2011 50th IEEE Conference on Decision and Control and
European Control Conference (CDC-ECC)
Orlando, FL, USA, December 12-15, 2011

Decentralized Adaptive Controller for Synchronization of Dynamical Networks With Delays And Bounded Disturbances

Alexander L. Fradkov, Grigoriy Grigoriev and Anton Selivanov

***Abstract*— An adaptive master-slave output feedback synchronization problem is studied firstly for a network of interconnected nonlinear dynamical systems with bounded disturbance and then for a network of systems with delayed couplings. The proposed structure of decentralized controller and adaptation algorithms in both cases is based on speed-gradient and passification methods. Synchronization conditions for systems with disturbances and for systems with delayed couplings are established. An example of synchronization of the network of Chua systems with bounded disturbances is given. The problem of convergence with prespecified accuracy is examined for the networks of dynamical systems with disturbances.** [1]

I. Introduction

An enormous interest is observed recently in adaptive control of networks. There were several researches made in this area during recent years, such as [3], [4], [9], [15]. It is motivated by a broad area of potential applications: formation control, cooperative control, control of power networks, communication networks, production networks, etc. However, only a restricted class of the problems in this area is currently solved. For example, existing papers are dealing mostly with disturbance-free systems [4], [9] or availability of the whole state vector for measurement as well as appearance of control in all equations for all nodes is assumed [3], [4], [15]. Nowadays a lot of new problems arise, such as taking into account uncertainties and a switching structure of the network topology, delayed couplings and disturbances. For the networks with delayed couplings some results have already been presented in [11]–[15]. However, in existing papers only a narrow class of networks, such as fully-controlled and fully-measured networks (e.g. [11], [15]), is examined, in [15] the algorithm is not decentralized. Some of these works and many others deal with systems with non-switching topology or provide non-adaptive control.

In the present work we overcome these restrictions and propose an adaptive decentralized algorithm for synchronization of networks of dynamic systems with delayed couplings, switching topology and bounded disturbances. These results are based on the speed-gradient method [1], [5] and passification theorem [6]. Also in this paper the problem of convergence with preliminarily specified accuracy is studied for systems with bounded disturbances. In contrast to the disturbance-free case, convergence of trajectories of the agents in this case is not possible. To avoid instability of the overall system adaptation algorithms are regularized by means of negative parametric feedback, similarly to [10]. The conditions, ensuring achievement of the control goal, are given and proven. The results are illustrated by example for a network of chaotic Chua circuits.

[1]A.L. Fradkov is with Institute of Problems in Mechanical Engineering, Russian Academy of Sciences, 61 Bolshoy ave V.O., St. Petersburg, 199178, Russia fradkov@mail.ru G. Grigoriev and A. Selivanov are with the Department of Theoretical Cybernetics, St.Petersburg State University Universitetsky prospekt, 28, Peterhof,198504, St. Petersburg, Russia {Grigoriy.Grigoriev,antonselivanov}@gmail.com

The work was supported by the Russian Foundation for Basic Research (RFBR), grant 11-08-01218; by the program of basic research of OEMPPU RAS #2 "Control and safety in energy and technical systems"; by the Russian Federal Program "Cadres", contracts 16.740.11.0042, 14.740.11.0942.

II. Problem statement

Consider a set of dynamical systems S consisting of d interconnected subsystems (agents) S_i, $i = 1, \dots, d$. Let S_i be described as follows:

$$\dot{x}_i = Ax_i + Bu_i + \varphi_0(x_i) + \sum_{j=1}^{d} \alpha_{ij}(t)\varphi_{ij}(x_i - x_j) + \sum_{j=1}^{N} \beta_{ij}(t)x_j(t-\tau) + f_i(t),$$
$$y_i = C^{\mathrm{T}} x_i \quad (1)$$

where $\tau > 0$ is the time delay, $x_i \in \mathbb{R}^n$, $u_i \in \mathbb{R}^1$, $\alpha_{ij}(t), \beta_{ij}(t) \colon [0,\infty) \to \mathbb{R}^1$ - piecewise continuous functions, $y_i \in \mathbb{R}^l$. Functions $\varphi_{ij}(\cdot), i = 1, \dots, d, j = 1, \dots, d$ are used to describe interaction among the subsystems, $f_i(t)$ is some bounded disturbance acting on subsystem S_i:

$$||f_i(t)|| \leqslant d_{f_i}, \quad (2)$$

and nonlinear part is presented by continuously differentiable function $\varphi_0(x,t) \colon \mathbb{R}^n \times [0,\infty) \to \mathbb{R}^n$. We will also consider isolated subsystems (agents) without interconnections and their local dynamics.

Suppose that $\varphi_{ii}(0) = 0, a_{ii} = 0, i = 1, \dots, d$. Also, we assume that A,B,C and $\varphi_0(\cdot)$ are well-determined, and φ_{ij}, $i = 1, \dots, d, j = 1, \dots, d$ depend on a vector of unknown parameters $\xi \in \Xi$, where Ξ is known.

To formulate the control goal the leading subsystem (leader) is introduced as follows

$$\dot{\overline{x}} = A\overline{x} + B\overline{u(t)} + \varphi_0(\overline{x}), \overline{y} = C^{\mathrm{T}}\overline{x}, \quad (3)$$

where input function $\overline{u(t)}$ is given.

