INTERNATIONAL JOURNAL OF ADAPTIVE CONTROL AND SIGNAL PROCESSING
Int. J. Adapt. Control Signal Process. 2013; **27**:729–740
Published online 28 September 2012 in Wiley Online Library (wileyonlinelibrary.com). DOI: 10.1002/acs.2343

Decentralized adaptive controller for synchronization of nonlinear dynamical heterogeneous networks

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SUMMARY

For a network of interconnected nonlinear dynamical systems, an adaptive leader–follower output feedback synchronization problem is considered. The proposed structure of decentralized controller and adaptation algorithm is based on speed gradient and passivity. Sufficient conditions of synchronization for one class of heterogeneous networks are established. An example of synchronization of the network of nonidentical Chua systems is analyzed. The main contribution of the paper is adaptive controller design and analysis under conditions of incomplete measurements, incomplete control, and uncertainty. Copyright © 2012 John Wiley & Sons, Ltd.

Received 9 September 2011; Revised 28 August 2012; Accepted 3 September 2012

KEY WORDS: synchronization; complex networks; heterogeneous networks; adaptive control; passivity

1. INTRODUCTION

Adaptive synchronization of dynamical networks has attracted a growing interest during recent years [1–4]. It is motivated by a broad area of potential applications as follows: formation control, cooperative control, control of power networks, communication networks, production networks, and so on. Existing works [1–4] and others are dealing with full state feedback and linear interconnections. The solutions are based on Lyapunov functions formed as the sum of Lyapunov functions for local subsystems. As for adaptive control algorithms, they are based on either local (decentralized) feedback [5–12] or neighbors-based (described by an information graph) feedback [13–16] strategies.

Despite a great interest in control of networks, only a restricted class of problems are currently solved. For example, in the existing papers, mainly linear models of subsystems are considered [13, 14]. In nonlinear case, only passive or passifiable systems are studied, and control is organized according to information graph, that is, not completely decentralized [15, 16]. Availability of the whole state vector for measurement as well as appearance of control in all equations for all controlled nodes is assumed in decentralized stabilization and synchronization problems [1–4, 17, 18]. In [19], synchronization problem for heterogeneous networks is considered. However, it seems that synchronization conditions of [19] are rather hard to analyze. Powerful passivity-based approaches are not well-developed for adaptive synchronization problems.

In this paper, we consider the problem of master–slave (leader–follower) synchronization in a heterogeneous network of systems (agents) in Lurie form (right-hand sides are split into linear and

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nonlinear parts). The case of identical agents (homogeneous network) is studied in [20]. Linearity of interconnections is not assumed; links between subsystems can also be nonlinear. In the contrary to known works on adaptive synchronization of networks, see [3, 4], only input of leader and some output functions are available and control may appear only in a part of the system equations. It is also assumed that some agent parameters are unknown. The leader subsystem is assumed to be isolated and the control objective is to approach the trajectory of the leader subsystem by all other ones under conditions of uncertainty. Interconnection functions are assumed to be Lipschitz continuous.

Adaptation algorithm is designed by the speed-gradient method extended to decentralized control problems in [11, 12, 21]. The results of [11, 12, 21] are employed both for adaptation algorithm design and for the derivation of synchronization conditions. The bounds for the interconnection strengths ensuring achievement of the control goal (synchronization) are found.

The results and operability of the proposed adaptive controller are illustrated by example of synchronization in the network of nonidentical Chua circuits with changing circuit parameters and changing interconnections.

2. AUXILIARY RESULTS

2.1. Yakubovich–Kalman Lemma

We need Yakubovich–Kalman Lemma in the following form, see [22].

Lemma 1
Let A, B, C be $n \times n, n \times m, n \times m$ real matrices and $u \in \mathbb{R}^m$, $\chi(s) = C^{\mathrm{T}}(sI_n - A)^{-1}B$, rank $B = m$. Then the following statements are equivalent

(1) There exists matrix $H = H^{\mathrm{T}} > 0$ such that

$$HA + A^{\mathrm{T}}H < 0, HB = C; \tag{1}$$

(2) Polynomial $\det(sI_n - A)$ is Hurwitz and the following frequency domain conditions hold

$$\mathrm{Re}\, u^{\mathrm{T}}\chi(i\omega)u > 0, \quad \lim_{\omega\to\infty} \omega^2\, \mathrm{Re}\, u^{\mathrm{T}}\chi(i\omega)u > 0$$

for all $\omega \in \mathbb{R}^1$, $u \in \mathbb{R}^m, u \neq 0$.

The aforementioned frequency domain conditions are equivalent to the following *strict positive realness* condition

$$\mathrm{Re}\, \chi(s) > 0, \text{ if } \mathrm{Re}\, s \geqslant 0,$$

where notation $\mathrm{Re}\, \chi = (\chi + \chi^*)/2$, and '*' for a matrix χ stands for transposed conjugate. They are also equivalent to the strict passivity of the system with transfer matrix $\chi(s)$.

2.2. Speed gradient algorithm in decentralized control

In order to present the main synchronization result of this paper, we need to formulate problem statement of decentralized control and Theorem 2.18 of [11] that differs insignificantly from Theorem 7.6 of [12] or Theorem 1 of [21].

Consider[‡] a system $\mathcal{S}$ consisting of d interconnected subsystems (agents) $\mathcal{S}_i$, dynamics of each being described by the following equation:

$$\dot{x}_i = F_i(x_i, \tau_i, t) + h_i(x, \tau, t), \quad i = 1, \ldots, d, \tag{2}$$

where $x_i \in \mathbb{R}^{n_i}$ is the state vector, and $\tau_i \in \mathbb{R}^{m_i}$ is the vector of inputs (tunable parameters) of subsystem. Vector-function $F_i(\cdot)$ describes local dynamics of subsystem S_i, and vectors $h_i(\cdot)$ describe interconnection between subsystems.

