Turbine blade vibration analysis using helicoidal shell model

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Abstract

Short turbine blades with comparable height and length are often used in turbine construction. In service, the blades are subject to various periodical impacts, and to prevent the fatigue failures, resonance vibrations should be avoided. Therefore, determination of natural frequencies and normal modes are of crucial importance. We consider free vibrations of a blade modeled by a helicoidal Kirchhoff shell. In the paper we use the variational approach based on the Lagrange mechanics in which the generalized coordinates play the role of factors for the displacement approximation. The displacement approximation are taken from the beam theory for the naturally twisted beam with an additional component taking into account the in-plane deformation of the cross-section. The algorithm of calculations by means of computer mathematics is proposed and an example of modal analysis of the helicoidal shell with variable section is presented. An example of modal analysis of the helicoidal shell with variable cross-section is provided, too. A three-dimensional computer model of the blade is developed and analyzed by ANSYS. Comparison of calculation results for shell and the three-dimensional model is demonstrated.

1 Introduction

The impacts of gas or fluid jets on a blade of the power plant is of the periodical character. Therefore, the blades are affected by the forced oscillations [1, 2, 3]. Some frequencies of these pulsating loads can be close to one of the normal frequencies of the blade, and resonance vibrations can cause the fatigue failure of the blade. In this regard, at the design should take into account the anticipated operating regimes and frequencies of alternating impacts, so that the blade could be "detuned" from the resonance frequencies by changing the mechanical and geometrical parameters of the blade. For this reason, the vibration analysis is an important part of the turbine design.
In power engineering, the engineers often use blades which have comparable length and width, therefore, from a mechanical perspective the blades can be considered shells.
In this work we propose to consider a short twisted blade as the helicoidal shell and apply the classical theory of thin shells [4, 5]. Based on the tensor equation of

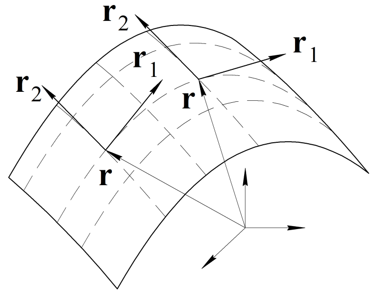

Figure 1: Shell surface

the shell theory, the scalar equations are derived and the energy relationships are obtained. Then the Lagrange equations, in which coefficients of the displacement approximation play the role of generalized coordinates, are used for construction of the global inertia and stiffness matrices for the further modal analysis. This approach was tested on the helicoidal blade in [6] in the present paper we give more attention to detailed consideration of the method. Natural frequencies and normal modes are to be calculated in the Wolfram Mathematica system [7]. It is reasonable to compare them with the results of the 3D-model finite element analysis by ANSYS [8], so that we judge the applicability of the approximations used.

2 Brief information from the theory of Kirchhof-fвЃЄs shells

According to the approach of the Lagrange analytical mechanics one must first determine the degrees of freedom of the examined object and introduce the generalized coordinates and then apply the principle of virtual work for determination of the generalized forces.

A classical shell can be presented as a deformable surface with material normal unit vectors which have three translational degrees of freedom and two rotational ones. It is assumed that the shell does not resist the rotation of the normal around its axis and therefore no corresponding degree of freedom is needed. The movement of the shell is then defined by the small displacement vector $\mathbf{u}$ and the small rotation vector $\boldsymbol{\theta}$ in the tangent plane [9].

It is convenient to use the vector of change of the normal $\mathbf{n}$ to the shell as a generalized coordinate: $\boldsymbol{\varphi} \equiv \boldsymbol{\theta} \times \mathbf{n} = \widetilde{\mathbf{n}}$. The sign of tilde implies a small increment with deformation.

Any surface is known to be defined by the dependence of the radius vector on the curvilinear coordinates $\mathbf{r}\,(\gamma^1,\gamma^2)$.