III. Networks with disturbances

Firstly consider the special case of the networks without delays in couplings, but in presence of bounded disturbances:

$$\dot{x}_i = Ax_i + Bu_i + \varphi_0(x_i) + \sum_{j=1}^{d} \alpha_{ij}(t)\varphi_{ij}(x_i - x_j) + f_i(t), y_i = C^{\mathrm{T}} x_i, i = 1, \ldots, d. \quad (4)$$

In this case the control goal is to achieve convergence of trajectories of all agents to some neighborhood of the trajectory of the leader:

$$\overline{\lim_{t\to\infty}} |x_i(t) - \overline{x}(t)| \leqslant \Delta_i \quad (5)$$

The problem is to find decentralized control $u_i(t) = \mathfrak{U}_i(y_i, t)$, $i = 1, \ldots, d$ ensuring (5) for all possible values of the vector of unknown parameters.

A. Control synthesis

Denote $z_i = x_i - \overline{x}$, $\tilde{u}_i = u_i - \overline{u}$. Dynamics of z_i can be described as follows:

$$\dot{z}_i = Az_i + B\tilde{u}_i + \varphi_0(x_i) - \varphi_0(\overline{x}) + f_i(t) + \sum_{j=1}^{d} \alpha_{ij}\varphi_{ij}(x_i - x_j),\ \tilde{y}_i = C^{\mathrm{T}} z_i, i = 1, \ldots, d. \quad (6)$$

We take linear control of the slave subsystem in the following form:

$$\tilde{u}_i = \theta_i^{\mathrm{T}}(t)\tilde{y}_i,\ \theta_i(t) \in \mathbb{R}^l,\ i = 1, \ldots, d, \quad (7)$$

where $\theta_i(t)$ are vectors of adjustable parameters. For determining $\theta_i(t)$, the speed gradient method [1], [5] is used consisting basically in reformulating the goal by means of a goal function Q, evaluating the speed $\dot{Q}$ of its changing along trajectories of (6) and adjusting adaptation parameters proportionally to the gradient of $\dot{Q}$ in u. Specify the following goal function:

$$Q(z_i) = \frac{1}{2} z_i^{\mathrm{T}} H z_i,\ H = H^{\mathrm{T}} > 0. \quad (8)$$

Applying the speed gradient procedure and passification theorem (see Lemma 1 below) we arrive at the adaptation law

$$\dot{\theta}_i(t) = -g^{\mathrm{T}}\tilde{y}_i(t)\Gamma_i\tilde{y}_i(t),$$

However this algorithm does not take into account the disturbance. Using ideas of [1], [2] introduce adaptation law with the deadzone:

$$\dot{\theta}_i(t) = \begin{cases} -g^{\mathrm{T}}\tilde{y}_i(t)\Gamma_i\tilde{y}_i(t),\ Q_i(x_i(t), t) > \Delta_i \\ 0\ ,\ Q_i(x_i(t), t) \leqslant \Delta_i. \end{cases}, \quad (9)$$

where $\Gamma_i = \Gamma_i^{\mathrm{T}} > 0$ - are matrices of size $l \times l$

B. Lipschitz-type nonlinearities

We need the definition of hyper-minimum-phase systems [1], [5], [6]

Definition 1: Let $W(S) = \beta(s)/\alpha(s)$ be a rational function, $\beta(s)$, $\alpha(s)$ are polynomials with real coefficients. $W(s)$ is *minimum-phase*, if its numerator $\beta(s)$ is a Hurwitz polynomial. $W(s)$ is called *hyper-minimum-phase* if it is minimum-phase and the number $\lim\limits_{s\to+\infty} sW(s)$ is positive.

To prove our further result we will need the following lemma, that is a modification of Lemma 2 from [9].

Lemma 1: For existence of $H = H^* > 0$ and θ_*, such that $HA_* + A_*^{\mathrm{T}}H < 0$, $HB = Cg$, where $A_* = (A + LI_n) + B\theta_*^{\mathrm{T}}C^{\mathrm{T}}$, it is necessary and sufficient that function $g^{\mathrm{T}}C^{\mathrm{T}}(sI_n - A - LI_n)^{-1}B$ is hyper-minimum-phase.

Further narration requires several assumptions.

Assumption 1: Consider real matrices $H = H^{\mathrm{T}} > 0$, g, θ_* of orders $n \times n$; $l \times 1$; $l \times 1$ respectively, and number $\rho > 0$ such that the following relations hold:

$$HA_* + A_*^{\mathrm{T}}H < -\rho H, HB = Cg, A_* = (A + LI_n) + B\theta_*^{\mathrm{T}}C^{\mathrm{T}}.$$

For some $g \in \mathbb{R}^l$ function $g^{\mathrm{T}}\chi(s - L)$ is hyper-minimum-phase, where $\chi(s)$ is the transfer function of (4), $\chi(s) = C^{\mathrm{T}}(sI_n - A)^{-1}B$. Then there exist matrices $H = H^{\mathrm{T}} > 0$, θ_* of sizes $n \times n$, $l \times 1$ respectively and a number $\rho > 0$, such that

$$HA_* + A_*^{\mathrm{T}}H < -\rho H, HB = Cg, A_* = (A + LI_n) + B\theta_*^{\mathrm{T}}C^{\mathrm{T}} \quad (10)$$

Assumption 2: $\varphi_0(\cdot)$ and $\varphi_{ij}(\cdot)$, $i = 1, \ldots, d$, $j = 1, \ldots, d$ are globally Lipschitz functions with respect to x:

$$||\varphi_0(x, t) - \varphi_0(x', t)|| \leqslant L||x - x'||,\ L > 0,$$

$$||\varphi_{ij}(x) - \varphi_{ij}(x')|| \leqslant L_{ij}||x - x'||,\ L_{ij} > 0.$$

Let $\lambda_* = \lambda_{max}(H)/\lambda_{min}(H)$ be condition number of matrix H, where $\lambda_{max}(H), \lambda_{min}(H)$ are maximum and minimum eigenvalues of matrix H respectively.