[‡]In this paper, norms are Euclidean, $\mathrm{col}(x_1, \ldots, x_d)$ stands for column vector with components consisting of components of $x_i, i = 1, \ldots, d$.

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DOI: 10.1002/acs

Let $Q_i(x_i,t) \geqslant 0, i = 1,\dots,d$ be local goal functions and let the control goal be

$$\lim_{t\to\infty} Q_i(x_i,t) = 0, \qquad i = 1,\dots,d. \tag{3}$$

For all $i = 1,\dots,d$, we assume an existence of smooth vector functions $x_i^*(t)$ such that $Q_i\left(x_i^*(t),t\right) \equiv 0$, that is, $x_i^* = \operatorname{argmin}_{x_i} Q_i(x_i,t)$. Decentralized speed-gradient algorithm is introduced as follows:

$$\dot{\tau}_i = -\Gamma_i \nabla_{\tau_i} \omega_i(x_i,\tau_i,t), \qquad i = 1,\dots,d, \tag{4}$$

where

$$\omega_i(x_i,\tau_i,t) = \frac{\partial Q_i}{\partial t} + \nabla_{x_i} Q_i(x_i,t)^{\mathrm{T}} F_i(x_i,\tau_i,t),$$

$\Gamma_i = \Gamma_i^{\mathrm{T}} > 0$, $m_i \times m_i$ - matrix.

Theorem 1

Suppose the following assumptions hold for the system $\mathcal{S}$.

(1) Functions $F_i(\cdot)$ are continuous in x_i, t, continuously differentiable in τ_i and locally bounded in $t > 0$; functions $Q_i(x_i,t)$ are uniformly continuous in the second argument for all x_i in bounded set, functions $\omega_i(x_i,\tau_i,t)$ are convex in τ_i; there exist constant vectors $\tau_i^* \in \mathbb{R}^{m_i}$ and scalar monotonically increasing functions $\kappa_i(Q_i)$, $\rho_i(Q_i)$ such that $\kappa_i(0) = \rho_i(0) = 0$, $\lim_{Q_i\to+\infty} \kappa_i(Q_i) = +\infty$

$$\omega_i\left(x_i,\tau_i^*,t\right) \leqslant -\rho_i(Q_i(x_i,t)), \tag{5}$$

and $Q_i(x_i,t) \geqslant \kappa_i\left(\|x_i - x_i^*(t)\|\right)$.

(2) Functions $h_i(x,\tau,t)$ are continuous and satisfy the following inequalities

$$|\nabla_{x_i} Q_i(x_i,t)^{\mathrm{T}} h_i(x,\tau,t)| \leqslant \sum_{j=1}^{d} \mu_{ij} \rho_j(Q_j(x_j,t)), \tag{6}$$

where matrix $M - I$ is Hurwitz, $M = \{\mu_{ij}\}$, $\mu_{ij} > 0$, I is identity matrix.

Then systems (2),(4) are globally asymptotically stable in variables $x_i - x_i^*(t)$, all trajectories are bounded on $t \in [0,+\infty)$ and satisfy (3).

The first group of conditions of Theorem 1 ensures achievement of goal (3) in every isolated subsystem $\mathcal{S}_i$, $i = 1,\dots,d$, whereas the second group of conditions defines allowable degree of interconnection strength between subsystems. Inequality (5) is primary in the first group of conditions and it means principal reachability of goal (3).

The Lyapunov function used in the proof of Theorem 1 is a convex combination of Lyapunov functions $V_i(\cdot)$ of subsystems $\mathcal{S}_i$ chosen as follows:

$$V_i(x_i,\tau_i,t) = Q_i(x_i,t) + \frac{1}{2}\left(\tau_i - \tau_i^*\right)^{\mathrm{T}} \Gamma_i^{-1}\left(\tau_i - \tau_i^*\right), \quad i = 1,\dots,d.$$

The coefficients of the convex combination are determined via matrix M, see [11].

In many cases, local goal functions $Q_i(x_i,t), i = 1,\dots,d$ can be chosen in the following form

$$Q_i(x_i,t) = \left(x_i - x_i^*\right)^{\mathrm{T}} H_i\left(x_i - x_i^*\right),$$

where matrices H_i are positive definite, $H_i = H_i^{\mathrm{T}} > 0$.

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3. MAIN RESULT

3.1. Problem statement and adaptive controller structure

Introduce the leader subsystem described by the equation

$$\dot{\overline{x}} = A_L\overline{x} + B_L(\overline{u} + \psi_0(\overline{y})), \quad \overline{y} = C^{\mathrm{T}}\overline{x}, \tag{7}$$

where $\overline{x} \in \mathbb{R}^n$ – state, $\overline{y} \in \mathbb{R}^l$ – measurement, $\overline{u}(t) \in \mathbb{R}^1$ is a piecewise continuous control function specified in advance, and $\psi_0 : \mathbb{R}^l \to \mathbb{R}^1$ is an internal nonlinearity. Suppose that A_L, B_L, C, and $\psi_0(\cdot)$ are known, that is, do not depend on the vector of unknown parameters $\xi \in \Xi$, where Ξ is the known set.

