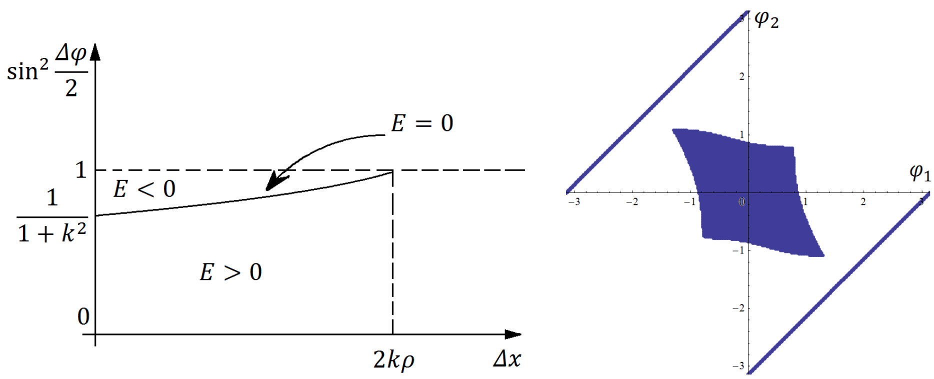

Figure 2: The analytical and the numerical parameter diagrams.

The conditions (4) can be displayed in the form

$$Ep^2 + B_1 p + C_1 \le 0, \qquad Ep^2 + B_2 p + C_2 \le 0, \tag{7}$$

where

$$E = (\Delta x)^2 + \sin^2 \Delta\varphi - 4k^2 \sin^4 \left(\frac{\Delta\varphi}{2}\right),$$

B_i, C_i are the functions of x_i, φ_i, x_C, y_C and z_C.

The boundaries between different regimes can be determined analytically. For example, in the case of $E < 0$, the solution exists, and can be obtained analytically [9], as shown in Fig. 2, on the left. Note that in this case it's limited to the range $\Delta x \leq 2k\rho$. In contract to this behavior, for $E \geq 0$ there is no such restriction and an additional step is required to address the question of the existence of the solution. At the point $(0, 0)$ we find $E = 0$, which means that two footholds are orthogonal to the cylinder axis. Here, two possible solution are either identical, or limited to a single diameter. In the latter case, point C and the reaction have to be in one plane, parallel to force Φ, and the problem has a solution.

For the desired legs or fingers configurations and given point C, the problem can be solved numerically. In Fig. 2, on the right, we present the numerical solution for the example when $x_2 = -x_1 = \rho = k = 1$. Note that in this case $E > 0$.

In the numerical simulations, we use a 300×300 array for the points (φ_1, φ_2), in the interval $[-\pi, \pi]$, and for each point verify the conditions (6), (7). Specifically, the condition (7) was analyzed in two cases, when $E = 0$ and $E > 0$, and when the solution of the problem does exist, the solutions were shown in the plot.

When $E = 0$, the reaction distribution problem reduces to the linear inequalities (6), (7) for the parameter p.

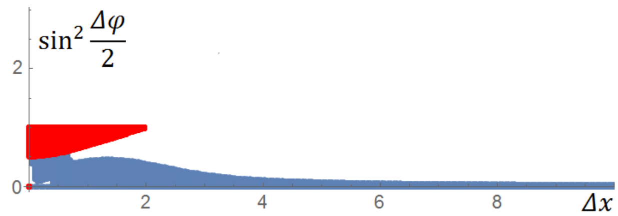

Figure 3: For $\alpha = \pi/4$; $x_2 = -x_1$, $\varphi_2 = -\varphi_1$.

For $E > 0$, we need to consider two conditions. First is the restriction on the determinants $D \geq 0$, while the second is the requirement of a non-empty intersection of the set of point of the intervals between the roots of quadratic equations. From this plot we see that, if two points are on one diameter, then the solution of the reaction distribution problem exists. The two lines in the plot, correspond to $\varphi_1 = \varphi_2 + \pi$ or $\varphi_1 = \varphi_2 - \pi$. The rhombus form represents the requirement on the determinants $D_i \geq 0$, while additional conditions further restrict the range [11].

