Non-isothermal steady power-law fluid flow through an axisymmetric sudden contraction

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Abstract

In this work, the laminar steady power-law fluid flow in a pipe with sudden contraction is considered taking into account viscous dissipation, dependence of the rheological characteristics on the temperature, and constant temperature on the rigid wall. The flow is described by the motion, continuity, and energy equations using stream function, vorticity, and temperature variables. This system of equations is enclosed by the Ostwald de Waele power law with a temperature dependence of the consistency coefficient defined by exponential law. For numerical solving, the finite-difference method based on the alternative directions scheme is used. The difference equations are solved by the sweep method.

As a result of parametrical calculations, the flow kinematic characteristics were studied depending on the power-law index and dimensionless criteria. The effect of viscous dissipation on the flow structure was determined, and the temperature field was obtained in a wide range of basic parameters. The calculated results were verified and compared with available data.

1 Introduction

Laminar fluid flows in pipes with sudden contraction are frequently encountered in various technical applications. In particular, these same flows are realized in the industrial equipment applied for a polymer processing by casting method. Arranging of efficient manufacturing procedures requires both a detailed investigation of the flow structure, head and rate determining, and estimation of hydraulic resistance provided by processing line components. In general, polymeric fluid flows are characterized by non-Newtonian rheological behavior and non-isothermal properties caused by mechanical energy dissipation, chemical heat sources, and various heat-transferring boundary conditions. In such processes, physical properties of the medium are temperature-dependent.

Since the flows through sudden contraction find a widespread industrial application, they attract much attention of the researchers, at least from the middle of last century. The results of investigation on the laminar isothermal fluid flow through contracting geometries available at that time were considered in [1, 2]. These papers include a detailed discussion on the flow structure and kinematics, pressure losses

depending on the Reynolds number and contraction ration for Newtonian and non-Newtonian fluids. The more recent studies were carried out and reflected in [3, 4, 5, 6].
The number of significant difficulties appears when solving the problem of non-isothermal non-Newtonian fluid flow taking into account mechanical energy dissipation and temperature-dependent rheological parameters. Therefore, in most cases, the theoretical studies of the flow and heat transfer with varying physical characteristics of the fluid are implemented using approximate solution or numerical methods at simplifying assumptions.
One-dimensional problems on the steady non-isothermal viscous flow can be solved analytically. The pioneering works containing analytic solutions of such problems appeared in the middle of last century [7, 8, 9, 10]. The stability of obtained stationary solutions and a corresponding phenomenon referred to as a hydrodynamic thermal explosion are discussed in [11, 12]. The research results regarding considered problem are described in the following monographies [13, 14, 15]. Non-isothermal Newtonian and non-Newtonian fluid flows in sudden contractions are studied in [16, 17, 18].
The purpose of this work is to simulate numerically a steady non-isothermal power-law fluid flow in a pipe with sudden contraction in order to evaluate the effect of viscous dissipation on the kinematics of the process, flow structure, and temperature and apparent viscosity distributions.

2 Formulation of the Problem

The laminar steady power-law fluid flow through an axisymmetric sudden contraction under non-isothermal conditions is considered. The flow region is depicted in Fig. 1.

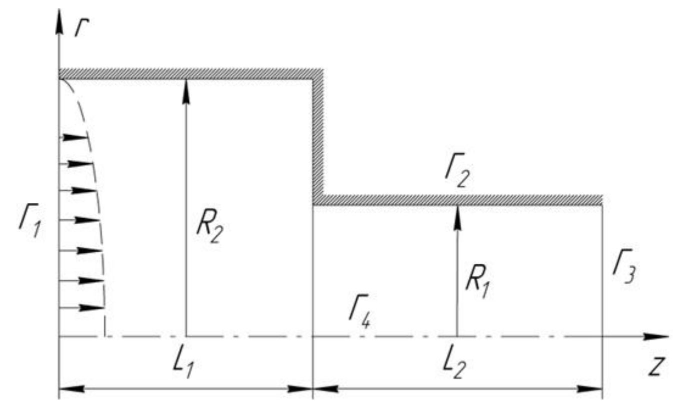

Figure 1: Flow region

The mathematical model of the flow includes the following dimensionless equations in terms of stream function, vorticity, and temperature [19]:

$$\frac{\partial(v\omega)}{\partial r} + \frac{\partial(u\omega)}{\partial z} = \frac{2^n \cdot B}{Re}\left(\Delta\omega - \frac{\omega}{r^2}\right) + \frac{2^n \cdot S}{Re}, \tag{1}$$

$$\Delta\psi - \frac{2}{r}\frac{\partial\psi}{\partial r} = -r\omega, \tag{2}$$

$$\frac{\partial(v\theta)}{\partial r} + \frac{\partial(u\theta)}{\partial z} = \frac{2}{Pe}\left(\Delta\theta + 2^{n-1}A^2B\cdot Br\right) - \frac{v\theta}{r}, \tag{3}$$

where, the source term (S) and the intensity of the rate of strain tensor (A) are given as

$$S = 2\frac{\partial^2 B}{\partial r\partial z}\left(\frac{\partial v}{\partial r} - \frac{\partial u}{\partial z}\right) + \left(\frac{\partial^2 B}{\partial z^2} - \frac{\partial^2 B}{\partial r^2}\right)\left(\frac{\partial u}{\partial r} + \frac{\partial v}{\partial z}\right) +$$

$$+2\frac{\partial B}{\partial z}\cdot\frac{\partial\omega}{\partial z} + 2\frac{\partial B}{\partial r}\cdot\frac{\partial\omega}{\partial r} + \frac{\partial B}{\partial r}\cdot\frac{\omega}{r},$$

$$A = \sqrt{2\left(\frac{\partial u}{\partial z}\right)^2 + 2\left(\frac{\partial v}{\partial r}\right)^2 + 2\left(\frac{v}{r}\right)^2 + \left(\frac{\partial u}{\partial r} + \frac{\partial v}{\partial z}\right)^2}.$$

The stream function (ψ) and vorticity (ω) are defined as follows:

$$v = -\frac{1}{r}\frac{\partial\psi}{\partial z},\ u = \frac{1}{r}\frac{\partial\psi}{\partial r}, \tag{4}$$

$$\omega = \frac{\partial v}{\partial z} - \frac{\partial u}{\partial r}. \tag{5}$$

The system of equations is completed by a rheological power law, which specifies temperature-dependent apparent viscosity by formula [19]:

$$B = \exp\left[-\theta\right]A^{n-1}, \tag{6}$$

where, n is the power-law index. For values of n between 0 and 1, the apparent viscosity is shear-thinning, for $n > 1$, the apparent viscosity is shear-thickening, and for $n = 1$, the Newtonian apparent viscosity model occurs.
In the equations stated above, v, u are the radial and axial velocity components, respectively, $\theta = \beta\left(T - T_1\right)$ is the dimensionless temperature, T and T_1 are the dimensional temperatures of the fluid in the flow and on a solid wall, respectively, and $D = 2R_1$ is the diameter of the downstream pipe. The typical scales for space, velocity, and apparent viscosity are the radius of the downstream pipe (R_1), the average velocity in the downstream pipe (U), and the value of $k_1\left(\frac{U}{R_1}\right)^{n-1}$, respectively.
The dimensionless Reynolds, Peclet, and Brinkman numbers are represented as

$$Re = \frac{\rho U^{2-n}D^n}{k_1},\ Pe = \frac{c\rho UD}{\lambda},\ Br = \frac{k_1 D^2\beta}{\lambda}\left(\frac{U}{D}\right)^{n+1}.$$

Here, $k_1 = k_0\exp\left[-\beta\left(T_1 - T_0\right)\right]$ is the consistency coefficient at T_1, k_0 is the consistency coefficient at T_0, β is the temperature dependency coefficient; ρ is the fluid density, c is the heat capacity, and λ is the thermal conductivity, which are considered to be constant.