Consider a network S of d interconnected subsystems S_i, $i = 1, \ldots, d, d \in \mathbb{N}, d > 1$. Let subsystem S_i be described by the following equation

$$\begin{aligned} \dot{x}_i &= A_i x_i + B_i u_i + B_L\psi_0(y_i) + \sum_{j=1}^{d} \alpha_{ij}\varphi_{ij}(x_i - x_j), \\ y_i &= C^{\mathrm{T}}x_i, i = 1, \ldots, d, \end{aligned} \tag{8}$$

where $x_i \in \mathbb{R}^n, u_i \in \mathbb{R}^1, \alpha_{ij} \in \mathbb{R}^1, y_i \in \mathbb{R}^l$. Functions $\varphi_{ij}(\cdot)$, $i = 1, \ldots, d$, $j = 1, \ldots, d$, describe interconnections between subsystems. We assume $\varphi_{ii} = (0, 0, \ldots, 0)^{\mathrm{T}}, i = 1, \ldots, d$. Let matrices A_i, B_i, and functions $\varphi_{ij}(\cdot)$, $i = 1, \ldots, d$, $j = 1, \ldots, d$, depend on the vector of unknown parameters $\xi \in \Xi$.

Vector ξ of unknown parameters includes coefficients of equations describing mathematical model of $\mathcal{S}$. It may also include coefficients describing dynamics of external disturbances. We will suppose that vector ξ is either constant or varying slowly, that is, slower than the flow of dynamic processes in system $\mathcal{S}$ and slower than the external disturbances. The set (class) Ξ characterizes information about system $\mathcal{S}$ that is available a priori: the less we know about parameters of system $\mathcal{S}$, the larger is the set Ξ. Network model (8) can describe, for example, interconnected electrical generators [23]. Then the vector ξ represent parameters of the generator models that are only partially known in practice.

Let the control goal be specified as convergence of all agent trajectories to the trajectory of the leader

$$\lim_{t\to+\infty} (x_i(t) - \overline{x}(t)) = 0, \qquad i = 1, \ldots, d. \tag{9}$$

The adaptive synchronization problem is to find a decentralized controller $u_i = \mathcal{U}_i(y_i, \overline{u}, \overline{y}, t)$ ensuring the goal (9) for all values of unknown parameters $\xi \in \Xi$. Moreover, the controller should not depend on these parameters.

Denote $\sigma_i(t) = \mathrm{col}(y_i(t), \overline{u}(t))$. Let the main loop of the adaptive system be specified as set of linear tunable local control laws

$$u_i(t) = \tau_i(t)^{\mathrm{T}}\sigma_i(t), \quad i = 1, \ldots, d, \tag{10}$$

where $\tau_i(t) \in \mathbb{R}^{l+1}, i = 1, \ldots, d$ are tunable parameters. By applying speed-gradient method [12], the following adaptation law is derived

$$\dot{\tau}_i = -g^{\mathrm{T}}(y_i - \overline{y})\Gamma_i\sigma_i(t), i = 1, \ldots, d, \tag{11}$$

where $\Gamma_i = \Gamma_i^{\mathrm{T}} > 0$ – $(l+1) \times (l+1)$ matrices, $g \in \mathbb{R}^l$ are some constant vectors, which will be specified in the next section. Derivation of (11) will be given in Section 3.3. The controller (10) and (11) is decentralized because no information from other nodes of the network is used for control of any fixed node. As for the leader, it is introduced just for the formulation of the common control goal for the nodes.

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3.2. Synchronization conditions

Introduce the following definition.

Definition 1
Let $G \in \mathbb{R}^l$. Function $f : \mathbb{R}^l \to \mathbb{R}^1$ is called G-monotonically decreasing if inequality $(x-y)^{\mathrm{T}} G\,(f(x) - f(y)) \leqslant 0$ holds for all $x, y \in \mathbb{R}^l$.

Remark 1
Apparently, for $l = 1, G = 1$ G-monotonic decrease of the function f is equivalent to incremental passivity [24] of the static system with characteristics $(-f)$. Definition 1 is readily extended to dynamical systems with the state vector $x \in \mathbb{R}^n$, input $u \in \mathbb{R}^m$, and output $y \in \mathbb{R}^l$. In this case, it corresponds to the existence of a smooth function $V(x_1, x_2)$ satisfying an integral inequality

$$\dot{V}(x_1, x_2) \leqslant (u_2 - u_1)^{\mathrm{T}} G(y_2 - y_1).$$

The corresponding property may be called incremental G-passivity by analogy with [25].

Consider real matrices $H = H^{\mathrm{T}} > 0, g$ of size $n \times n, l \times 1$ correspondingly and a number $\rho > 0$ such that

$$HA_L + A_L^{\mathrm{T}} H < -\rho H, \quad HB_L = Cg. \tag{12}$$

Denote $\lambda_* = \lambda_{max}(H)/\lambda_{min}(H)$ condition number of matrix H, where $\lambda_{max}(H), \lambda_{min}(H)$ are maximum and minimum eigenvalues of matrix H.

For analysis of the system dynamics, the following assumptions are made.

(A1) The functions $\varphi_{ij}(\cdot), i = 1, \ldots, d,\ j = 1, \ldots, d$ are globally Lipschitz

$$\|\varphi_{ij}(x) - \varphi_{ij}(x')\| \leqslant L_{ij}\|x - x'\|, \quad L_{ij} > 0.$$

The function $\psi_0(\cdot)$ is such that the existence and uniqueness of solutions of (7) holds.

(A2) (Matching conditions, [26]). For each $\xi \in \Xi$, there exist vectors $\nu_i = \nu_i(\xi) \in \mathbb{R}^l$ and numbers $\theta_i = \theta_i(\xi) > 0$ such that for $i = 1, \ldots, d$

$$A_L = A_i + B_i \nu_i^{\mathrm{T}} C^{\mathrm{T}}, B_L = \theta_i B_i. \tag{13}$$

Denote $\chi(s) = C^{\mathrm{T}}(sI_n - A_L)^{-1} B_L$. For the case when matrix A_L is Hurwitz, introduce notation ρ_* for stability degree of the function's $g^{\mathrm{T}}\chi(s)$ denominator, that is, $\rho_* = \min_{k=1,\ldots,n} |\mathrm{Re}\lambda_k(A_L)|$ where $\lambda_k(A_L)$ are eigenvalues of A_L.