Thus, each point of this surface is the point of intersection of two coordinate curves (Fig. 1). Vectors of derivatives of the radius vector are tangential to these curves and form the basis:

$$\mathbf{r}_\beta = \partial\mathbf{r}/\partial\gamma^\beta = \partial_\beta\mathbf{r}. \tag{1}$$

The unit normal vector to the surface formed by these two vectors is

$$\mathbf{n} = \frac{\mathbf{r}_1 \times \mathbf{r}_2}{H}, \; H \equiv |\mathbf{r}_1 \times \mathbf{r}_2| \,. \tag{2}$$

Any vector is known to be decomposed into components in both covariant and contravariant bases as follows: $\boldsymbol{\chi} = \chi_\beta \mathbf{r}^\beta + \chi_n \mathbf{n} = \chi^\beta \mathbf{r}_\beta + \chi_n \mathbf{n}$. Here the components χ_β of the vector $\boldsymbol{\chi}$ are called "covariant" and χ^β are called "contravariant".
The contravariant basis is constructed according to the condition $\mathbf{r}_\alpha \cdot \mathbf{r}^\beta = \delta_\alpha^\beta$ where δ_α^β is the Kronecker symbol.
It is necessary to introduce both bases to obtain the sought-for equations. Firstly, we can find the expression for the Hamilton operator:

$$\nabla = \mathbf{r}^\beta \partial_\beta. \tag{3}$$

Then we compose expressions for the first and second metric tensors

$$\mathbf{a} = \nabla \mathbf{r}, \; \mathbf{b} = -\nabla \mathbf{n}. \tag{4}$$

It is known that the surface form is completely defined by covariant components of these metric tensors [10]. Therefore, the surface deformation is quite defined by the small increments of these components and can be determined by two symmetrical tensors in the tangent plane:

$$\boldsymbol{\varepsilon} \equiv \frac{1}{2} \widetilde{a_{\alpha\beta}} \mathbf{r}^\alpha \mathbf{r}^\beta, \; \boldsymbol{\kappa} \equiv \widetilde{b_{\alpha\beta}} \mathbf{r}^\alpha \mathbf{r}^\beta, \; a_{\alpha\beta} = \mathbf{r}_\alpha \cdot \mathbf{r}_\beta, \; b_{\alpha\beta} = -\partial_\alpha \mathbf{n} \cdot \mathbf{r}_\beta. \tag{5}$$

The first of these tensors defines the change of lengths and angles on the surface whereas the second one defines the change of its curvature. Accounting for $\mathbf{u} \equiv \widetilde{\mathbf{r}}$ and eq. (5) we obtain

$$\boldsymbol{\varepsilon} = (\nabla \mathbf{u})_\perp^{\mathrm{S}}, \; \boldsymbol{\kappa} = -(\nabla \mathbf{u})_\perp + \mathbf{b} \cdot \nabla \mathbf{u}^{\mathrm{T}}. \tag{6}$$

Here $(\ldots)_\perp$ denotes the part of the tensor in the tangential plane, the signs $(\ldots)^{\mathrm{S}}$ and $(\ldots)^{\mathrm{T}}$ denote symmetrization and transposition, respectively.
It follows from KirchhoffвЂ™s kinematic hypothesis that the rotation is related with the displacement by the orthogonality condition:

$$\mathbf{r} \cdot \mathbf{n} = 0 \Rrightarrow \boldsymbol{\varphi} = -\nabla \mathbf{u} \cdot \mathbf{n}. \tag{7}$$

The strain energy and the kinetic energy of the shell are integrals over the surface area:

$$\Pi = \int \widehat{\Pi} \mathrm{d}o, \; K = \int \rho h \left| \dot{\mathbf{u}} \right|^2 \mathrm{d}o, \tag{8}$$

where ρ is the mass density, $\mathrm{d}o = H \mathrm{d}\gamma^1 \mathrm{d}\gamma^2$ is the surface element, $(\ldots)^{\cdot}$ denotes differentiation with respect to time t, $\widehat{\Pi}$ is the specific strain energy of the isotropic shell, which is the function of both strain tensors $\boldsymbol{\varepsilon}$, $\boldsymbol{\kappa}$:

$$\widehat{\Pi}(\boldsymbol{\varepsilon},\boldsymbol{\kappa}) = \frac{Eh}{2(1-\nu^2)} \left[\nu \varepsilon^2 + \left(1-\nu^2\right) \boldsymbol{\varepsilon} \cdot\cdot \boldsymbol{\varepsilon} + \frac{h^2}{12} \left(\nu \kappa^2 + (1-\nu) \boldsymbol{\kappa} \cdot\cdot \boldsymbol{\kappa} \right) \right], \tag{9}$$

where E is the Young modulus, ν is the Poisson ratio, and ε, κ are the traces of respective strain tensors.

