5th IFAC International Workshop on Periodic Control Systems
The International Federation of Automatic Control
July 3-5, 2013. Caen, France

Combined Speed-gradient Controlled Synchronization of Multimachine Power Systems ⋆

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Abstract: Controlled synchronization problem for multimachine power system with losses is considered. Conditions of existence of invariants in the system are obtained. Synchronization algorithm based on the speed-gradient method is designed. Simulation results evaluating performance of the closed loop system are presented.

Keywords: Transient Stability, Speed Gradient Algorithm, Multimachine Power System

1. INTRODUCTION

Stable synchronous behavior of complex energy systems is important for reliable energy generation. Mathematical formulation of proper behavior of energy systems is based on the notion of transient stability introduced by J.L.Willems in 1974. Transient stability is concerned with a power system ability to reach an acceptable steady-state following a fault, e.g., a short circuit or a generator outage, that is later cleared by the protective system operation [Anderson, Fouad (1977)], [Pai (1981)], [Kundur (1994)]. The fault modifies the circuit topology driving the system away from the stable operating point and the question is whether the trajectory will remain in the basin of attraction of this (or other) equilibrium after the fault is cleared.

Studying transient stabilization of power systems is based on the use of aggregated reduced network models that represent the system as an n port system described by a set of nonlinear ordinary differential equations. Several excitation controllers that establish Lyapunov stability of the desired equilibrium of these models have been reported. These nonlinear controller design techniques include feedback linearization [Guo, Hill, Wang (2001)], interconnection and damping assignment passivity based control, speed-gradient-passivity control, see [Gordon, Hill (2006)], [Dib et al. (1999)], [Ortega et al. (2005)], [Poromsky, Fradkov, Hill (1996)].

Insofar an issue of transient stability is associated with a large-disturbance rotor angle stability [Kundur et al. (2004)], the main task in this paper will be to develop a control algorithm providing synchronization of the generators in the system after some faults, whereas the reason of these faults is not known. The model of the multimachine power system from [Ortega et al. (2005)] will be considered, where, in contrast to [Poromsky, Fradkov, Hill (1996)], the model of a single generator includes three variables – angle and angular velocity of the rotor of the generator and the internal voltage of the generator. It is suggested that the control input is carried out only in the internal voltage of the generator.

The key idea of our approach is using an invariant function (or an invariant functional), depending on the system variables for control design. It makes the task of control design much easier. For the system under consideration conditions under which in the system exists such invariant functional will be presented. Synchronization algorithm based on controlling of the invariants with using the speed-gradient method [Fradkov (1979)] and conditions for their applicability will be obtained.

The structure of the paper is as follows. In section 2 a speed-gradient algorithm and a technique based on it are given. The mathematical model of the multimachine power system is presented in Section 3. Section 4 includes the application of the proposed technique for solving the task of transient stabilization. Section 5 presents the results of the numerical simulation. Some notes and conclusions are provided in Section 6.

2. METHODS

In this section a brief overview of the speed-gradient method is provided and synchronization algorithm is designed.

2.1 Speed Gradient Algorithm

For completeness a scheme of speed gradient algorithm is presented here. Details can be found in [Miroshnik,

⋆ This work was supported by Russian Foundation for Basic Research (project 11-08-01218), and Russian Federal Program "Cadres" (agreements 8846, 8855).

Nikiforov, Fradkov (2000)] or [Fradkov (2007)]. Consider the controlled system

$$\dot{x} = F(x, u, t), t \geq 0, \quad (1)$$

where $x \in R^n$ are state variables, $u \in R^m$ are control inputs, a vector function $F(\cdot) : R^{n+m+1} \to R^n$ is piecewise continuous in t and continuously differentiable in x, u. Introduce the control goal

$$Q(t) \leq \Delta, \; t > t_*, \quad (2)$$

where Q_t is a goal functional (values Δ, t_* can be specified or not, depending on the task). We distinguish between two primary types of the goal functional: the local functional

$$Q_L = Q_L(x(t), t), \quad (3)$$

where $Q_L(x, t) \in R^{n+1}$ is a scalar function; and the integral functional

$$Q_I = \int_0^t R(x(\tau), u(\tau), \tau)\, d\tau, \quad (4)$$

where $R(x, u, t) \in R^{n+m+1}$ is a scalar function.