Theorem 2
Let $B_L \neq 0$, matrix A_L be Hurwitz and for some $g \in \mathbb{R}^l$, the following frequency domain conditions hold

$$\mathrm{Re}\, g^{\mathrm{T}}\chi(i\omega) > 0, \quad \lim_{\omega\to\infty} \omega^2\, \mathrm{Re}\, g^{\mathrm{T}}\chi(i\omega) > 0 \tag{14}$$

for all $\omega \in \mathbb{R}^1$. Then there exist $H = H^{\mathrm{T}} > 0, \rho > 0$ such that relations (12) hold.

Let, for all $\xi \in \Xi$, Assumptions A1, A2 hold, function $\psi_0(\cdot)$ be g-monotonically decreasing and the following inequalities hold

$$\sum_{j=1}^{d} \left|\alpha_{ij} L_{ij}\right| < \gamma \quad i = 1, \ldots, d, \tag{15}$$

where $\gamma = \rho_*/(4d\lambda_*)$, λ_* is the condition number of matrix H.

Then for all $\xi \in \Xi, i = 1, \ldots, d$ adaptive controller (10) and (11) ensures the achievement of the goal (9) and the boundedness of functions $\tau_i(t)$ on $[0, \infty)$ for all solutions of the closed-loop system (7), (8), (10), and (11).

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Proof

Let us apply Lemma 1. Note that in our case, $m = 1$, that is, u is scalar. Let us choose Cg instead of C in (1). Then the statement of the Lemma 1 and conditions of Theorem 2 ensure the existence of matrix $H = H^{\mathrm{T}} > 0$ such that

$$HA_L + A_L^{\mathrm{T}} H < 0, \quad HB_L = Cg.$$

Therefore, there exists number $\rho > 0$ such that the following is true:

$$HA_L + A_L^{\mathrm{T}} H < -\rho H, \quad HB_L = Cg. \tag{16}$$

Denoting $z_i = x_i - \overline{x}$ introduce auxiliary error subsystems

$$\begin{aligned} \dot{z}_i &= A_i x_i + B_i u_i + B_L \psi_0(y_i) + \sum_{j=1}^{d} \alpha_{ij} \varphi_{ij}(x_i - x_j) - (A_L \overline{x} + B_L(\overline{u} + \psi_0(\overline{y}))), \\ \tilde{y}_i &= C^{\mathrm{T}} z_i, \quad i = 1, \dots, d. \end{aligned} \tag{17}$$

Here, we choose $u_i(t)$ same as in (10).

Let us choose the goal functions $Q_i(z_i) = \frac{1}{2} z_i^{\mathrm{T}} H z_i$ and apply Theorem 1. We need to evaluate the derivatives of $Q_i(z_i)$ along the trajectories of isolated (i.e., without interconnections) auxiliary subsystems (17)

$$\omega_i(x_i, \overline{x}, \tau_i) = z_i^{\mathrm{T}} H \left[A_i x_i + B_i \tau_i^{\mathrm{T}}(t) \sigma_i(t) + B_L \psi_0(y_i) - A_L \overline{x} - B_L(\overline{u} + \psi_0(\overline{y})) \right]. \tag{18}$$

Denote $\tau_i^* = \mathrm{col}(\nu_i, \theta_i), i = 1, \dots, d$ and replace τ_i with $\tau_i^*, i = 1, \dots, d$. Then

$$\begin{aligned} \omega_i \left(x_i, \overline{x}, \tau_i^* \right) &= z_i^{\mathrm{T}} H \left[A_i x_i + B_i \left(\nu_i C^{\mathrm{T}} x_i + \theta_i \overline{u} \right) + B_L \psi_0(y_i) - A_L \overline{x} - B_L \overline{u} - B_L \psi_0(\overline{y}) \right] \\ &= z_i^{\mathrm{T}} H \left[A_L x_i + B_L \overline{u} + B_L \left(\psi_0(y_i) - \psi_0(\overline{y}) \right) - A_L x_i - B_L \overline{u} \right] \\ &= z_i^{\mathrm{T}} H \left[A_L z_i + B_L (\psi_0(y_i) - \psi_0(\overline{y})) \right]. \end{aligned}$$

Further, for $i = 1, \dots, d$

$$z_i^{\mathrm{T}} H B_L (\psi_0(y_i) - \psi_0(\overline{y})) = z_i^{\mathrm{T}} C g (\psi_0(y_i) - \psi_0(\overline{y})) = (y_i - \overline{y})^{\mathrm{T}} g (\psi_0(y_i) - \psi_0(\overline{y})) \leqslant 0.$$

The last inequality holds because $\psi_0(\cdot)$ is g-monotonically decreasing. Therefore,

$$\omega_i \left(x_i, \overline{x}, \tau_i^* \right) \leqslant \frac{1}{2} z_i^{\mathrm{T}} \left(HA_L + A_L^{\mathrm{T}} H \right) z_i.$$

Taking into account (16), we conclude

$$\omega_i \left(x_i, \overline{x}, \tau_i^* \right) \leqslant -\rho \, Q_i(z_i).$$

By taking $\rho_i(Q) = \rho \cdot Q$, we ensure that (5) holds for $i = 1, \dots, d$. Other conditions from the first part of Theorem 1 hold, because the right-hand side of the system (17) and function $Q_i(z_i)$ are continuous in z_i and piecewise continuous in t for any $i = 1, \dots, d$. Convexity condition is valid because the right-hand side of (18) is linear in τ_i.

