On axial movement and transverse vibrations of layered thin-walled membrane-plate structures and the problems of stability

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Abstract

This study concentrated on stability analysis and optimization of thermoelastic web travelling between two rollers. It is presented a model for a layered travelling web (continuous layered panel composed from isotropic materials) restricting the consideration on one open draw. The web is mechanically simply supported at the inflow and outflow ends of the span with the rest boundaries of the span unsupported. The considered part of the layered web is effectively isotropic, homogeneous and occupies the domain having a rectangular shape in plan. The web is symmetrically composed with respect to a middle plane and it is consisted of thermoelastic layers characterized by some important parameters (mass per unit area, Young modulus, Poisson ratio and distances from the middle plane). The movement of layered membrane-plate structure with constant axial velocity is considered [1]. Various mechanical and temperature actions and characteristic properties of the moving media are taken into account [2], [8]. Transverse vibrations arising in the process of axial movement are supposed to be small. The loss of stability of thin-walled thermoelastic plate-like moving structures is studied in the static form (divergence) and the stability domain is determined in the space of basic considered parameters.

1 Introduction

In this paper we present mechanical models for a homogeneous (continuous, isotropic) and for a layered (effectively isotropic and homogeneous) travelling webs, restricting the consideration to one open draw. The webs are mechanically simply. supported at the inflow ($x = 0$) and outflow ($x = l$) ends of the span. The webs are travelling at a constant velocity V_0 in the x-direction of the rectangular global coordinate system xz and are loaded by axial tension T_0 and thermal loads. The length l and the total thickness H are supposed to be given, while $0 < x < l$ and $-H/2 < z < H/2$. For given problem parameters we study stability problems and derive the expressions for critical temperature and critical web velocity. As a result we find safety domain of stability.

2 Homogeneous thermoelastic web

Free transverse vibrations of homogeneous web axially moving with constant velocity and loaded by axial tension and heated by some temperature are described by the following equation for transverse displacement w and simply supported boundary conditions

$$m\left(\frac{\partial^2 w}{\partial t^2}+2V_0\frac{\partial^2 w}{\partial x\partial t}+V_0^2\frac{\partial^2 w}{\partial x^2}\right)=\left(T-\frac{EH}{1-\nu}\varepsilon_\theta\right)\frac{\partial^2 w}{\partial x^2}-D\frac{\partial^4 w}{\partial x^4}, \tag{1}$$

$$(w)_{x=0}=0,\quad \left(\frac{\partial^2 w}{\partial x^2}\right)_{x=0}=0,\quad (w)_{x=l}=0,\quad \left(\frac{\partial^2 w}{\partial x^2}\right)_{x=l}=0, \tag{2}$$

where m, E, ν, D are, respectively, the mass per unit area, Young's modular, Poisson ratio, bending rigidity ($D=EH^3/12(1-\nu^2)$) and the deformation ε_θ is defined as

$$\varepsilon_\theta=\alpha_\theta\theta,\quad \theta=\theta_a-\theta_0. \tag{3}$$

Here α_θ is the linear expansion coefficient, θ is the temperature discrepancy, θ_0 is the temperature of zero deformation, θ_a is the actual temperature.
In the stationary case, when

$$\frac{\partial w}{\partial t}=\frac{\partial^2 w}{\partial t^2}=0, \tag{4}$$

the transverse displacement $w=w(x)$ satisfies the equation

$$\frac{d^4 w}{dx^4}+\lambda\frac{d^2 w}{dx^2}=0, \tag{5}$$

where parameter λ (eigenvalue) is given by the expression

$$\lambda=\frac{1}{D}\left(mV_0^2+\frac{EH}{1-\nu}\alpha_\theta\theta-T_0\right)\equiv f\left(V_0^2,\theta\right). \tag{6}$$