In practice sometimes it is convenient to use the combined functionals, and the most widely used one is the sum of local (3) and integral (4) functionals:

$$Q = Q_L(x, u, t) + \alpha \int_0^t R(x, u, \tau)\, d\tau. \quad (5)$$

The speed gradient algorithm for the functional (5) in differential form is

$$\frac{du}{dt} = -\Gamma \nabla_u [\omega(x, u, t) + \alpha R(x, u, t)], \quad (6)$$

with symmetric positive definite matrix gain $\Gamma = \Gamma^T \geq 0$. The finite-differential form of speed-gradient algorithm for the functional (5) is

$$\frac{d[u + \gamma\psi(x, u, t)]}{dt} = -\Gamma \nabla_u \omega(x, u, t). \quad (7)$$

where a symmetric matrix $\Gamma = \Gamma^T \geq 0$, a function $\omega(\cdot)$ is a speed of changing of the local functional (3) along trajectories of the system (1), and a function $\psi(\cdot)$ satisfies pseudogradient condition $\psi^T \bigtriangledown_u \omega \geq 0$.

2.2 Control algorithm design. Key lemma

Consider a system

$$\dot{z} = f(z) + g(z)\, u, \quad (8)$$

where $z = (x, y)^T \in R^{k+m}$ are state variables, where $x \in R^k$, $y \in R^m$; $u \in R^m$ are control inputs. Set the control inputs as

$$u_i = \dot{y}_i, \; i = 1, \ldots, m. \quad (9)$$

Thereby the system (8) has m controlled variables y, and k uncontrolled variables x.

Introduce a functional

$$V = \sum_{i=1}^{N} \left[\frac{1}{2} (y_i - y_{di})^2 + \right. \\ \left. + \int_0^t \left((y_i - y_{di} - p_i)^2 - p_i u_i \right) d\tau \right], \quad (10)$$

where $p_i = p_i(x)$ are smooth functions, $z = (x, y)^T \in R^{k+m}$, $y_{di} = const > 0$, $i = 1, \ldots, m$.

This functional (10) is a particular case of the general formula (5).

Proposition 1. [Pchelkina, Fradkov (2012)] The functional V is an invariant of the system (8), (9) closed by controls $u_i = u_{i*}$, where

$$u_{i*} = -[y_i - y_{di} + p_i(x)], \; i = 1, \ldots, m. \quad (11)$$

Proof. Calculate the time derivative of the function (10) along trajectories of the system (8) and choose control inputs as (11):

$$\dot{V} = \sum_{i=1}^{m} \left[(y_i - y_{di}) \cdot u_i + \left((y_i - y_{di} - p_i)^2 - p_i u_i \right) \right] = \\ = \sum_{i=1}^{m} \left[(y_i - y_{di} - p_i) \cdot u_i + (y_i - y_{di} - p_i)^2 \right] = \\ = \sum_{i=1}^{m} \left[-(y_i - y_{di} - p_i)^2 + (y_i - y_{di} - p_i)^2 \right] \equiv 0. \quad (12)$$

Now introduce a functional Q with real $V_d > 0$

$$Q = |V(z) - V_d|. \quad (13)$$

Using the functional (13) as the goal function, design the speed-gradient algorithm:

$$u_i = -\gamma_i sign(V - V_d) [y_i - y_{di} + p_i(x)], \quad (14)$$

where $i = 1, \ldots, m$.

Lemma. Let $\gamma_i > 1$, $i = 1, \ldots, m$. Then $V(t) \to V_d$, or $p_i(\delta, \omega) \to const$ in the closed loop system (8), (9), (14).