The interconnection condition (6) in our case reads

$$\left| \nabla_{z_i} Q(z_i)^{\mathrm{T}} \sum_{j=1}^{d} \alpha_{ij} \varphi_{ij}(z_i - z_j) \right| \leqslant \sum_{j=1}^{d} \mu_{ij} \rho \cdot Q(z_j), \tag{19}$$

where $i = 1, \dots, d$, and matrix $M - I$ should be Hurwitz ($M = \{\mu_{ij}\}$, $\mu_{ij} > 0$). For $i = 1, \dots, d$, rewrite (19) as follows:

$$\left| z_i^{\mathrm{T}} H \sum_{j=1}^{d} \alpha_{ij} \varphi_{ij}(z_i - z_j) \right| \leqslant \frac{\rho}{2} \sum_{j=1}^{d} \mu_{ij} \, z_j^{\mathrm{T}} H z_j. \tag{20}$$

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Evaluate the left-hand side of (20)

$$\begin{aligned}\left|z_i^{\mathrm{T}} H \sum_{j=1}^{d} \alpha_{ij}\varphi_{ij}(z_i - z_j)\right| &\leqslant \sum_{j=1}^{d} \left|z_i^{\mathrm{T}} H\alpha_{ij}\varphi_{ij}(z_i - z_j)\right| \\ &\leqslant \sum_{j=1}^{d} |\alpha_{ij} L_{ij}| \cdot \|z_i\| \cdot \|H\| \cdot \|z_i - z_j\| \\ &\leqslant \sum_{j=1}^{d} |\alpha_{ij} L_{ij}| \cdot \lambda_{max}(H) \cdot (\|z_i\|^2 + \|z_i\| \cdot \|z_j\|),\end{aligned}$$

for $i = 1, \dots, d$. Then for $i = 1, \dots, d$, evaluate lower bound of the right-hand side of (20)

$$\frac{\rho}{2} \sum_{j=1}^{d} \mu_{ij} z_j^{\mathrm{T}} H z_j \geqslant \frac{\rho}{2} \sum_{j=1}^{d} \mu_{ij} \lambda_{min}(H) \|z_j\|^2.$$

It is seen that for $i = 1, \dots, d$ to ensure (6), it is sufficient to impose an inequality

$$\sum_{j=1}^{d} |\alpha_{ij} L_{ij}| \cdot \lambda_{max}(H) \cdot \left(\|z_i\|^2 + \|z_i\| \cdot \|z_j\|\right) \leqslant \frac{\rho}{2} \sum_{j=1}^{d} \mu_{ij} \lambda_{min}(H) \|z_j\|^2,$$

or for $i = 1, \dots, d$,

$$\sum_{j=1}^{d} |\alpha_{ij} L_{ij}| \cdot \left(\|z_i\|^2 + \|z_i\| \cdot \|z_j\|\right) \leqslant \frac{\rho}{2} \frac{\lambda_{min}(H)}{\lambda_{max}(H)} \sum_{j=1}^{d} \mu_{ij} \|z_j\|^2. \tag{21}$$

Denote $\zeta = \rho/(\alpha_{max} \cdot 2\lambda_*)$, where

$$\alpha_{max} = \max_{i:1 \leqslant i \leqslant d} \sum_{j=1}^{d} \left|\alpha_{ij} L_{ij}\right|.$$

Noting that ρ in (12) can be chosen arbitrarily close to ρ_* and taking into account (15), we conclude that $\zeta > 2d$.

The left-hand side of (21) can be evaluated as follows:

$$\begin{aligned}\sum_{j=1}^{d} |\alpha_{ij} L_{ij}| \left(\|z_i\|^2 + \|z_i\| \cdot \|z_j\|\right) &\leqslant \left(\sum_{j=1}^{d} |\alpha_{ij} L_{ij}|\right) \cdot \left(\sum_{j=1}^{d} \left(\|z_i\|^2 + \|z_i\| \cdot \|z_j\|\right)\right) \\ &\leqslant \frac{1}{2} \alpha_{max} \left(3d \, \|z_i\|^2 + \sum_{j=1}^{d} \|z_j\|^2\right), \quad i = 1, \dots, d.\end{aligned}$$

Thus, if following inequality holds for $i = 1, \dots, d$, then (6) is ensured as follows:

$$3d \, \|z_i\|^2 + \sum_{j=1}^{d} \|z_j\|^2 \leqslant 2\zeta \cdot \sum_{j=1}^{d} \mu_{ij} \|z_j\|^2. \tag{22}$$

Introduce matrix $M = \{\mu_{ij}\}$ as follows:

$$M = \begin{pmatrix} \mu_{11} & \mu_{12} & \dots & \mu_{1d} \\ \mu_{21} & \mu_{22} & \dots & \mu_{2d} \\ \vdots & \vdots & \ddots & \vdots \\ \mu_{d1} & \mu_{d2} & \dots & \mu_{dd} \end{pmatrix}, \qquad \mu_{ij} = \begin{cases} \frac{1}{2\zeta}(3d + 1), & i = j; \\ \frac{1}{2\zeta}, & i \neq j. \end{cases}$$

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Apparently, such a choice of M ensures (22) and M is symmetric. If matrix $I - M$ is positive definite, then $M - I$ is Hurwitz. Diagonal elements of $I - M$ are positive because $d > 1$ and $\zeta > 2d$. By taking into account that

$$1 - \frac{1}{2\zeta}(3d + 1) - (d - 1)\frac{1}{2\zeta} > 0$$

and applying Gershgorin circle theorem, we conclude that $I - M$ is positive definite.

Thus, the statement of the Theorem 2 follows from Theorem 1. □

Remark 2
The value of γ can be evaluated by solving LMI (16) by means of one of the existing software packages.