In Fig. 3 we present the results for $E > 0$ and $E \leq 0$, when $x_2 = -x_1$, $\varphi_2 = -\varphi_1$ and shows the case of $\alpha = \pi/4$. The figures for $\alpha = 0$ and increased to $\pi/2$ are shown in [12]. Note that when $\alpha = \pi/2$, the solutions exists only for diametrical footholds.

For two-finger robot when E is negative, the solution exists, and obtained analytically [13]. Using numerical simulations we explain the reaction distribution problem existing and build this problem solution existing fields for given footholds and point C position. For example, for two-foothold phase, we consider symmetric, about

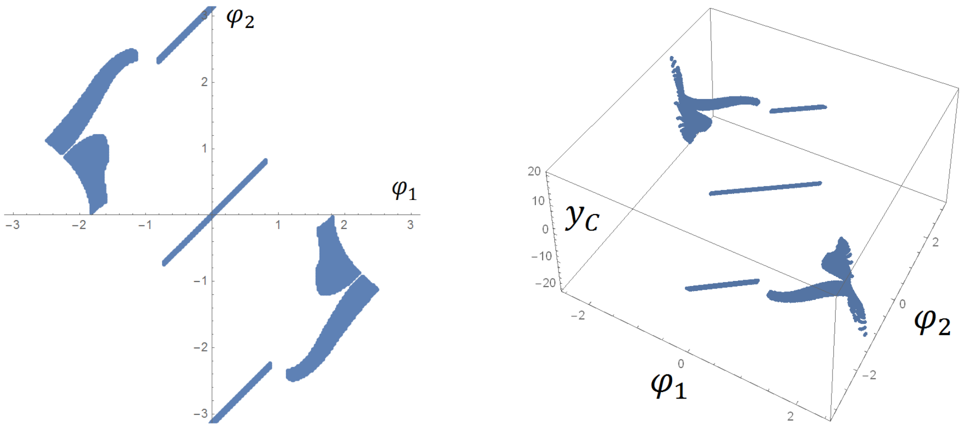

Figure 4: Admissible area for $\alpha = \pi/3$; $\Delta x = 1,1$.

point C, along and orthogonal cylinder axis, robot configurations. For first of these configurations examined three cases with nonnegative E coefficient, for distance x, between point C and footholds: $0,9$; 1 and $1,1$ at ρ and k equal 1, α from 0 to π (in all 13 different values cylinder inclination angles). Reactions distribution problem solution existing fields constructed on the two angles plane, correspond to footholds projections on the cylinder base and three dimensional fields which supplement this plane by point C z-coordinate altitude. When α equals to 0, x equals to 1, the field consist of three separate situated subregions. On the angle plane each of pair parallel lines corresponds to support on the cylinder diameter plane section contained point C [14]. There is connected field between these lines. It contains the line segment corresponding to the angles equality, robot supported above on the line which is parallel to cylinder axis and satisfy force direction deviation restriction. The indicated segment on the plot disappear when x equals to $0,9$ for α equals $\pi/4$, and at increasing x, later, for $4\pi/9$. It corresponds to the robot beginning sliding down the cylinder. When x equals to $1,1$ for α equals $\pi/3$ in three-dimensional fields observed bundles of separate points, Fig. 4. That means that the point C altitude position more harsh change while changing the angles [15].

3 Conclusion

During the robot motion, one-supporting point and two-supporting point phases are changed. The reaction distribution problem have a solution in following cases.
1. One-supporting point phase. So the motion existing condition is reaction is equal to force $\mathbf{\Phi}$ and supporting point and the point C are on the line along $\mathbf{\Phi}$. And the angle between $\mathbf{\Phi}$ and the normal not exceed friction angle.
1.1 If the grasp inside the surface then point C is under the surface. In opposite case the grasp is under the surface. Then point C is inside the surface.
1.2 If m is even. And one of each par of the supporting points is on and another is in the thin surface such that we consider them like one geometrical point. Then it does not matter where the point C is on the line.
2. Two-supporting point phases. In case when the grasp is inside the cylinder. The

point C and the reactions have to be in the plane parallel to force $\mathbf{\Phi}$.
2.1 If supporting points are on one diameter.
2.2 When coefficient $E < 0$. And in some fields with connected set of points, when $E \geq 0$.
So the robot can transfer the cylinder by one or two fingers.

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