Theorem 1: Let assumptions 1 and 2 hold. Denote $\delta_i = \frac{\rho}{2}\frac{\lambda_{min}(H)}{\lambda_{max}(H)} - \sum_{j=1}^{d} |\alpha_{ij}L_{ij}|$. If for any $i = 1, \ldots, d$ the following condition is fulfilled:

$$\delta_i > 0, \quad (11)$$

then for any $i = 1, \ldots, d$ adaptive control (7), (9) provides achievement of the control goal (5) for all Δ_i, that satisfy:

$$\Delta_i > \frac{d_{f_i}\lambda_{max}(H)}{2\rho\delta_i}, i = 1, \ldots, d \quad (12)$$

meanwhile the vector of adjustable parameters θ is bounded for all solutions of the closed-loop system (4), (3), (7), (9).

Proof. To prove this theorem we need an auxiliary lemma. This lemma is a modification of the theorem 2.19 from [2].

Lemma 2: Consider system that consists of N interconnected subsystems, where each one is described as:

$$\dot{x}_i = F_i(x_i, \theta_i, t) + h_i(x, \theta, t), \quad i = 1, \ldots, K, \quad (13)$$

$$\dot{\theta}_i(t) = \begin{cases} -\Gamma_i\nabla_{\theta_i}\omega_i(x_i, \theta_i, t),\ Q_i(x_i(t), t) > \Delta_i \\ 0\ ,\ Q_i(x_i(t), t) \leqslant \Delta_i. \end{cases} \quad (14)$$

where $x_i \in \mathbb{R}^{n_i}, \theta_i \in \mathbb{R}^{m_i}$,

$$\omega_i(x_i, \theta_i, t) = \frac{\partial Q_i}{\partial t} + \nabla Q_i(x_i, t)^{\mathrm{T}} F_i(x_i, \theta_i, t),$$

here $Q_i(\cdot)$ - is some objective function, $N = \sum n_i$, $m = \sum m_i$, $x = col(x_1, \dots, x_l) \in \mathbb{R}^N$. Assume that for (13) the following groups of conditions hold:

1) Functions $F_i(\cdot)$ are continuous with respect to x_i and t_i, are continuously differentiable with respect to θ_i and locally bounded in time $t > 0$; functions $\omega_i(x_i, \theta_i, t)$ are convex by θ_i; there exist vectors $\theta_i^* \in \mathbb{R}^{m_i}$ and scalar continuous growing functions $k_i(Q)$, $\rho_i(Q)$ such that $k_i(0) = \rho_i(0) = 0$, $k_i(Q) \to +\infty$ and $\rho_i(Q) \to \infty$ when $Q \to +\infty$.

$$\omega_i(x_i, \theta_i^*, t) \leqslant -\rho_i(Q_i(x_i, t)) \quad (15)$$

and

$$Q_i(x_i, t) \geqslant k_i(||x_i - x_i^*(t)||),$$

where $x_i^* = argmin_{x_i}(Q_i(x_i, t))$ and $Q_i(x_i^*(t), t) \equiv 0$

2) Functions $h_i(x, \theta, t)$ are continuous and the following inequalities hold

$$|\nabla_{x_i} Q_i(x_i, t)^{\mathrm{T}} h_i(x, \theta, t)| \leqslant \sum_{j=1}^{l} \mu_{ij} \rho_j(Q_j(x_j, t)) + d_i \quad (16)$$

where $M - I$ is Hurwitz matrix, $M = \{\mu_{ij}\}$, $\mu_{ij} \geqslant 0$, I is identity matrix, $d_i > 0$, and Δ_i in (14) satisfy the inequalities:

$$\rho_i(\Delta_i) > r_i, \quad (17)$$

where $r = (I - M)^{-1} d, r = col(r_1, \dots, r_l), d = col(d_1, \dots, d_l)$.

Then all trajectories of the system (13), (14) are bounded and the control goal

$$\overline{\lim_{t \to \infty}} Q_i(x_i(t), t) \leqslant \Delta_i, \ i = 1, \dots, l$$

is achieved.

Consider the first group of conditions of Lemma 2. Local boundedness for $t > 0$ is met, because for any $i = 1, \dots, d$ right-hand side of the system (6) and $Q(z_i)$ are continuous functions, not depending from t, and $f(t)$ is bounded. Convexity condition is satisfied because right-hand side of $\dot{Q}_i$ is linear by Q_i. Let's take function $Q \to \rho \cdot Q$ as $\rho_i(\cdot)$, $i = 1, \dots, d$, from Lemma 2. It can be shown that existence of $\theta_* \in \mathbb{R}^l$ and ρ, such that $\omega_i(z_i, \theta_*) \leqslant -\rho Q(z_i)$, is provided by hyper-minimum-phase restriction for function $g^{\mathrm{T}} \chi(s)$. Indeed, according to the Lemma 1 if $g^{\mathrm{T}} \chi(s)$ is hyper-minimum-phase then exist $H = H^{\mathrm{T}} > 0$ and θ_* such that.