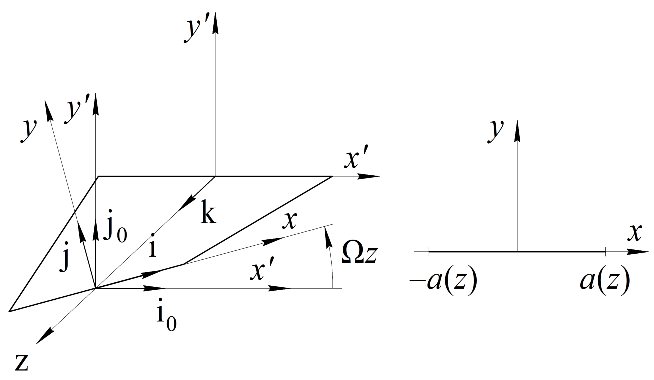

Figure 2: Helicoid model

3 Geometry and deformation of the blade

Helicoid blade is formed by helical motion of a thin strip with the thickness h and width $2a(z)$ (Fig. 2) rotating about axis z by angle per unit length Ω. Axis z is passing through the center of mass of all cross-sections. Cartesian axes x', y' (with unit vectors $\mathbf{i}_0$, $\mathbf{j}_0$) are fixed, while axes x, y (with the unit vectors $\mathbf{i}$, $\mathbf{j}$) are rotating together with the section (Fig. 2). The unit vectors of both coordinate systems are related by the following relations:

$$\mathbf{i}_0 = \cos(\Omega z)\,\mathbf{i} - \sin(\Omega z)\,\mathbf{j},\ \mathbf{j}_0 = \sin(\Omega z)\,\mathbf{i} + \cos(\Omega z)\,\mathbf{j}. \tag{10}$$

The radius vector is defined as

$$\mathbf{r}\,(z,x) = z\mathbf{k} + x\mathbf{i}\,(z)\,,\ \ -a\,(z) \le x \le a\,(z)\,. \tag{11}$$

Accounting for the following formulas for differentiating the unit vectors $\mathbf{i}' = \Omega\mathbf{j}$, $\mathbf{j}' = -\Omega\mathbf{i}$, we can form the basis in the tangent plane:

$$\mathbf{r}_1 = \partial_z\mathbf{r} = \mathbf{k} + \Omega x\mathbf{j} \equiv H\mathbf{e},\ \mathbf{r}_2 = \partial_x\mathbf{r} = \mathbf{i}, \tag{12}$$

where $H = |\mathbf{r}_1 \times \mathbf{r}_2| = \sqrt{1 + \Omega^2 x^2}$. Therefore, the unit normal vector is

$$\mathbf{n} = \frac{1}{H}\left(\mathbf{j} - \Omega x\mathbf{k}\right). \tag{13}$$

Now we can construct the contravariant basis:

$$\mathbf{r}^1 = \frac{1}{H}\mathbf{e} = \frac{1}{H^2}\left(\mathbf{k} + \Omega x\mathbf{j}\right),\ \mathbf{r}^2 = \mathbf{i}, \tag{14}$$

and obtain the Hamilton operator afterward:

$$\nabla = \mathbf{r}^\beta\partial_\beta = \frac{1}{H}\mathbf{e}\partial_z + \mathbf{i}\partial_x. \tag{15}$$

Now we can derive the relations between the unit vectors:

$$\left\{\begin{array}{l} H\mathbf{n} = \mathbf{j} - \Omega x\mathbf{k}, \\ H\mathbf{e} = \Omega x\mathbf{j} + \mathbf{k}, \end{array}\right. \Rightarrow \left\{\begin{array}{l} \mathbf{k} = \dfrac{1}{H}\left(\mathbf{e} - \Omega x\mathbf{n}\right), \\ \mathbf{j} = \dfrac{1}{H}\left(\mathbf{n} + \Omega x\mathbf{e}\right). \end{array}\right. \tag{16}$$

