Numerical simulation of non-Newtonian fluid flow in a T-shaped channel under the given pressure boundary conditions

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Abstract

The planar steady-state flow of non-Newtonian incompressible fluid in a T-shaped channel is considered. The motion of the fluid is caused by a given pressure difference between inlet/outlet boundaries. The flow is described by momentum and continuity equations written in dimensionless variables. On the solid walls, no slip boundary conditions are assigned. The viscosity of non-Newtonian fluid is determined by the Ostwald-de Waele power law. The problem is solved numerically using the finite difference method based on the SIMPLE procedure. The parametric studies of the flow kinematics depending on the pressure values given at the inlet/outlet boundaries have been performed. The typical flow regimes characterized by redistribution and reversal of the fluid flow have been found. The effect of main parameters on the kinematic and dynamic characteristics has been estimated.

1 Introduction

Pipelines networks using for transportation of fluids and gases consist of branched or connected elements. One such element is a T-shaped channel. The fluid flow in a T-channel is characterized by separation of the flow into two parts. In engineering practice, it is essential to understand the main characteristics of the flow in separating and reattaching flows [1, 2].

Nowadays, a large number of investigations of the flows of both Newtonian [3, 4, 5, 6, 7] and non-Newtonian [1, 2, 8, 9, 10] fluids with given flow rate at the boundaries of a T-shaped channel were carried out. There are a few works in which values of the pressure are given at the boundaries of a T-shaped channel. Among these results we find the works [11, 12] where numerical simulation of the flow of a Newtonian incompressible fluid in channels of complex geometry including fluid flow in a T-channel was performed.

The primary purpose of this work is to investigate characteristics of the flow of a power-law fluid in a T-shaped channel under the given pressure difference between boundary sections.

2 Problem Formulation

The planar steady-state flow of a non-Newtonian incompressible fluid in a T-channel is investigated. The flow region is limited by solid walls *MKF*, *EDC*, *AB* (Fig.1). The fluid flow is driven by pressure difference between boundary sections *AM*, *FE*, *BC* of the T-shaped channel. Mathematical problem statement includes momentum and continuity equations which in dimensionless vector form are written as follows:

$$(\mathbf{U} \cdot \nabla)\,\mathbf{U} = -\nabla p + \nabla \cdot (2\eta \mathbf{E})\,, \tag{1}$$

$$\nabla \cdot \mathbf{U} = 0. \tag{2}$$

Here, U is dimensionless velocity vector with components (u,v) in the Cartesian coordinate system $(x,\ y)$, p is dimensionless pressure, E is the strain rate tensor.

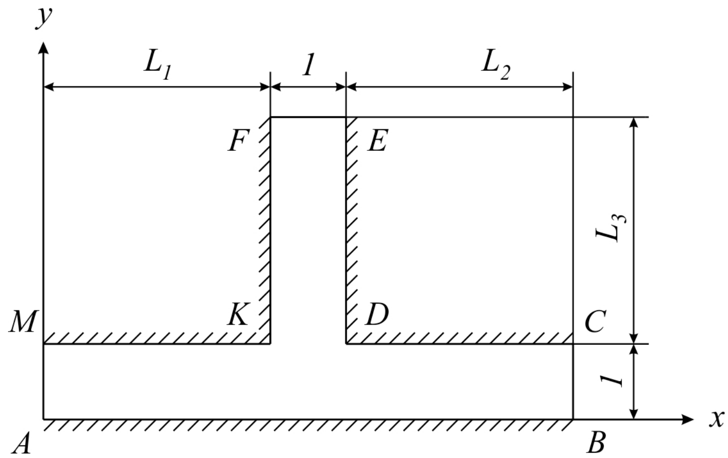

Figure 1: Flow region

The viscosity of non-Newtonian fluid is determined by the Ostwald-de Waele power law [13]:

$$\eta = (A)^{n-1}, \tag{3}$$

where $A = \sqrt{2\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2 + 2\left(\frac{\partial v}{\partial y}\right)^2}$ is dimensionless intensity of the strain rate tensor, n is the power-law index. Needless to say that the model describes the rheology of Newtonian fluids at $n=1$.

To scale the length and the velocity, L (the width of the boundary section *AM*) and $U_0 = \left(\frac{k}{\rho L^n}\right)^{\frac{1}{2-n}}$ are used, respectively. Dimensionless pressure is prescribed by following expression:

$$p = (P - P_{FE}) \,/ \left(\frac{k^2}{\rho^n L^{2n}}\right)^{\frac{1}{2-n}},$$

where k is the power-law consistency index, ρ is the fluid density, P is dimension pressure, P_{FE} is dimension pressure in the cross-section *FE*.