$$HA_* + A_*^{\mathrm{T}} H < 0 \ HB = Cg,$$

where $A_* = (A + LI_n) + B\theta_*^{\mathrm{T}} C^{\mathrm{T}}$.

$$F_i = Az_i + B\tilde{u}_i + \varphi_0(x_i) - \varphi_0(\overline{x})$$

Taking derivative of Q_i due to error equation for the ith node (6), it can be shown that:

$$\dot{Q}_i = \omega_i(z_i, \theta_*) \leqslant z_i^{\mathrm{T}} H(A + B\theta_i^{\mathrm{T}} C^{\mathrm{T}}) z_i + \\ + ||z_i^{\mathrm{T}}|| \cdot ||H|| \cdot L \cdot ||z_i|| \leqslant z_i^{\mathrm{T}} H(A + B\theta_i^{\mathrm{T}} C^{\mathrm{T}}) z_i + \\ + L \frac{\lambda_{max}(H)}{\lambda_{min}(H)} z_i^{\mathrm{T}} H z_i = \frac{1}{2} z_i^{\mathrm{T}} [HA_* + A_*^{\mathrm{T}} H] z_i, \quad (18)$$

where $A_* = (A + LI_n) + B\theta_*^{\mathrm{T}} C^{\mathrm{T}}$. Negativity of $HA_* + A_*^{\mathrm{T}} H$ implies existence of $\rho > 0$, such that $HA_* + A_*^{\mathrm{T}} H \leqslant -\rho H$, and therefore the condition:

$$\omega_i(z_i, \theta_*) \leqslant -\rho Q(z_i), \ i = 1, \dots, d$$

is met. Consider the conditions on connections between the systems (second group of conditions in Lemma 2). In our particular case they can be written down as:

$$|z_i^{\mathrm{T}} H[\sum_{j=1}^{d} \alpha_{ij} \varphi_{ij}(z_i - z_j) + f_i(t)]| \leqslant \\ \leqslant \frac{\rho}{2} \sum_{j=1}^{d} \mu_{ij} z_j^{\mathrm{T}} H z_j + d_i, \ i = 1, \dots, d \quad (19)$$

where $M - I$ is Hurwitz matrix, $M = \{\mu_{ij}\}$, $\mu_{ij} \geqslant 0$, I is identity matrix. The left-hand side of (19) can be estimated as follows:

$$\left| z_i^{\mathrm{T}} H[\sum_{j=1}^{d} \alpha_{ij} \varphi_{ij}(z_i - z_j) + f_i(t)] \right| \leqslant \\ \leqslant \sum_{j=1}^{d} |\alpha_{ij} L_{ij}| \cdot \lambda_{max}(H) \cdot (||z_i||^2 + ||z_i|| \cdot ||z_j||) + \\ + \frac{1}{2} \sigma_i ||z_i||^2 \lambda_{max}(H) + \frac{d_{f_i}}{2\sigma_i} \lambda_{max}(H), \ i = 1, \dots, d,$$

where $\sigma_i > 0$, $i = 1, \dots, d$, are some numbers. It can be shown that the lower bound of the right-hand side of (19) is:

$$\frac{\rho}{2} \sum_{j=1}^{d} \mu_{ij} z_j^{\mathrm{T}} H z_j + d_i \geqslant \\ \geqslant \frac{\rho}{2} \sum_{j=1}^{d} \mu_{ij} \lambda_{min}(H) ||z_j||^2 + d_i, \ i = 1, \dots, d$$

Thereby, it is sufficient to demand fulfilment of the following inequalities:

$$\begin{cases} \frac{d_{f_i}}{2\sigma_i} \lambda_{max}(H) \leqslant d_i \\ \sum_{j=1}^{d} |\alpha_{ij} L_{ij}| \cdot (||z_i||^2 + ||z_i|| \cdot ||z_j||) + \\ + \frac{1}{2} \sigma_i ||z_i||^2 \leqslant \frac{\rho}{2} \frac{\lambda_{min}(H)}{\lambda_{max}(H)} \sum_{j=1}^{d} \mu_{ij} ||z_j||^2, \end{cases} \quad (20)$$

where $i = 1, \dots, d$. Consider the following notations: $\mathbf{z} = col(||z_1||, ||z_2||, \dots, ||z_3||)$, $\nu_i^{(1)}$, $\nu_i^{(2)}$, η_i are described as follows:

$$\nu_i^{(1)} = \begin{pmatrix} 0 & \dots & 0 & \dots & 0 \\ \vdots & \ddots & \vdots & & \vdots \\ 0 & \dots & \sum_{j=1}^d |\alpha_{ij} L_{ij}| + \frac{1}{2}\sigma_i & \dots & 0 \\ \vdots & & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & \dots & 0 \end{pmatrix}, \tag{21}$$

here the non-zero element is on the main diagonal in the i-th row. Assuming that $\alpha_{ii} = 0$ for all $i = 1, \dots, d$, we denote $\nu_i^{(2)}$ as:

$$\nu_i^{(2)} = \begin{pmatrix} 0 & \dots & 0 & \dots & 0 \\ \vdots & \ddots & \vdots & & \vdots \\ |\alpha_{i1} L_{i1}| & \dots & 0 & \dots & |\alpha_{id} L_{id}| \\ \vdots & & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & \dots & 0 \end{pmatrix}.$$

Where non-zero elements are in the i-th row, and the zero is on the main diagonal. Choose η_i as follows:

$$\eta_i = \frac{\rho}{2} \frac{\lambda_{min}(H)}{\lambda_{max}(H)} \begin{pmatrix} \mu_{i1} & 0 & \dots & 0 \\ 0 & \mu_{i2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \mu_{id} \end{pmatrix}.$$