If we introduce new unknown variable $\psi(x)$ as

$$\psi(x)=\frac{d^2 w}{dx^2},\quad 0\le x\le l, \tag{7}$$

we obtain the following spectral problem

$$\frac{d^2\psi}{dx^2}+\lambda\psi=0, \tag{8}$$

$$\psi(0)=0,\quad \psi(l)=0. \tag{9}$$

Here the value λ plays the role of an eigenvalue. Nontrivial solution of the formulated eigenvalue problem can be represented as

$$\psi(x)=C_1\sin\left(\sqrt{\lambda}x\right)+C_2\cos\left(\sqrt{\lambda}x\right) \tag{10}$$

with two arbitrary coefficients C_1, C_2 and unknown value λ. Taking into account (9), (10) we will have $C_2 = 0$ and

$$\lambda = \left(\frac{j\pi}{l}\right)^2, \quad j = 1, 2, ..., \tag{11}$$

$$\psi(x) = C_1 sin\left(\frac{j\pi x}{l}\right) \tag{12}$$

with arbitrary constant C_1.
Thus, for given problem parameters D, E, ν, H, l, T_0, m, V_0, α_θ we obtain the critical temperature θ^{div} of instability (divergence or buckling)

$$\theta^{div} = \frac{1-\nu}{EH\alpha_\theta}\left[D\left(\frac{\pi}{l}\right)^2 + T_0 - mV_0^2\right] \tag{13}$$

and

$$\lambda_{min} = \left(\frac{\pi}{l}\right)^2 \tag{14}$$

corresponding the minimal $j = 1$ in (11). Analogously we find the critical instability velocity (squared) $(V_0^2)^{div}$ as

$$\left(V_0^2\right)^{div} = \frac{1}{m}\left[D\left(\frac{\pi}{l}\right)^2 + T_0 - mV_0^2\right], \tag{15}$$

where D, E, ν, H, l, T_0, m, θ, α_θ are considered as a given positive parameters.
Safety domain for stability in the values (θ, V_0^2) is defined by the inequality

$$f\left(V_0^2, \theta\right) < \lambda_{min} = \left(\frac{\pi}{l}\right)^2 \tag{16}$$

that is reduced to the condition

$$F\left(V_0^2, \theta\right) \equiv \frac{1}{D}\left(\frac{l}{\pi}\right)^2 f\left(V_0^2, \theta\right) = C_V V_0^2 + C_\theta \theta - C_0 < 0, \tag{17}$$

where

$$C_V = \frac{m}{D}\left(\frac{l}{\pi}\right)^2, \quad C_\theta = \frac{EH\alpha_\theta}{D(1-\nu)}\left(\frac{l}{\pi}\right)^2, \quad C_0 = \frac{T_0}{D}\left(\frac{l}{\pi}\right)^2 + 1.$$

Safety domain of the values V_0^2, θ has a triangular shape OAB shown in Fig. 1

3 Layered thermoelastic web

Consider the layered web that is symmetrically composed with respect to a middle plane (Fig. 2) and consisted of $2n+1$ (odd number) thermoelastic layers characterized by mass per unit area m_i, Young's modulus E_i, Poisson's ratio ν_i, coefficient