At the inlet boundary (Γ_1), the velocity and temperature profiles are calculated corresponding to a fully developed one-dimensional non-isothermal fluid flow with a specified constant flow rate in the infinite pipe. Based on the obtained velocity profile, the stream function and vorticity are calculated according to Eqs.(4,5), and assigned as an inlet boundary conditions. On the rigid walls (Γ_2), the no-slip boundary conditions are realized, and the dimensionless temperature is set to zero. At the output boundary (Γ_3), the derivatives of stream function, vorticity, and temperature with respect to z are set to zero. The inlet and outlet boundaries are supposed to be remote from contraction plane to exclude the effect of the latter on the flow behavior in the vicinity of inlet and outlet sections. Along the axis of symmetry (Γ_4), the symmetry conditions are applied.
Consequently, the boundary conditions are written as follows:

$$\Gamma_1 : u = f_1(r), \quad \psi = \int_0^r urdr, \quad \omega = -\frac{\partial u}{\partial r}, \quad \theta = f_2(r), \quad z = 0, \quad 0 \le r \le \frac{R_2}{R_1};$$

$$\Gamma_2 : \psi = const, \quad \omega = -\frac{1}{r}\frac{\partial^2 \psi}{\partial r^2}, \quad \theta = 0, \quad r = \frac{R_2}{R_1}, \quad 0 \le z \le \frac{L_1}{R_1},$$

$$\psi = const, \quad \omega = -\frac{1}{r}\frac{\partial^2 \psi}{\partial z^2}, \quad \theta = 0, \quad 1 \le r \le \frac{R_2}{R_1}, \quad z = \frac{L_1}{R_1},$$

$$\psi = const, \quad \omega = -\frac{\partial^2 \psi}{\partial r^2}, \quad \theta = 0, \quad r = 1, \quad \frac{L_1}{R_1} \le z \le \frac{L_1}{R_1} + \frac{L_2}{R_1};$$

$$\Gamma_3 : \frac{\partial \psi}{\partial z} = 0, \quad \frac{\partial \omega}{\partial z} = 0, \quad \frac{\partial \theta}{\partial z} = 0, \quad z = \frac{L_1}{R_1} + \frac{L_2}{R_1};$$

$$\Gamma_4 : \psi = 0, \quad \omega = 0, \quad \frac{\partial \theta}{\partial r} = 0, \quad r = 0.$$

3 Method of Solution

During numerical simulation of the considered flow, an asymptotic time solution of the unsteady flow equations was obtained in order to yield a steady-state solution of the initial problem [20]. The finite-difference method based on the alternative directions scheme is used to implement the difference approximation for governing equations [21]. The obtained difference equations are solved by the sweep method [20].
A set of calculations was carried out on the sequence of square grids intended to validate the numerical algorithm and to verify the approximating convergence. Distributions of the axial velocity and temperature along contraction plane are presented at various grid steps (h) in Fig. 2. A resulting data analysis shows an approximating convergence of the method. The following calculations are implemented for a difference grid with h=0.025.