Remark 3
The information graph of the network S can be introduced as the directed graph with nodes and arcs defined as follows. Cardinality of a set of nodes is d and ith node is associated with subsystem S_i for any $i = 1, \dots, d$. The arc from ith node to jth node belongs to the set of arcs if $\varphi_{ij}(\cdot)$ is not zero function. By weighted in-degree of ith node, we denote the following number: $\sum_{j=1}^{d} \left|\alpha_{ij} L_{ij}\right|$. If each nonzero addend from the last sum is equal to 1, then introduced definition of weighted in-degree of the node coincides with the definition of in-degree of digraph's node. Then the inequality (15) in Theorem 2 can be interpreted as follows: weighted in-degree of each node of interconnections graph must be less than γ.

Remark 4
If $\overline{x}$ is bounded, then all trajectories of closed-loop systems (7), (8), (10), and (11) are bounded too.

Remark 5
Seemingly, restrictive Assumption A2 in fact is nothing but a possibility of equalizing the linear parts of the agent dynamics by a linear output feedback. Let us demonstrate by example that it is not that restrictive. Consider the case when $\theta_i = 1$ for all $i = 1, \dots, d$, vectors $B_L = \mathrm{col}(0, \dots, 0, 1)$, $C = \mathrm{col}(\mathcal{M}_0, \dots, \mathcal{M}_m, 0, \dots, 0)$ for some $m \leqslant n - l$ and

$$A_L = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \\ 0 & 0 & 0 & \dots & 1 \\ -\mathcal{N}_0 & -\mathcal{N}_1 & -\mathcal{N}_2 & \dots & -\mathcal{N}_{n-1} \end{pmatrix}.$$

Let $\mathcal{M}(s) = \mathcal{M}_m s^m + \ldots + \mathcal{M}_0$, $\mathcal{N}(s) = s^n + \mathcal{N}_{n-1} s^{n-1} + \ldots + \mathcal{N}_0$. Then the transfer function $\chi(s)$ is as follows:

$$\chi(s) = C^{\mathrm{T}}(sI_n - A_L)^{-1} B_L = \frac{\mathcal{M}(s)}{\mathcal{N}(s)}.$$

Consider new output $\tilde{y}_i = \mathrm{col}\left(y_i, \dot{y}_i, \dots, y_i^{(l-1)}\right)$ of ith isolated system. It is possible because $m + l \leqslant n$. Denote $\overline{\mathcal{M}} = \mathcal{M}(s)\left(1, s, \dots, s^{l-1}\right)^{\mathrm{T}}$ and define the transfer function of ith isolated system for $i = 1, \dots, d$ as follows:

$$\chi_i(s) = \frac{\mathcal{M}(s)}{\mathcal{N}(s) - \nu_i^{\mathrm{T}}\overline{\mathcal{M}}(s)}.$$

Because $\nu_i \in \mathbb{R}^l$ is an arbitrary vector, transfer function $\chi_i(s) = \mathcal{M}(s)/(\mathcal{N}(s) - \mathcal{M}(s)\mathcal{N}_i(s))$, where $\mathcal{N}_i(s)$ is an arbitrary polynomial of degree $l - 1$. Thus, if $l = n, m = 0$, then

$$\chi_i(s) = \frac{\mathcal{M}(s)}{\mathcal{K}_i(s)},$$

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where $\mathcal{K}_i(s)$ is an arbitrary polynomial of degree n with the highest coefficient equal to 1. Therefore, Assumption A2 is not restrictive for the considered case.

3.3. Derivation of adaptation law

To design adaptation laws for $\tau_i(t), i = 1, \dots, d$, the speed-gradient method is used. According to the speed-gradient method, we need to choose a non-negative goal function such that its convergence to zero corresponds to the achievement of the goal (9). In our case, the following goal function is suitable (same as in the proof of Theorem 2):

$$Q(z_i) = \frac{1}{2} z_i^{\mathrm{T}} H z_i, \quad H = H^{\mathrm{T}} > 0.$$

Then, we need to evaluate the derivative of $Q(z_i)$ along trajectories of isolated subsystems.

$$\omega_i(x_i, \overline{x}, \tau_i) = z_i^{\mathrm{T}} H \left[A_i x_i + B_i \tau_i^{\mathrm{T}}(t)\sigma_i(t) + B_L \psi_0(y_i) - A_L \overline{x} - B_L(\overline{u} + \psi_0(\overline{y}))\right].$$

The next step is evaluating of the gradient in τ_i

$$\nabla_{\tau_i} \omega_i(x_i, \overline{x}) = z_i^{\mathrm{T}} H B_i \sigma_i(t), \quad i = 1, \dots, d.$$

Using (13) and (16), we can rewrite it as follows:

$$\nabla_{\tau_i} \omega_i(x_i, \overline{x}) = \frac{1}{\theta_i} z_i^{\mathrm{T}} H B_L \sigma_i(t) = \frac{1}{\theta_i} z_i^{\mathrm{T}} C g \sigma_i(t), \quad i = 1, \dots, d.$$

We arrive at the following adaptation algorithm

$$\dot{\tau}_i = -\hat{\Gamma}_i \frac{1}{\theta_i} z_i^{\mathrm{T}} C g \sigma_i(t), \quad i = 1, \dots, d,$$

where $\hat{\Gamma}_i = \hat{\Gamma}_i^{\mathrm{T}} > 0$ are positive definite $(l+1) \times (l+1)-$ matrices. For $i = 1, \dots, d$, we can take $\Gamma_i = \frac{1}{\theta_i} \hat{\Gamma}_i$, because the choice of $\hat{\Gamma}_i$ is arbitrary. Noting that $z_i^{\mathrm{T}} C g$ is scalar, we finally obtain (11).