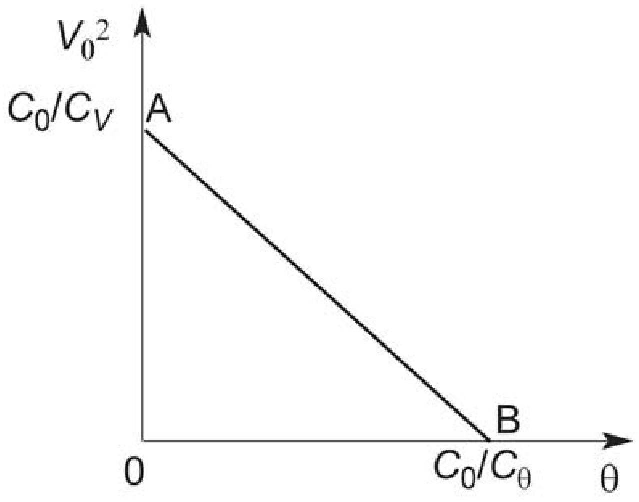

Figure 1: Safety domain OAB

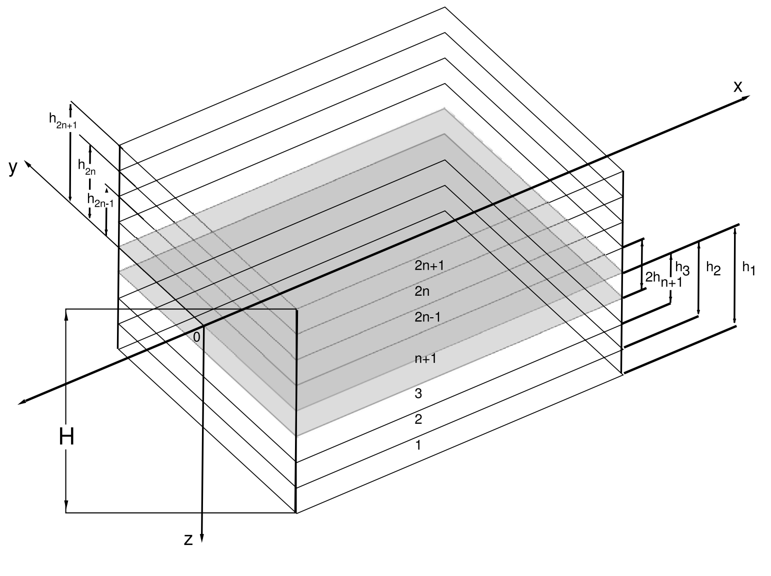

Figure 2: Layered web

$(\alpha_\theta)_i$, and distances h_i from the middle plane. We will take into account the symmetry of internal web structure, i.e.

$$E(z) = E(-z), \quad \nu(z) = \nu(-z), \quad \alpha_\theta(z) = \alpha_\theta(-z) \tag{18}$$

and derive the expressions for effective moduli D^{ef}, ν^{ef}, ε_θ^{ef} and m^{ef}. To this end we apply the formulas for stresses and strains and the expression for bending moment

$$\int_{-H/2}^{H/2} \sigma_x z dz = \left[2 \int_0^{H/2} \frac{z^2 E(z) dz}{1 - (\nu(z))^2} \right] \frac{d^2 w}{dx^2} = D^{ef} \frac{d^2 w}{dx^2}. \tag{19}$$

Thus we find the expression for effective bending rigidity in the form

$$D^{ef} = 2\int_0^{H/2} \frac{z^2 E(z) dz}{1 - (\nu(z))^2}. \tag{20}$$

Using mechanical and geometric characteristics of the web layers E_i, ν_i, h_i we evaluate the integral in (20). We will have the following formula

$$D^{ef} = \frac{2}{3}\frac{E_{n+1}}{1-\nu_{n=1}^2}h_{n+1}^3 + \frac{2}{3}\sum_{i=1}^{n}\frac{E_i}{1-\nu_i^2}\left(h_i^3 - h_{i+1}^3\right). \tag{21}$$

In analogous manner we derive the formulas for effective Poisson's ratio ν^{ef} and for effective thermal deformation ε_θ^{ef} of nonhomogeneous isotropic layered web. We have

$$\nu^{ef} = \frac{2}{D^{ef}}\int_0^{H/2}\frac{z^2\nu(z)E(z)}{1-(\nu(z))^2}dz = \frac{2}{3D^{ef}}\left[\frac{\nu_{n+1}E_{n+1}h_{n+1}^3}{1-\nu_{n+1}^2} + \sum_{i=1}^{n}\frac{E_i\nu_i}{1-\nu_i^2}\left(h_i^3 - h_{i+1}^3\right)\right], \tag{22}$$