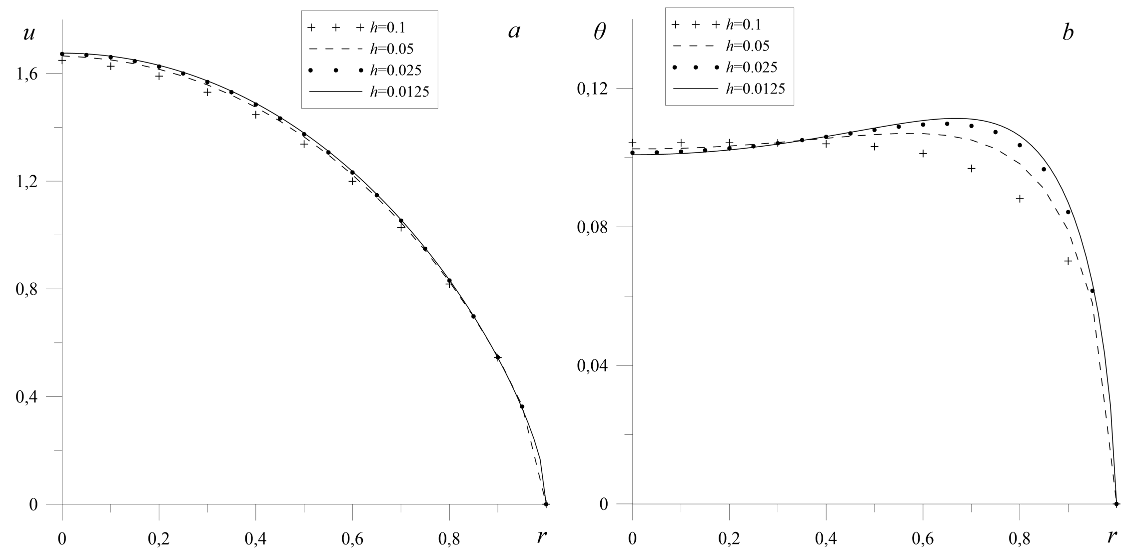

Figure 2: Distributions of the (a) velocity and (b) temperature along contraction plane at Re=1, Pe=100, n=0.8, and $\frac{R_2}{R_1}$=2

4 Results and Discussion

In the considered problem of the power-law fluid flow in a pipe with sudden contraction ($\frac{R_2}{R_1}$=2), the lengths of upstream and downstream pipes are assumed to be $\frac{L_1}{R_1}$=8 and $\frac{L_2}{R_1}$=40, respectively. Due to the fact that such flows are typically characterized by small Reynolds numbers, the value of Re is fixed at 1 in this work. The Brinkman number was stated to be Br=1, providing an existence of stable stationary solution. In general, the flow field observed in a pipe with sudden contraction consists of three distinct flow regions in both isothermal and non-isothermal cases. The first is a one-dimensional flow zone appearing near the inlet section, which is referred to as a fully developed flow and is recognized by the parallel to the wall streamlines (Fig. 3). The second is a two-dimensional flow zone occurring upstream and downstream of contraction plane including recirculating region at the corner. This zone is characterized by distortion of the streamlines towards the centerline as the contraction plane is approached. The extension of this region is strongly dependent upon the governing parameters such as Reynolds number, Peclet number, power-law index, and contraction ratio. As the downstream pipe is entered, the flow tends to reach the fully developed flow conditions, where the third (one-dimensional) zone appears.

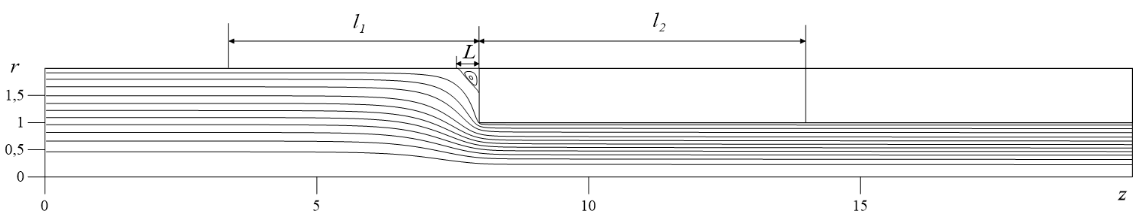

Figure 3: Non-isothermal flow pattern (Pe=100, n=0.8)

Based on the obtained flow pattern, the non-dimensional geometrical characteristics are imposed in order to implement a quantitative analysis of the flow. These are the length of recirculating flow region (L), and the length of upstream (l_1) and downstream (l_2) two-dimensional flow zones (Fig. 3). Since the flow structure is strongly dependent upon rheology and temperature, it is of major interest to consider both isothermal and non-isothermal flow formation at varying power-law index. The results of parametric study revealing the effect of power-law index on the lengths of two-dimensional flow zones at various thermal conditions are presented in Table 1.