Using this notations we can state that for fulfilment of (19) it is sufficient that for any $i = 1, \dots, d$ the following inequality is true:

$$\mathbf{z}^{\mathrm{T}}(\nu_i^{(1)} + \nu_i^{(2)}) \leqslant \mathbf{z}^{\mathrm{T}} \eta_i \mathbf{z}, \tag{22}$$

i.e. matrix $\eta_i - \nu_i^{(1)} - \nu_i^{(2)}$ for any $i = 1, \dots, d$ should be positively defined. Choose the following diagonal matrix as $M = \{\mu_{ij}\}$:

$$0 < \mu_{ii} < 1, \mu_{ij} = 0, i \neq j, i = 1, \dots, d, j = 1, \dots, d.$$

Apparently $M - I$ is Hurwitz matrix. Non-negative determinacy of $\eta_i - \nu_i^{(1)} - \nu_i^{(2)}$ implies that for $i = 1, \dots, d$:

$$\mu_{ii} \geqslant \left(\frac{\rho}{2} \frac{\lambda_{min}(H)}{\lambda_{max}(H)}\right)^{-1} \left(\sum_{j=1}^d |\alpha_{ij} L_{ij}| + \frac{1}{2}\sigma_i\right). \tag{23}$$

Owing to $\mu_{ii} < 1$, we obtain the condition (11). Now, consider (17), together with (23). Assume $\delta_i = \frac{\rho}{2} \frac{\lambda_{min}(H)}{\lambda_{max}(H)} - \sum_{j=1}^d |\alpha_{ij} L_{ij}|$. The (20) (23) can be rewritten as

$$\begin{cases} \frac{d_{f_i} \lambda_{max}(H)}{2\delta_i} \leqslant d_i < \rho \Delta_i, \\ \delta_i > 0, \end{cases} \tag{24}$$

which matches the conditions of the Theorem. Thereby, we can apply Lemma 2, which proves the Theorem.□

Condition (12) of the Theorem 1 says that the higher the disturbance level is, the bigger is the size of the limit set of the network.

Remark 1: Let ρ_* denote degree of stability of nominator of the function $g^{\mathrm{T}}\chi(s - L)$. Using results of [6] it can be shown that if function $g^{\mathrm{T}}\chi(s - L)$ is hyper-minimum-phase, then as θ_* and ρ in (10) we can take $\theta_* = \kappa g$ and any $\rho : 0 < \rho < \rho_*$, where $\kappa > 0$ is sufficiently big number. Thereby, inequality (11) can be replaced with

$$\sum_{j=1}^d |\alpha_{ij} L_{ij}| < \gamma, \tag{25}$$

where $\gamma = \frac{\rho_*}{2\lambda_*}$.

C. Matched nonlinearity

Let us study the case when $\varphi_0(x_i) = B\psi_0(y_i)$, $\psi_0 : \mathbb{R}^l \to \mathbb{R}^1$. Then the subsystem (4) can be rewritten as

$$\dot{x_i} = Ax_i + B(u_i + \psi_0(y_i)) + \sum_{j=1}^d \alpha_{ij} \varphi_{ij}(x_i - x_j) + f_i(t),$$
$$y_i = C^{\mathrm{T}} x_i, \tag{26}$$

and the leading system can be rewritten in the following form:

$$\dot{\overline{x}} = A\overline{x} + B(\overline{u} + \psi_0(\overline{y})),\ \overline{y} = C^{\mathrm{T}} \overline{x}, \tag{27}$$

where $\overline{u} \in \mathbb{R}^1$ - is the given control which is assumed to be known. For formulating the ongoing results, we need the following definition:

Definition 2: Let $G \in \mathbb{R}^l$. Function $f : \mathbb{R}^l \to \mathbb{R}^1$ is called *G-monotonically decreasing*, if for any $x, y \in \mathbb{R}^l$ the following inequality is true: $(x - y)^{\mathrm{T}} G(f(x) - f(y)) \leqslant 0$.

Assumption 3: φ_{ij} are globally Lipschitz functions, with constants $L_{ij} > 0$, and $\psi_0(\cdot)$ is such, that existence and uniqueness of solutions of all the subsystems.

We can choose (7),(9) again as the control input. Consider real matrices $H = H^{\mathrm{T}} > 0$, g, θ_* of sizes $n \times n$, $l \times 1$, $l \times 1$ respectively, and number $\rho > 0$ such that:

$$HA_* + A_*^{\mathrm{T}} H < -\rho H,\ HB = Cg,\ A_* = A + B\theta_*^{\mathrm{T}} C^{\mathrm{T}}. \tag{28}$$

Note that this is equivalent to (10) with $L = 0$.