In the through-flow sections AM, FE, and BC, zero tangential components of the velocity vector and values of the pressure are specified

$$v = 0,\ p_{AM} = p_1,\ x = 0,\ 0 \leq y \leq 1$$
$$u = 0,\ p_{FE} = 0,\ L_1 \leq x \leq L_1 + 1,\ y = L_3 + 1 \tag{4}$$
$$v = 0,\ p_{BC} = p_3,\ x = L_1 + L_2 + 1,\ 0 \leq y \leq 1$$

On the solid walls, the no slip boundary conditions hold

$$\mathbf{U} = 0. \tag{5}$$

The problem solution is reduced to finding both the velocity and pressure fields which satisfy Eqs. (1)-(3) with given boundary conditions (4)-(5).

3 Numerical Method and Validation

The problem is solved numerically. An asymptotic time solution of the unsteady flow equation is used to obtain steady-state velocity and pressure fields [20]. Such method of solution assumes the addition of time derivative of the function U in Eq.(1). The obtained system is discretized by the finite difference method based on the SIMPLE procedure [15]; rectangular staggered grid is used.
The rheological model for shear-thinning fluid ($n < 1$) has peculiarity of "infinite" viscosity, as $A \to 0$. To ensure the stability and accuracy of calculations in the regions of small values of A, the modified model of the rheological equation is used [16-17]. According to this model, the viscosity is determined by expression

$$\eta = (A + \varepsilon)^{n-1},$$

where ε is the regularization parameter. The approximate convergence of the method of calculating with using regularized rheological model is presented in [16, 5].

4 Results and Discussion

The flow characteristics of the problem are depending on geometric sizes of the channel and three parameters: the power-law index (n) and values of the pressure given at boundaries AM and BC, respectively, (p_1 and p_3). In present work, all calculations have been performed in the T-shaped channel with branches of the same width equal to one dimensionless unit and the same length L_1=L_2=L_3=3 (Fig. 1). Investigation of the flow characteristics depending on the parameters p_1 and p_3 at n=0.8 has been carried out.
In Fig. 2, distribution of the flow characteristics at p_1=-300 and p_3=-400 are presented. The fluid flow enters through the boundary section FE. The inlet flow divides into two parts in the vicinity of the junction of the branches and leaves the channel through the boundary sections AM and BC. The planar-parallel flow of the non-Newtonian fluid with the fully developed velocity profile occurs near the

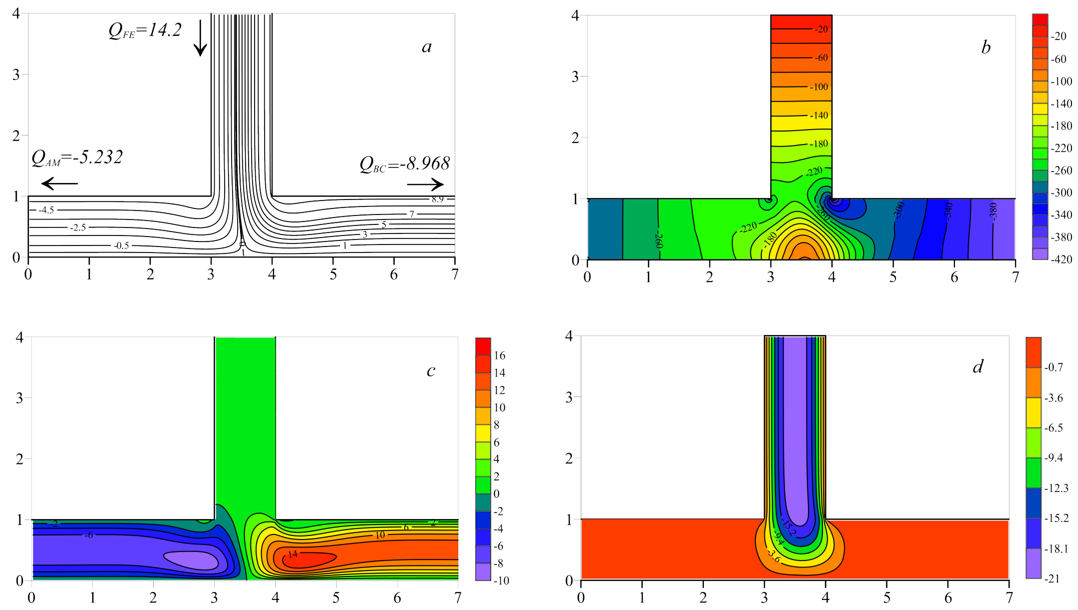

Figure 2: Distribution of the flow characteristics at n=0.8, p_1=-300 and p_3=-400 (a — the stream function contours, b — the pressure field, c — the field of velocity u, d — the field of velocity v)

through-flow sections AM, FE, and BC. Transient regions of the flow appear in the vicinity of the sections with corner points K and D.