Table 10: The lengths of two-dimensional flow zones versus power-law index

	isothermal flow			non-isothermal flow (Pe=10)			non-isothermal flow (Pe=100)		
n	0.8	1.0	1.2	0.8	1.0	1.2	0.8	1.0	1.2
l_1	3.85	3.225	2.75	3.975	3.375	2.875	3.875	3.25	2.775
l_2	1.325	1.025	0.875	4.275	4.725	5.025	12.775	16.225	19.025
L	0.3811	0.4854	0.5955	0.3470	0.4162	0.4876	0.3963	0.4501	0.5589

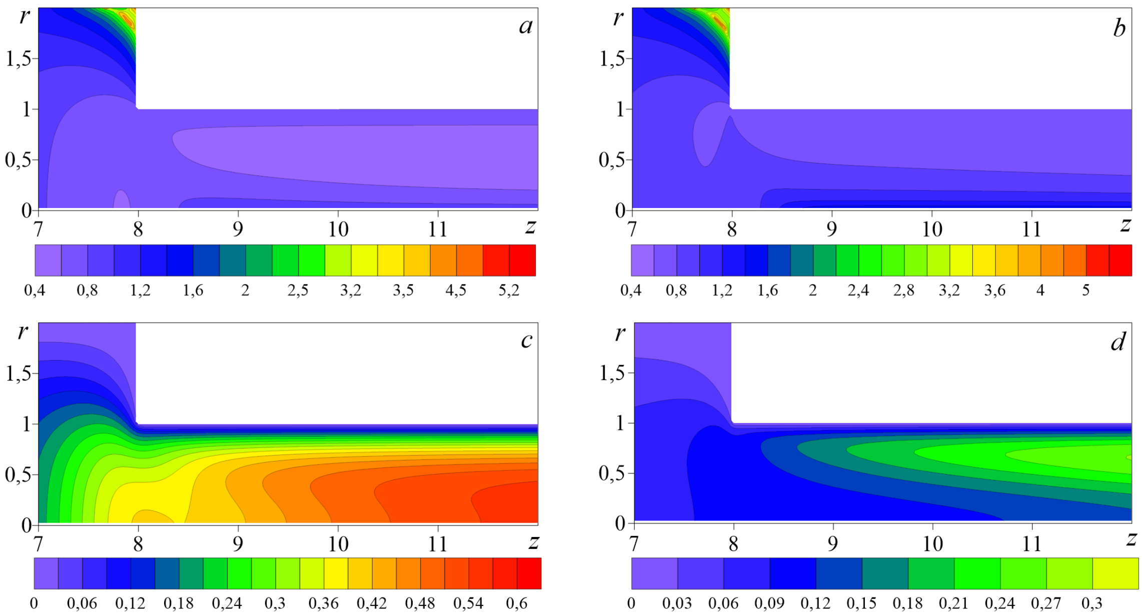

Figure 4: Distribution of the apparent viscosity (a,b) and temperature (c,d) in the vicinity of contraction plane at n=0.8: (a,c) Pe=10 and (b,d) Pe=100

Comparing isothermal and non-isothermal flows, the tendencies for l_1 and L are found to be similar, i.e. increasing power-law index leads to a decrease in the length of upstream two-dimensional flow zone and to an increase in recirculating flow region. However, behavior of the downstream two-dimensional flow zone is significantly affected by non-isothermality. The values of l_2 obtained for isothermal flow are different from those for non-isothermal flow: in the first case, l_2 is found to

decrease in size with increasing power-law index, and in the second case, an increase in l_2 is observed.
Figures 4,5 demonstrate the effect of power-law index and Peclet number on the apparent viscosity and temperature distribution in the immediate vicinity of contraction plane. It is evident that increase in Pe provides changes in the apparent viscosity and temperature fields, and those are more significant for the latter. An increased heat transfer due to convection (as compared to conduction) displaces the heated region towards outlet section and changes the temperature maximum in the flow field. Thus, the fully developed temperature distribution is reached at a certain distance from contraction plane, and this distance increases with higher Peclet number.