One obtains

$$\mathbf{k}_{\perp} = \frac{1}{H}\mathbf{e},\, \mathbf{j}_{\perp} = \frac{\Omega x}{H}\mathbf{e} \tag{17}$$

from (16) to separate the tangential components of $\nabla\mathbf{u}$ and $\nabla\boldsymbol{\varphi}$ when constructing the strain tensors (6). Hence the first and second metric tensors are

$$\mathbf{a} = \nabla\mathbf{r} = \mathbf{r}^{\beta}\mathbf{r}_{\beta} = \mathbf{ee} + \mathbf{ii},\, \mathbf{b} = -\nabla\mathbf{n} = \frac{\Omega}{H^2}\left(\mathbf{ei} + \mathbf{ie}\right). \tag{18}$$

We define the displacement vector as

$$\mathbf{u} = u_x\mathbf{i} + u_y\mathbf{j} + u_z\mathbf{k}, \tag{19}$$

after what, in accordance with (7), we can introduce the rotation vector of the normal:

$$\boldsymbol{\varphi} = -\nabla\mathbf{u}\cdot\mathbf{n} = \varphi_x\mathbf{i} + \varphi_y\mathbf{j} + \varphi_z\mathbf{k}. \tag{20}$$

Taking (13) and (15) into account we obtain the components of $\boldsymbol{\varphi}$:

$$\begin{aligned}
\varphi_x &= \frac{1}{H}\left(\Omega x\partial_x u_z - \partial_x u_y\right),\\
\varphi_y &= \frac{\Omega x}{H^3}\left(\Omega x u_z' - u_y' - \Omega u_x\right),\\
\varphi_z &= \frac{1}{H^3}\left(\Omega x u_z' - u_y' - \Omega u_x\right).
\end{aligned} \tag{21}$$

Hereafter $(\ldots)'$ denotes the differentiation with respect to coordinate z.
It is convenient to introduce the strain tensors $\boldsymbol{\varepsilon}$ and $\boldsymbol{\kappa}$ as follows:

$$\begin{aligned}
&\boldsymbol{\varepsilon} = \varepsilon_x\mathbf{ii} + \varepsilon_1\mathbf{ee} + \varepsilon_{1x}\left(\mathbf{ei} + \mathbf{ie}\right),\\
&\boldsymbol{\kappa} = \kappa_x\mathbf{ii} + \kappa_1\mathbf{ee} + \kappa_{1x}\left(\mathbf{ei} + \mathbf{ie}\right),\\
&\varepsilon_x = \mathbf{i}\cdot\boldsymbol{\varepsilon}\cdot\mathbf{i} = \partial_x u_x,\\
&\varepsilon_1 = \mathbf{e}\cdot\boldsymbol{\varepsilon}\cdot\mathbf{e} = \frac{1}{H^2}\left[\Omega x\left(\Omega u_x + u_y'\right) + u_z'\right],\\
&\varepsilon_{x1} = \mathbf{i}\cdot\boldsymbol{\varepsilon}\cdot\mathbf{e} = \varepsilon_{1x} = \frac{1}{2H}\left[u_x' - \Omega u_y + \Omega x\partial_x u_y + \partial_x u_z\right],\\
&\kappa_x = \mathbf{i}\cdot\boldsymbol{\kappa}\cdot\mathbf{i} = -\partial_x\varphi_x + \frac{\Omega}{H^3}\left[\Omega x\partial_x u_y + \partial_x u_z\right],\\
&\kappa_1 = \mathbf{e}\cdot\boldsymbol{\kappa}\cdot\mathbf{e} = -\frac{1}{H^2}\left[\Omega x\left(\Omega\varphi_x + \varphi_y'\right) + \varphi_z'\right] + \frac{\Omega}{H^3}\left(u_x' - \Omega u_y\right),\\
&\kappa_{x1} = \mathbf{i}\cdot\boldsymbol{\kappa}\cdot\mathbf{e} = \kappa_{1x} = \frac{1}{H}\left[-\varphi_x' + \Omega\varphi_y\right] + \frac{\Omega}{H^2}\partial_x u_x.
\end{aligned} \tag{22}$$

4 Displacement approximations and vibration analysis

Following the approach proposed in [6, 12], we use the Lagrange equations for vibration analysis of the shell. Having applied the Lagrange equations

$$\left(\frac{\partial K}{\partial \dot{q}_i}\right)^{\cdot} - \frac{\partial K}{\partial q_i} = -\frac{\partial \Pi}{\partial q_i} + Q_i, \tag{23}$$

we obtain the system of equation in terms of generalized coordinates q_i. However we have to choose the displacement approximation first. In this paper we derive the bending vibrations of the shell by taking the displacement vector as

$$\mathbf{u} = \mathbf{U}(z,t) - \mathbf{U}'(z,t) \cdot x\mathbf{ik} + S(z,t)\left[x^2 - a^2(z)\right]\mathbf{j}. \tag{24}$$