The Reynolds Number is imposed to analyse the results and use the similarity theory as follows:

$$\mathrm{Re} = \frac{\rho {U_{avg}}^{2-n} L^n}{k}.$$

Here, U_{avg} is the average velocity in the cross section of the channel which is characterized by maximum flow rate. For case plotted in Fig. 2, Re $= |Q_{FE}|^{2-n} =$ 24.1.

The research has been carried out over the range of values of the pressure -2000$\leq$ p_1, p_3 $\leq$2000. Four characteristic flow regimes have been determined for this range of main parameters. Regime I (Fig. 3a) corresponds to the case describing above. The fluid flow enters through the boundary section FE and leaves the channel through the boundary sections AM and BC. This regime is observed when values of the pressure given in the boundary sections AM and BC are less than the value of the pressure in the section FE. The increase of the pressure in the through-flow section AM, (p_1 >0), leads to reversal of the flow in the branch containing the through-flow section AM if all other parameters remaining equal (Regime II). The fluid flow enters through two boundary sections AM and FE. After confluence of entering flows, the fluid leaves the channel through the boundary section BC (Fig. 3b). As the parameter p_1 is further increased, the fluid flow changes the direction in the branch containing the through-flow section FE (Regime III). The fluid flow entering through the boundary section AM divides into two parts in the vicinity of the junction of the branches and leaves the channel through the boundary sections FE and BC (Fig. 3c). Regime IV (Fig. 3d) corresponds to the case when two fluid flows entering through the

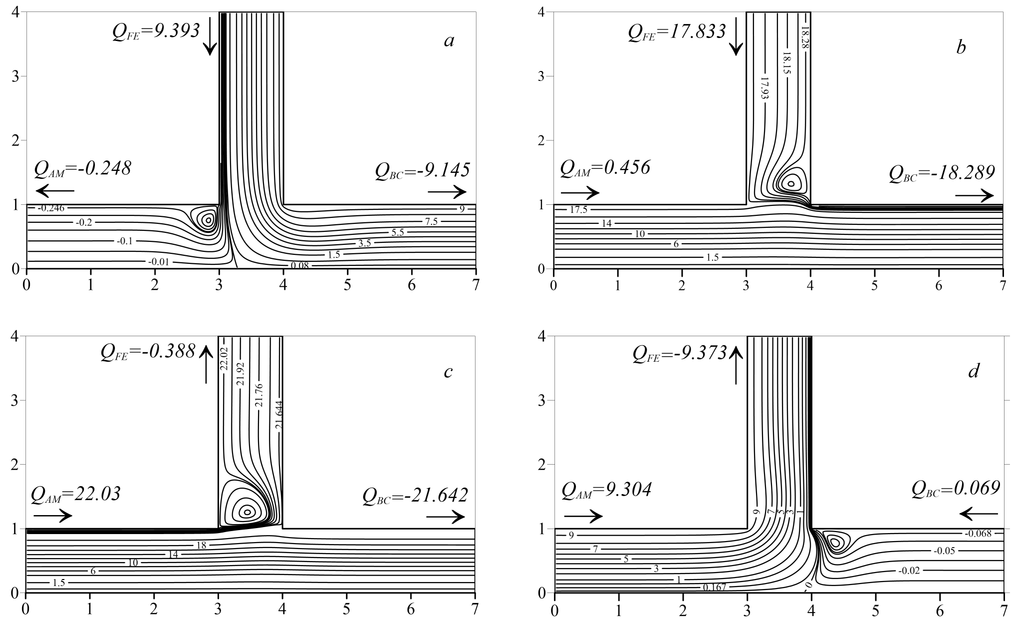

Figure 3: Flow regimes at n=0.8 (a — p_1=-160 and p_3=-400, b — p_1=250 and p_3=-400, c — p_1=320 and p_3=-400, c — p_1=400 and p_3=280)

boundary sections AM and BC merge into one in the vicinity of the the junction of the branches and run out through the boundary sections FE. This regime is observed for positive values of p_1 and p_3.