4. EXAMPLE. NETWORK OF CHUA CIRCUITS

4.1. System description and theoretical study

Chua circuit is the well-known example of a simple nonlinear system possessing complex chaotic behavior [27]. Its trajectories are unstable for some parameter values and it is represented in the Lurie form. Let us apply our results to synchronization with leader in the network of five interconnected nonidentical Chua systems. Let $m_0 = -8/7$, $m_1 = -5/7$, $p = 15.6$, $q = 30$, $b = 1$, and $g = 1$. Let the leader subsystem be described by the equation

$$\dot{\overline{x}} = A_L \overline{x} + B_L(\overline{u} + \psi_0(\overline{y})), \quad \overline{y} = C^{\mathrm{T}} \overline{x},$$

where $\overline{x} \in \mathbb{R}^3$ is the state vector of the system, $\overline{y} \in \mathbb{R}^1$ is the output available for measurement, and $\overline{u}$ is the scalar control variable, $\psi_0(\overline{y}) = pv(\overline{y})/b$, where $v(x) = -0.5(m_0 - m_1)(|x+1| - |x-1| - 2x)$. Further, let

$$A_L = \begin{pmatrix} -1 & 0 & 0 \\ 1 & -1 & 1 \\ 0 & -q & 0 \end{pmatrix},$$

$B_L = \mathrm{col}(b, 0, 0), C = \mathrm{col}(1, 0, 0)$.

Transfer function $\chi(s) = C^{\mathrm{T}}(sI - A_L)^{-1} B_L = (s^2 + s + 30)/(s^3 + 2s^2 + 31s + 30)$. It is seen from the Nyquist plot of $\chi(i\omega) \forall \omega \in \mathbb{R}^1$ presented on Figure 1 that first frequency domain inequality of (14) holds. The second frequency domain inequality of (14) also holds because the relative degree of $\chi(s)$ is equal to one and the highest coefficient of its numerator is positive.

Obviously, $\psi_0(\cdot)$ is g-monotonically decreasing.

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DOI: 10.1002/acs

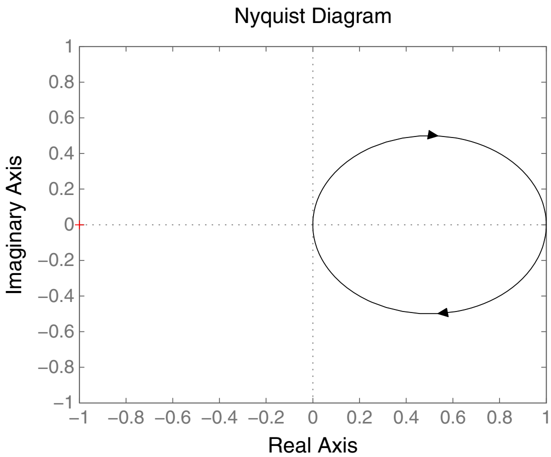

Figure 1. Nyquist plot of $\xi(i\omega), \omega \in \mathbb{R}^1$.

Let the set Ξ consist of two elements: $\Xi = \{\xi_1, \xi_2\}$. Let subsystem S_i for $i = 1, \dots, 5$ be described by (8) with $u_i, \alpha_{ij} \in \mathbb{R}^1$. By choosing $(\nu_1(\xi_1), \nu_2(\xi_1), \nu_3(\xi_1), \nu_4(\xi_1), \nu_5(\xi_1)) = (3, 1, 4, 1, 5)$, $(\nu_1(\xi_2), \nu_2(\xi_2), \nu_3(\xi_2), \nu_4(\xi_2), \nu_5(\xi_2)) = (-9, -2, -6, -5, -3)$, $\theta_i(\xi_1) = \theta_i(\xi_2) = 1/i$, $i = 1, \dots, 5$ and using (13), we obtain matrices A_i, B_i for $i = 1, \dots, 5$, which are not equal, that is, nodes are nonidentical. Let $\varphi_{ij}(x_i - x_j)$ be identically equal to $(0, 0, 0)^{\mathrm{T}}$ for $(i, j) = \{(1, 4), (2, 5), (3, 2), (4, 2), (4, 5), (5, 2), (5, 3)\}$. Further, let

$$
\begin{aligned}
&\varphi_{12}(\xi_1) = (\sin(x_{11} - x_{21}), 0, 0)^{\mathrm{T}}, \quad \varphi_{13}(\xi_1) = (0, x_{12} - x_{32}, 0)^{\mathrm{T}},\\
&\varphi_{15}(\xi_1) = (0, 0, \sin(x_{13} - x_{53}))^{\mathrm{T}}, \quad \varphi_{21}(\xi_1) = (x_{21} - x_{11}, 0, x_{23} - x_{13})^{\mathrm{T}},\\
&\varphi_{23}(\xi_1) = (0, \sin(x_{22} - x_{32}), 0)^{\mathrm{T}}, \quad \varphi_{24}(\xi_1) = (0, x_{22} - x_{42}, 0)^{\mathrm{T}},\\
&\varphi_{31}(\xi_1) = (\sin(x_{31} - x_{11}), 0, 0)^{\mathrm{T}}, \quad \varphi_{34}(\xi_1) = (\sin(x_{31} - x_{41}), 0, 0)^{\mathrm{T}},\\
&\varphi_{35}(\xi_1) = (x_{31} - x_{51}, x_{32} - x_{52}, x_{33} - x_{53})^{\mathrm{T}}, \quad \varphi_{41}(\xi_1) = (0, \sin(x_{42} - x_{12}), 0)^{\mathrm{T}},\\
&\varphi_{43}(\xi_1) = (\sin(x_{41} - x_{31}), 0, 0)^{\mathrm{T}}, \quad \varphi_{51}(\xi_1) = (x_{51} - x_{11}, 0, x_{53} - x_{13})^{\mathrm{T}},\\
&\varphi_{54}(\xi_1) = (0, x_{52} - x_{42}, 0)^{\mathrm{T}}.
\end{aligned}
$$

Further, let $\varphi_{ij}(\xi_2) = \varphi_{ji}(\xi_1), i = 1, \dots, 5, j = 1, \dots, 5$. Lipschitz constants of all φ_{ij} are equal to one.