Theorem 2: Suppose that for every $\xi \in \Xi$ assumption 3 is true, and assumption 1 with $L = 0$ holds for the system (26). Also assume that function $\psi_0(\cdot)$ is g-monotonically decreasing. Denote $\delta_i = \frac{\rho}{2} \frac{\lambda_{min}(H)}{\lambda_{max}(H)} - \sum_{j=1}^d |\alpha_{ij} L_{ij}|$. If for all $i = 1 \dots d$ the following condition holds:

$$\sum_{j=1}^d |\alpha_{ij} L_{ij}| < \gamma, \tag{29}$$

where $\gamma = \rho_*/(2\lambda_*)$, λ_* - condition number of matrix H, ρ_* - degree if stability of nominator of $g^{\mathrm{T}}\chi(s)$. Then for all $i = 1, \dots, d$ adaptive control (7),(9) provides fulfilment of the control goal:

$$\overline{\lim_{t \to \infty}} |x_i(t) - \overline{x}(t)| \leqslant \Delta_i, \tag{30}$$

for all Δ_i, that satisfy the following inequality:

$$\Delta_i > \frac{d_{f_i} \lambda_{max}(H)}{2\rho\delta_i},$$

meanwhile the vector of adjustable parameters θ_i remains bounded on $[0, \infty)$ for all solutions of the closed loop system (7),(9),(26),(27).

Proof. The proof of this theorem is similar to the proof of Theorem 1. To prove it one should apply lemmas 1 and 2 with $L = 0$.

To this end prove that $\omega(z_i, \theta_*) \leqslant -\rho Q(z_i)$ for $i = 1, \dots, d$.

Using previously made assumptions it can be shown that:

$$\begin{aligned}\omega_i(z_i, \theta_*) &= z_i^{\mathrm{T}} H[Az_i + B(\tilde{u}_i + \psi_0(y_i) - \psi_0(\overline{y}))] = \\ &= z_i^{\mathrm{T}} H(A + B\theta_i^{\mathrm{T}} C^{\mathrm{T}})z_i + (y_i - \overline{y})^{\mathrm{T}} g(\psi_0(y_i) - \psi_0(\overline{y})) \leqslant \\ &\leqslant z_i^{\mathrm{T}} H(A + B\theta_i^{\mathrm{T}} C^{\mathrm{T}})z_i, \quad (31)\end{aligned}$$

The latter inequality holds since $\psi(\cdot)$ is g-monotonically decreasing. Then,

$$\begin{aligned}\omega_i(z_i, \theta_*) &\leqslant z_i^{\mathrm{T}} H[A + B\theta_*^{\mathrm{T}} C^{\mathrm{T}}]z_i = \\ &= \frac{1}{2} z_i^{\mathrm{T}}[HA_* + A_*^{\mathrm{T}} H]z_i, i = 1, \dots, d.\end{aligned}$$

Here $A_* = A + B\theta_*^{\mathrm{T}} C^{\mathrm{T}}$. Since $HA_* + A_*^{\mathrm{T}} H$ is negatively determined, exists such $\rho > 0$, that $HA_* + A_*^{\mathrm{T}} H \leqslant -\rho H$, and that provides fulfilment of the following inequality

$$\omega_i(z_i, \theta_*) \leqslant -\rho Q(z_i),\ i = 1, \dots, d.$$

Repeating further the proof of the Theorem 1 and taking into account Remark 1, ends the proof. □

IV. Networks with delayed couplings

Now study the case when the agents are not influenced by disturbances, but there are delayed couplings instead:

$$\begin{aligned}\dot{x}_i &= Ax_i + bu_i + \varphi_0(x_i, t) + \sum_{j=1}^{N} \alpha_{ij}(t)x_j + \\ &\sum_{j=1}^{N} \beta_{ij}(t)x_j(t - \tau), \\ y_i(t) &= C^{\mathrm{T}} x_i(t), \qquad i = 1, \dots, d.\end{aligned} \tag{32}$$

Let matrices $\alpha(t) = (\alpha_{ij}(t))$ and $\beta(t) = (\beta_{ij}(t))$ satisfy the following conditions:

1) If the i-th node is connected with the j-th node ($i \neq j$) at time $t \geq 0$, then $\alpha_{ij}(t) > 0, \beta_{ij}(t) > 0$;
2) If the i-th node is not connected with the j-th node ($i \neq j$) at time $t \geq 0$, then $\alpha_{ij}(t) = 0, \beta_{ij}(t) = 0$;
3) $\alpha_{ii}(t) = -\sum_{j=1, j\neq i}^{N} \alpha_{ij}(t), \quad \beta_{ii}(t) = -\sum_{j=1, j\neq i}^{N} \beta_{ij}(t) \quad \forall i = 1, \dots, N \quad \forall t \geq 0$.

Let $C([-\tau, 0], \mathbb{R}^n)$ be the Banach space of continuous functions mapping the interval $[-\tau, 0]$ into $\mathbb{R}^n$ with the norm $\|\phi\|_C = \sup_{-\tau \leq z \leq 0} \|\phi(z)\|$. Initial conditions of the system (32) are given by functions $\varphi_i(t) \in C([-\tau, 0], \mathbb{R}^n)$: $x_i(t) = \varphi_i(t) \quad \forall t \in [-\tau, 0]$.

Here we treat the issue of synchronization, i. e. the control goal is to make the trajectories of all the subsystems converge to the trajectory of the leader system:

$$\lim_{t\to\infty} (x_i(t) - \bar{x}(t)) = 0, \quad i = 1, \dots, d. \tag{33}$$

The problem is to find control functions $u_i = U_i(y_i, t) \quad i = 1, \dots, d$ ensuring achievement of the goal (33).