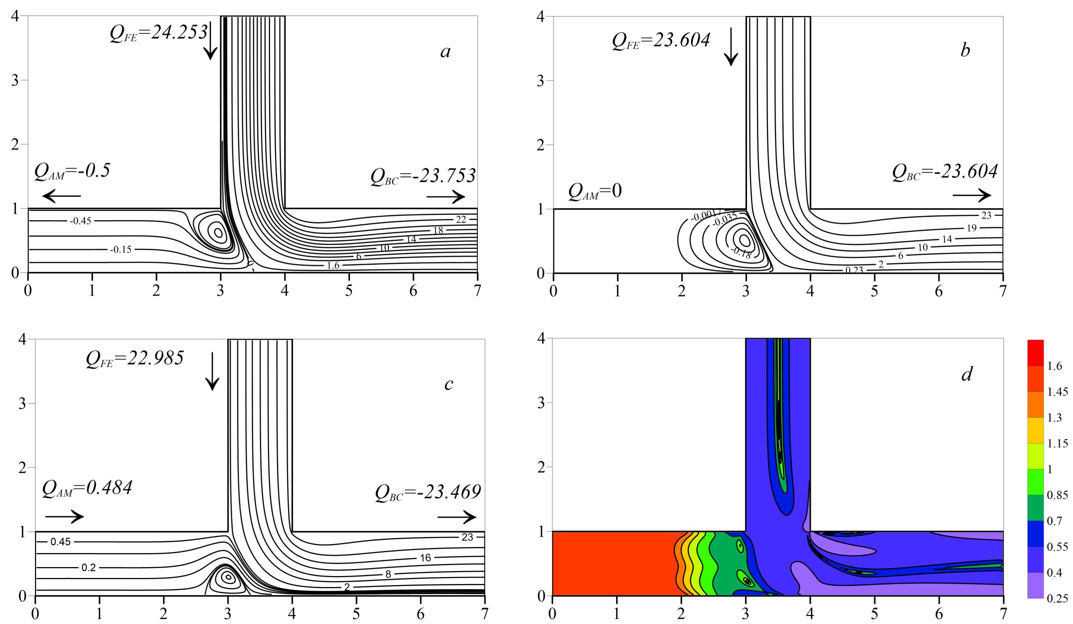

Figure 4: Distribution of the streamlines (a,b,c) and the viscosity (d) at different values of p_1 (n=0.8, p_3=-1000, a — p_1=-230, b,d — p_1=-214.95, c — p_1=-200)

Fig. 4 demonstrates change of the flow kinematics during transition from regime I to regime II at critical pressure $p_1 = p_{crit}$ and fixed value p_3 =-1000. Regime I without recirculation zone in the flow of the fluid is observed at $p_1 \ll p_{crit}$. The recirculation zone in the vicinity of the corner point K appears with increasing parameter p_1 (Fig. 4a). As the pressure p_1 is further enhanced, sizes of formed recirculation zone become larger and, subsequently, reach its maximum at $p_1 = p_{crit}$ =-214.95 (Fig. 4b). Thus, the recirculation zone closes the cross-section, and the flow rate through the boundary section AM attains zero. The viscosity field for this case is plotted in Fig. 4d. It can be seen that the apparent viscosity of the fluid is maximum in the branch containing the cross section AM. Regime II is observed at $p_1 > p_{crit}$ (Fig. 4c). The recirculation zone turns around, decreases and shifts to the solid wall AB with further increase of the parameter p_1. Similarly, the change of other regimes occurs; and the recirculation zone appears in the branch of the T-shaped channel in which the reorientation of the flow happens.

5 Conclusions

The planar flow of the power-law incompressible fluid in the T-channel has been studied. The fluid flow is driven by pressure difference between boundary sections AM, FE, and BC of the T-shaped channel. On the solid walls, the no slip boundary conditions have been used.
Investigation of the flow characteristics depending on values of the pressure given in the through-flow sections AM and BC (-2000$\leq p_1, p_3 \leq$2000) has been carried out. The range of change of these parameters has been chosen so that the planar-parallel flow of the non-Newtonian fluid with the fully developed velocity profile has been realized in the vicinity of the through-flow sections AM, FE, and BC.
As a result of the parametric studies, four regimes of the flow have been determined for this range of parameters p_1 and p_3. Estimation of the influence of main parameters on the flow pattern has been performed. Characteristics of the flow for these regimes have been presented. The results describing transition from one regime to another have been demonstrated.

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