It follows from Theorem 2 that decentralized adaptive control (10) provides synchronization goal (9) if for all $i = 1, \dots, 5$ inequality $\sum_{j=1}^{5} |\alpha_{ij}| < \gamma$ holds, i.e. if interconnections are sufficiently weak.

4.2. Simulation results

Consider the following control of leader subsystem $\overline{u} = \frac{1}{b}[(-(1 + m_0)p + 1)\overline{x}_1 + p\overline{x}_2]$. Such $\overline{u}$ ensures chaotic behavior of leader subsystem. Let us put $\Gamma_i = I, i = 1, \dots, d$, where I is the identity matrix and

$$
\begin{aligned}
&\overline{x}_1(0) = 0.5, \quad \overline{x}_2(0) = 0, \quad \overline{x}_3(0) = 0,\\
&x_1(0) = (7, 14, 0.4)^{\mathrm{T}}, \quad x_2(0) = (0, 4, 4)^{\mathrm{T}}\\
&x_3(0) = (1, -1, 4.5)^{\mathrm{T}}, \quad x_4(0) = (3, -4, 0.2)^{\mathrm{T}}\\
&x_5(0) = (2, 8, 15).
\end{aligned}
$$

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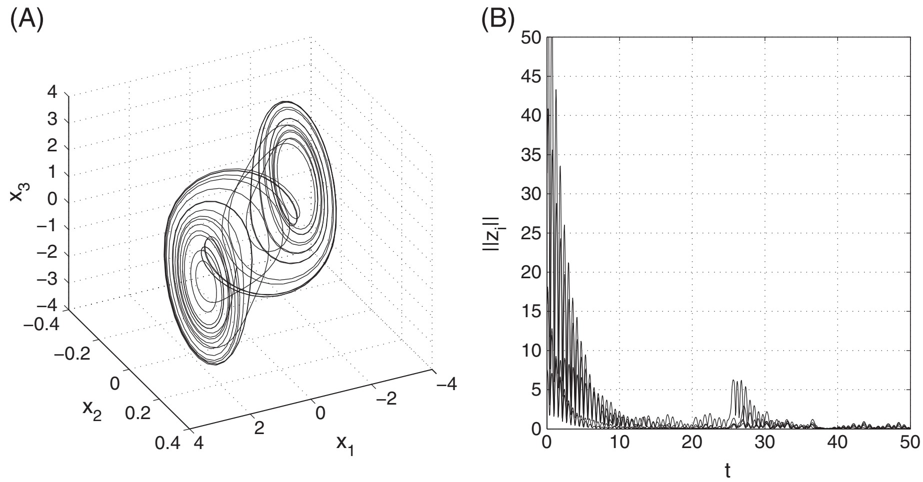

Figure 2. (A) Phase portrait of the leader; (B) Time histories of $||Z_i(t)||, i = 1, \ldots, 5$.

Denoted by α 5×5 matrix with element α_{ij} lying in the ith row and the jth column, i, $j = 1, \ldots, 5$, and

$$\widehat{\alpha} = \begin{pmatrix} 0 & 0.0051 & 0.1395 & 0 & 0.1676 \\ 0.0662 & 0 & 0.0921 & 0.0065 & 0 \\ 0.2013 & 0 & 0 & 0.2271 & 0.1430 \\ 0.0907 & 0 & 0.0675 & 0 & 0 \\ 0.0663 & 0 & 0 & 0.2773 & 0 \end{pmatrix}.$$

Let

$$\xi = \begin{cases} \xi_1, & 0 \leqslant t < 25, \\ \xi_2, & 25 \leqslant t \leqslant 50, \end{cases}$$

that is, parameters of plants and interconnections change at $t = 25$ in the way, described in the previous subsection. Let us choose adaptive control $u_i, i = 1, \ldots, 5$ as in (10) and apply Theorem 2.

For $\alpha = \widehat{\alpha}$, simulation shows that $\|z_i\| \to 0, i = 1, \ldots, 5$, that is, synchronization is achieved: the difference between the state vectors of the nodes and the state vector of the leader subsystem converges to zero, see Figure 2(B). We see that the designed controller achieves synchronization for two values of ξ. Phase portraits of the leader subsystem and $\|z_i\|, i = 1, \ldots, 5$ found by simulation for 50 s are shown on Figure 2.

5. CONCLUSIONS

In contrast to a large number of previous results, in this paper, the decentralized adaptive control algorithm and synchronization conditions are obtained for heterogeneous networks consisting of nonlinear systems with incomplete measurement, incomplete control, and incomplete information about system parameters and coupling. The design of the control algorithm providing synchronization property is based on speed-gradient method [12], whereas derivation of synchronizability conditions is based on Yakubovich–Kalman lemma and the result presented in [11]. Detailed derivation of adaptation law and example of synchronization in network of nonidentical Chua circuits with changing plants parameters and changing interconnections are also presented.

The designed adaptive controller uses only input and output of leader and outputs of followers and does not use agent or interconnection parameters. It allows designer to apply the proposed controller to different classes of networks under weaker conditions than other existing methods.

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ACKNOWLEDGEMENTS

This research was supported by the Russian Foundation for Basic Research (RFBR) with grant no. 11-08-01218, the Programs of Basic Research of OEMPPU RAS with grant no. 10,14, and the Russian Federal Program 'Cadres' (Contracts 16.740.11.0042, 14.740.11.0942).

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