A. Control synthesis

The error equation in this case can be written as follows:

$$\begin{aligned}\dot{z}_i &= Az_i + \varphi_0(x_i, t) - \varphi_0(\bar{x}, t) + \sum_{j=1}^{d} \alpha_{ij} z_j + \\ &\sum_{j=1}^{d} \beta_{ij} z_j(t - \tau) + B\tilde{u}_i, \\ \tilde{y}_i &= C^{\mathrm{T}} z_i, \quad i = 1, \dots, d.\end{aligned} \tag{34}$$

Again applying the speed gradient method we obtain the following control law:

$$\begin{aligned}u_i &= -\theta_i(y_i - \bar{y}) + \bar{u}, \\ \dot{\theta}_i &= \gamma_i(y_i - \bar{y})^{\mathrm{T}} g(y_i - \bar{y}).\end{aligned} \tag{35}$$

B. Lipschitz-type nonlinearities

In this part the systems with Lipschitz-type nonlinearities are considered. First, reformulate assumption 1 for the current system type.

Assumption 4: There exists $g \in \mathbb{R}^l$ such that the transfer function $g^{\mathrm{T}} C^{\mathrm{T}}(sI_n - A)^{-1}B$ is hyper-minimum-phase.
Under this assumption according to Lemma 1, there exist $H > 0$, θ_* and $\rho > 0$ such that conditions

$$HA_* + A_*^{\mathrm{T}} H < -\rho H, HB = C^{\mathrm{T}} g, A_* = A - B\theta_*^{\mathrm{T}} C^{\mathrm{T}}, \tag{36}$$

hold.

Introduce the following numbers:

$$\mu = \sup_{t\in[0,\infty)} \max_{i\in\{1,\dots,d\}} \sum_{j=1, j\neq i}^{d} (\alpha_{ji}(t) - \alpha_{ij}(t));$$

$$\nu = \sup_{t\in[0,\infty)} \max_{i\in\{1,\dots,d\}} \sum_{j=1}^{d} (|\beta_{ij}(t)| + |\beta_{ji}(t)|).$$

Remark 2: It is clear that $\mu \geq 0, \nu \geq 0$. for any matrices $\alpha(t)$ and $\beta(t)$ satisfying conditions 1—3.
The value μ has the meaning of maximum asymmetricity of the matrix $\alpha(t)$. Thus if the matrix $\alpha(t)$ is symmetric at any time $t \geq 0$, then $\mu = 0$.

Theorem 3: Suppose the assumption 4 holds and $\varphi_0(x, t)$ is Lipschitz with respect to x with a Lipschitz constant η. Then, if $2\eta + \mu + \nu < \rho$ the control algorithm (35) ensures the achievement of the goal (33). Moreover, all tunable parameters $\theta_i(t)$ will be bounded over the time interval $[0, \infty)$ for all $i = 1, \dots, d$.
Proof. According to assumption 4, for some $H > 0$, θ_* and $\rho > 0$ the conditions (36) hold. Lets consider the following function

$$V(z_i) = \sum_{i=1}^{d} [z_i^{\mathrm{T}}(t) H z_i(t) + \frac{1}{\gamma_i}(\theta_i(t) - \theta_*)^2 + \int_{t-\tau}^{t} z_i^{\mathrm{T}}(s) P_i z_i(s) ds] \geq 0, \tag{37}$$

where $P_i = \sum_{j=1}^{d} |\beta_{ji}(t)| H \geq 0$. By taking the derivative of $V(\cdot)$ along the trajectories of the system (34), and bounding sums with coefficients α_{ij} and β_{ij} by means of inequality $2x^{\mathrm{T}}y \le x^{\mathrm{T}}Qx + y^{\mathrm{T}}Q^{-1}y$ and using the Lipschitz condition for $\varphi_0(x,t)$, the following estimate for $\dot{V}$ is obtained:

$$\begin{aligned}\dot{V} \le & \sum_{i=1}^{d} z_i^{\mathrm{T}} \left[A_*^{\mathrm{T}} H + H A_*\right] z_i + 2\eta \sum_{i=1}^{d} z_i^{\mathrm{T}} H z_i + \\ & \mu \sum_{i=1}^{d} z_i^{\mathrm{T}} H z_i + \sum_{i=1}^{d} z_i^{\mathrm{T}} \left[\sum_{j=1}^{d} |\beta_{ij}| H + P_i\right] z_i + \\ & \sum_{i=1}^{d} z_i^{\mathrm{T}}(t-\tau) \left[\sum_{j=1}^{z} |\beta_{ji}| H - P_i\right] z_i(t-\tau),\end{aligned} \tag{38}$$

where A_* is from (36). Substituting $P_i = \sum_{j=1}^{d} |\beta_{ji}| H$ and using the first inequality from (36) we obtain:

$$\dot{V} \le (2\eta + \mu + \nu - \rho) \sum_{i=1}^{d} z_i^{\mathrm{T}} H z_i \le 0, \tag{39}$$

At the same time if $\exists i \in \{1,\ldots,d\} : z_i \ne 0$, then $\dot{V} < 0$. Thus, it was shown that the function $V(\cdot)$ is a Lyapunov function for the error system. Therefore $z_i(t) = 0$ is asymptotically stable solution which means that $x_i(t) - \bar{x}(t) \to 0$ while $t \to \infty$ for $i = 1,\ldots,d$. It is obvious that if $\exists i \in \{1,\ldots,d\} : \theta_i(t) \to \infty$ while $t \to \infty$, then $V \to \infty$, which is not possible because $V(\cdot)$ is a bounded function. This proves the uniform boundedness of $\theta_i(t)$ and ends the proof of the Theorem 3.□

C. Matched nonlinearity

Let nonlinearities satisfy the matching condition $\varphi_0(x,t) = B\psi_0(C^{\mathrm{T}}x,t)$, where $\psi_0 \colon \mathbb{R} \times [0,\infty) \to \mathbb{R}$ is some function.