$$\varepsilon_\theta^{ef} = \frac{2}{H}\int_0^{H/2}\alpha_\theta(z)\theta dz = \frac{2}{H}\left[(\alpha_\theta)_{n+1}\theta_{n+1}h_{n+1} + \sum_{i=1}^{n}(\alpha_\theta)_i\theta_i(h_i - h_{i+1})\right], \tag{23}$$

if $\theta_1 = \theta_2 = ... = \theta_{n+1} = \theta$ then

$$\varepsilon_\theta^{ef} = \frac{2\theta}{H}\left[(\alpha_\theta)_{n+1}h_{n+1} + \sum_{i=1}^{n}(\alpha_\theta)_i(h_i - h_{i+1})\right], \tag{24}$$

$$m^{ef} = m_{n+1} + 2\sum_{i=1}^{n} m_i. \tag{25}$$

If we define

$$a = \frac{EH}{1-\nu}\varepsilon_\theta, \quad a = a(z) = \frac{E(z)H}{1-\nu(z)}\varepsilon_\theta(z) = \frac{HE(z)}{1-\nu(z)}\alpha_\theta(z)\theta(z),$$

then

$$a^{ef} = 2\int_0^{H/2}\frac{E(z)\alpha_\theta(z)\theta(z)}{1-\nu(z)}dz = 2\left[\frac{(\alpha_\theta)_{n+1}E_{n+1}\theta_{n+1}}{1-\nu_{n+1}}h_{n+1} + \sum_{i=1}^{n}\frac{(\alpha_\theta)_iE_i\theta_i}{1-\nu_i}(h_i - h_{i+1})\right]. \tag{26}$$

If the temperature of each layer are the same, then

$$a^{ef} = 2\theta\left[\frac{(\alpha_\theta)_{n+1}E_{n+1}h_{n+1}}{1-\nu_{n+1}} + \sum_{i=1}^{n}\frac{(\alpha_\theta)_iE_i(h_i - h_{i+1})}{1-\nu_i}\right]. \tag{27}$$

In this case the domain for stability in the values (θ, V_0^2) is defined analogy (17)

$$F\left(V_0^2, \theta\right) \equiv \frac{1}{D^{ef}}\left(\frac{l}{\pi}\right)^2 f\left(V_0^2, \theta\right) = C_V^{ef}V_0^2 + C_\theta^{ef}\theta - C_0^{ef} < 0,$$

where

$$C_V^{ef} = \frac{m^{ef}}{D}\left(\frac{l}{\pi}\right)^2, \quad C_0^{ef} = \frac{T_0}{D^{ef}}\left(\frac{l}{\pi}\right)^2 + 1.$$

$$C_\theta^{ef} = \frac{2}{D^{ef}}\left(\frac{l}{\pi}\right)^2 \left[\frac{(\alpha_\theta)_{n+1}E_{n+1}h_{n+1}}{1-\nu_{n+1}} + \sum_{i=1}^{n}\frac{(\alpha_\theta)_i E_i (h_i - h_{i+1})}{1-\nu_i}\right].$$

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References

[1] Nikolay Banichuk, Juha Jeronen, Pekka Neittaanmaki, Tytti Saksa, Tero Tuovinen. Mechanics of Moving Materials. 2014. Springer International Publishing Switzerland.

[2] Kovalenko, A.D. Thermoelasticity. 1975. Kiev: Vishcha Shkola. 216p. (in Russian).

[3] Timoshenko, S.P., Woinowsky-Kriger, S. Theory of plates and shells, 2nd edn. 1959. New York: Tokyo: McGraw-Hill.

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[1] Nikolay Banichuk, Juha Jeronen, Pekka Neittaanmaki, Tytti Saksa, Tero Tuovinen. Mechanics of Moving Materials. Springer International Publishing Switzerland. 2014. 253p. [2] Timoshenko, S.P., Woinowsky-Kriger, S. Theory of plates and shells, 2nd edn. New York: Tokyo: McGraw-Hill. 1959. 595p. [3] Kovalenko, A.D. Thermoelasticity. Kiev: Vishcha Shkola.1975. 216p. (in Russian).