The two first terms in (24) correspond to the elementary beam theory where deflection vector $\mathbf{U}$ is

$$\mathbf{U}(z,t) = U_x(z,t)\,\mathbf{i}_0 + U_y(z,t)\,\mathbf{j}_0, \tag{25}$$

cf. [11]. The third term in (24) determines the deformation of the cross-section in its plane. We supposed that the deformed cross-section takes a parabolic shape. The components of $\mathbf{u}$ according to the relations (10) are

$$\begin{aligned} u_x &= U_x(z,t)\cos(\Omega z) + U_y(z,t)\sin(\Omega z),\\ u_y &= U_y(z,t)\cos(\Omega z) - U_x(z,t)\sin(\Omega z) + S(z,t)\left[x^2 - a^2(z)\right],\\ u_z &= -x\left[U_x'(z,t)\cos(\Omega z) + U_y'(z,t)\sin(\Omega z)\right]. \end{aligned} \tag{26}$$

Now we can proceed to the discrete model by approximating the introduced functions U_x, U_y, S in accordance with the Ritz method:

$$\begin{aligned} U_x(z,t) &= \sum_{i=1}^{N} U_{xi}\,\Phi_i(z) = U_x^{\mathrm{T}}\,\Phi,\\ U_y(z,t) &= \sum_{i=1}^{N} U_{yi}\,\Phi_i(z) = U_y^{\mathrm{T}}\,\Phi,\\ S(z,t) &= \sum_{i=1}^{N} S_i\,\Psi_i(z) = S^{\mathrm{T}}\,\Psi. \end{aligned} \tag{27}$$

We have thus presented the generalized coordinates whose roles are played by functions U_{xi}, U_{yi}, S_i. Then Φ and Ψ in (28) are columns of the coordinate functions satisfying the boundary conditions. The "rigid support" in the root cross-section $z = 0$ corresponds to the following condition

$$\Phi(0) = \Phi'(0) = \Psi(0) = 0. \tag{28}$$

For instance, the power functions satisfy these conditions:

$$\Phi_1 = z^2,\ \Phi_2 = z^3,\ \Psi_1 = z,\ \Psi_2 = z^2,\ \ldots \tag{29}$$

The number of the coordinate functions is denoted by N in (24).
Now we can obtain the potential and kinetic energy. The area element is $\mathrm{d}o = \mathbf{r}_1 \times \mathbf{r}_2 = H\mathrm{d}z\mathrm{d}x$; then according to (8)

$$\Pi = 2\int_0^L\int_0^{a(z)} \widehat{\Pi} H\mathrm{d}z\mathrm{d}x,\ K = 2\int_0^L\int_0^{a(z)} \rho h\left({\dot{u}_x}^2 + {\dot{u}_y}^2 + {\dot{u}_z}^2\right) H\mathrm{d}z\mathrm{d}x, \qquad (30)$$

where L is the blade length.
The final equations for energies expressed in terms of the generalized coordinates are obtained by means of the Wolfram Mathematica. They are too cumbersome to be presented in the paper.
Now the Lagrange equations (23) are given by:

$$M\ddot{U} + CU = Q(t), \qquad (31)$$

in which the block columns and matrices are used.

$$M \equiv \begin{pmatrix} M_{xx} & M_{xy} & M_{xs} \\ M_{xy} & M_{yy} & M_{ys} \\ M_{xs} & M_{ys} & M_{ss} \end{pmatrix},\ U \equiv \begin{pmatrix} U_x \\ U_y \\ S \end{pmatrix},$$
$$C \equiv \begin{pmatrix} C_{xx} & C_{xy} & C_{xs} \\ C_{xy} & C_{yy} & C_{ys} \\ C_{xs} & C_{ys} & C_{ss} \end{pmatrix},\ Q(t) \equiv \begin{pmatrix} Q_x \\ Q_y \\ Q_s \end{pmatrix}. \qquad (32)$$