Theorem 4: Suppose assumption 4 holds and $\varphi_0(x,t) = B\psi_0(C^{\mathrm{T}}x,t)$, where $\psi_0(C^{\mathrm{T}}x,t)$ is a g-monotonically decreasing function for any $t \in [0,\infty)$. Then, if $\mu + \nu < \rho$, then the control algorithm (35) ensures achievement of the goal (33). Moreover, all tunable parameters $\theta_i(t)$ will be bounded in the time interval $[0,\infty)$ for all $i = 1,\ldots,d$.

Proof. Consider function (37). Taking derivative along the trajectories of the system (34) and using estimations from the proof of the theorem 3 bounds $\dot{V}$ as follows:

$$\begin{aligned}\dot{V} \le & (\mu + \nu - \rho) \sum_{i=1}^{d} z_i^{\mathrm{T}} H z_i + \\ & + 2 \sum_{i=1}^{d} (y_i - \bar{y})^{\mathrm{T}} g \left[\psi_0(y_i,t) - \psi_0(\bar{y},t)\right].\end{aligned} \tag{40}$$

By conditions of the theorem 4 function ψ_0 is g-monotonically decreasing. Therefore

$$\dot{V} \le (\mu + \nu - \rho) \sum_{i=1}^{d} z_i^{\mathrm{T}} P z_i \le 0. \tag{41}$$

Similarly to the proof of the theorem 3, we conclude that $x_i - \bar{x} \to 0$ as $t \to \infty$ and $i = 1,\ldots,d$ and $\theta_i(t)$ are uniformly bounded. That ends the proof of the theorem 4.□

V. Conclusions

In this paper in contrast to the existing results the solution of the problem of convergence with pre-specified accuracy is proposed. Synchronization conditions for delayed coupling networks with switching topology consisting of nonlinear systems under incomplete measurement, incomplete control, incomplete information about system parameters are obtained. The design of the control algorithm providing synchronization property is based on speed-gradient method, while derivation of synchronizability conditions is based on the passification theorem.

Further research may be aimed at application more sophysticated adaptive control techniques [16] to synchronization of networks.

References

[1] *Fradkov A. L.*. Adaptive control in complex systems. Moscow: Nauka, 1990.

[2] *Druzhinina M.V., Fradkov A.L.* Adaptive decentralized control of interconnected systems // Proc. of 14th IFAC World Congress, 1999, Vol.L, pp.175-180.

[3] *Jinhu Lu, Guanrong Chen.* A Time-Varying Complex Dynamical Network Model and Its Controlled Synchronization Criteria // IEEE Trans. Autom. Control, V. 50 (6), 2005.

[4] *Zhou J., Lu J., Lu J.* Adaptive synchronization of an uncertain complex dynamical network // IEEE Trans. Autom. Control, vol. 51, no. 4, pp. 652-656, Apr. 2006.

[5] *Fradkov A.L., Miroshnik I.V., Nikiforov V.O.* Nonlinear and Adaptive Control of Complex Systems. Dordrecht: Kluwer Academic Publ., 1999.

[6] *Fradkov A.L.* Passification of nonsquare linear systems and feedback Yakubovich-Kalman-Popov Lemma // Europ. J. Contr. No 6, 2003, pp. 573-582.

[7] *Chai Wah Wu, Chua L. O.* Synchronization in an array of linearly coupled dynamical systems. IEEE Trans. Circuits and Systems-I. Vol. 42, no. 8, 1995, pp. 430-447.

[8] *Alligood K., Sauer T., Yorke J.* Chaos: an introduction to dynamical systems. Springer, 1996.

[9] *Dzhunusov I.A., Fradkov A.L.* Adaptive synchronization of a network of interconnected nonlinear Lur'e systems. Automation and Remote Control, 2009, V.70, (7), pp.1190Ű1205.

[10] *Fradkov A.L., Razuvaeva I.V., Grigoriev G.K.* Passification Based Adaptive Control Under Coordinate-Parametric White Noise Disturbances. Prepr. 8th IFAC Symposium NOLCOS 2010, Bologna, 2010. pp.659-664.

[11] *Jing Yao and Zhi-Hong Guan and David J. Hill* Passivity-based control and synchronization of general complex dynamical networks. 2009, Automatica V.45, (9), pp.2107-2113.

[12] *Wang Y.W., and Xiao J.W., and Wang H.O.* Global synchronization of complex dynamical networks with network failures. International Journal Of Robust And Nonlinear Control. 2010, V.20, (15), pp.1667-1677.

[13] *Nuno E., Ortega R., Basanez L. and Hill D.* Synchronization of Networks of Nonidentical Euler-Lagrange Systems with Uncertain Parameters and Communication Delays. , IEEE Trans. Automatic Control V. 56 (4), 2011, pp.935-941.

[14] *Tao Liu, and Hill D.J., and Jun Zhao* Synchronization of dynamical networks by network control. Proc. 48th IEEE Conf. Dec. Control, 2009, pp.1684-1689.

[15] *Yao J., Hill D.J., Guan Z.H. and Wang H.O.* Synchronization of complex dynamical networks with switching topology via adaptive control. Proc. 45th IEEE Conf. Dec. Control, 2006, pp.2819-2824.

[16] Jiang Z.P. Hill D.J. and Fradkov A L. A passification approach to adaptive nonlinear stabilization Systems & Control Letters V. 28 (2), pp. 73-84, 1996.