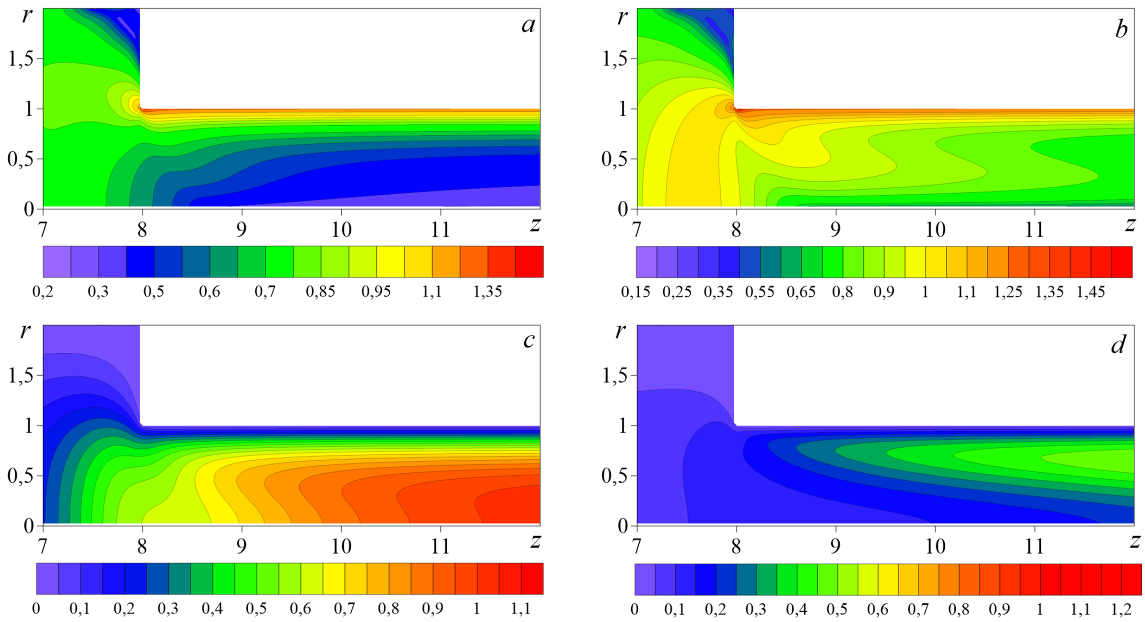

Figure 5: Distribution of the apparent viscosity (a,b) and temperature (c,d) in the vicinity of contraction plane at n=1.2: (a,c) Pe=10 and (b,d) Pe=100

5 Conclusion

A steady power-law fluid flow through an axisymmetric sudden contraction under non-isothermal conditions was numerically simulated using the finite-difference method. The flow kinematic characteristics such as the lengths of two-dimensional flow regions were obtained for shear-thinning, Newtonian, and shear thickening fluids, and compared with those determined in the isothermal case. It was revealed that the accounted heat generation due to viscous dissipation had little effect on the length of both recirculating region and upstream two-dimensional flow zone, and, in contrast, it provided a significant increase in the length of downstream two-dimensional flow zone, which enhanced with higher power-law index.
A parametric study was implemented in order to evaluate the impact of rheology and thermo-physical properties on the apparent viscosity distribution and heat transfer

behavior. An increase in the Peclet number was found to extend a distance from contraction plane to the region where the fully developed temperature distribution occurred.

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