Here M and C are the global matrices of the inertia and stiffness, U and Q are the global columns of the unknowns and the generalized forces, respectively. The latter can be found from the expression for the virtual work of the external load.
In the case of the free harmonic oscillations we have a zero column in the right part of (31) and can replace $\ddot{U}$ by $-\omega^2 U$. The result is the generalized eigenvalue problem:

$$\left(C - \omega^2 M\right) U = 0 \qquad (33)$$

When the global matrices of inertia and stiffness are constructed we can use such special commands as *Eigenvalues* and *Eigenvectors* for solving (33) in the Wolfram Mathematica. In the result we can compute values of the first N normal frequencies.
We have analyzed the blade with the following parameters: $\Omega \approx 1.745$ (the tip cross-section is turned by a 30° angle against the root section), $h = 0.01$ m, $L = 0.3$ m, the width ranging from 0.4 m to 0.2 m. The material characteristics are: $\rho = 7800$ kg/m^3, $\nu = 0.3$, $E = 2 \cdot 10^{11}$ Pa.
For our calculation we took $N = 5$.
The results of the finite element analysis for the two first normal modes are shown in Fig. 3.
Values of the two first natural frequencies (Hz) are as follows:

	3D-model	shell model
f_1	109.2	101.5
f_2	552.5	498.3

The difference between the results of the two approaches is less than ≈ 10 %.

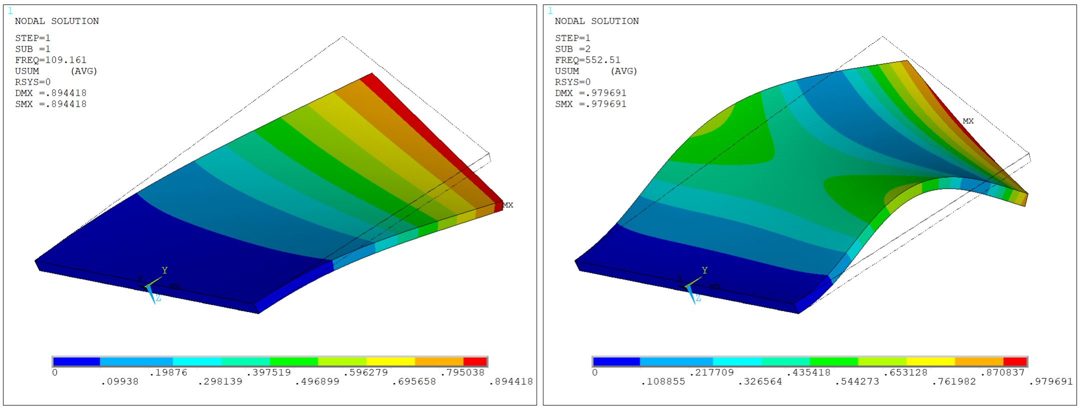

Figure 3: The first and second normal modes and their natural frequencies calculated by ANSYS

5 Conclusion

In this paper we presented a brief review of formalism of the Kirchhoff shells. The geometry of a blade is described as a helicoidal shell. The translational and rotational displacements were found from the tensor equations of the shell theory. The approximation for the displacement was considered as for the twisted beam with an additional term that takes into account the in-plane deformation of the cross-section. The displacement was approximated in terms of the chosen functions and the Ritz method was applied. It should be noted that the number of the coordinate functions N, as well as the type of these functions should be a subject for tests. For example, the results of the authors' analyses of the long blades' vibrations [12, 13] performed by using several different techniques showed that the increase in N may not necessarily lead to a more accurate result.

Having approximated the displacement functions, we proceeded to the discrete model and introduced the generalized coordinates. After that we derived the expression of potential and kinetic energies and then obtain the system of the Lagrange equations. The formulated generalized eigenvalue problem allows one to carry out the modal analysis for the blade. The problem was solved by means of special functions embedded in the Wolfram Mathematica.

We compared the results of our approach with the ANSYS finite element analysis of a 3D-model. The certain difference between these results could be explained by the properties of the coordinate functions. However, the actual choice of the type and number of coordinate functions is a topic of separate study which the authors intend to perform in the nearest future.

The presented algorithm allows us to analyze forced vibrations: the generalized forces can be derived from the expression for the virtual work. One can also consider the transient oscillations in the manner suggested in [12] for the long blades handled as rods.

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