Proof. Calculate the speed of changing of the functional (13) :

$$\dot{Q} = sign(V - V_d) \cdot \dot{V} = sign(V - V_d) \times \\ \times \sum_{i=1}^{m} [(y_i - y_{di}) u_i + y_i - y_{di} - p_i - p_i u_i] = \\ = sign(V - V_d) \cdot \sum_{i=1}^{m} (-\gamma_i sign(V - V_d) + 1) \times \\ \times (p_i - y_i + y_{di}) \leq -(\gamma_i - 1) \sum_{i=1}^{m} (p_i - y_i + y_d i)^2 \leq 0, \quad (15)$$

where

$$\gamma = \min_i \gamma_i > 0. \quad (16)$$

From (15) we have $Q(t) \geq 0$, therefore $\lim Q(t) = Q_*$ as $t \to \infty$ exists. Then both $\lim V(t) = V_*$ and $\lim y(t) = y_*$ exist. If $V_* = V_d$, then Lemma is proven.

Let $V_* \neq V_d$. Then from the inequality

$$\int_0^t (p_i - y_i + y_{0i})^2 \, d\tau \leq Q_0 / (\gamma - 1), \quad (17)$$

valid for all $t > 0$, and from Barbalat lemma [Miroshnik, Nikiforov, Fradkov (2000)], it follows that

$$p_i(x(t)) - y_i(t) + y_{0i} \to 0, \quad (18)$$

and we have $p_i(x(t)) \to const$. Lemma is proven.

Proposition 2. Take $p_i(x) = \mu_i \omega_i + \delta_i$, where $\mu_i > 0$, $x = (\delta, \omega)^T \in R^k$, $\delta \in R^{\frac{k}{2}}$, $\omega \in R^{\frac{k}{2}}$ (for even k) and $\dot{\delta}_i = \omega_i$. Then from properties of stable linear systems we obtain $\delta_i \to const$ and $\omega_i \to 0$.

3. MULTIMACHINE POWER SYSTEMS

In this section, a new solution to the problem of transient stabilization of multimachine power systems with nonnegligible transfer conductances is given. More specifically, the full $3N$ dimensional model of the generator system with lossy transmission lines and loads is considered. A nonlinear state feedback law for the generator excitation field that ensures asymptotic stability of the operating point is proposed. To design the control law the proposed speed-gradient technique (see Section 2) is applied.

3.1 Mathematical Model

Consider the multimachine power system consisting of N generators interconnected through a transmission network which is assumed to be lossy, that is, the presence of transfer conductances is taken into account. The dynamics of the i-th machine with excitation is represented by the classical three-dimensional flux decay model [Ortega et al. (2005)]:

$$\begin{cases} \dot{\delta}_i = \omega_i, \\ \dot{\omega}_i = -D_i \omega_i + P_{mi} - G_i E_i^2 - \\ -E_i \sum\limits_{j=1, j\neq i}^{N} E_j Y_{ij} \sin(\delta_i - \delta_j + \alpha_{ij}), \\ \dot{E}_i = f_i + v_i, \ i = \overline{1, N}. \end{cases} \quad (19)$$

where $i = 1, \ldots, N$, N is a number of generators. The state variables of this subsystem are the rotor angle δ_i, the rotor speed ω_i and the quadrature axis internal voltage E_i. The control input is the field excitation signal v_i. The parameters G_{ij}, B_{ij}, and G_{ii} are, respectively, the conductance, susceptance and self-conductance of the ith generator:

$$Y_{ij}^2 = G_{ij}^2 + B_{ij}^2, \tan \alpha_{ij} = \frac{G_{ij}}{B_{ij}}.$$

D_i represents the damping coefficient, P_{mi} – the mechanical power, which is assumed to be constant.
f_i – the known function:

$$f_i = -a_i E_i + b_i \sum_{j=1, j\neq i}^{N} E_j \cos(\delta_i - \delta_j + \alpha_{ij}) + E_{fi},$$

whereas E_{fi} represents the constant component of the field voltage,

$$a_i = (1 - B_{ij}(x_{id} - x'_{id})) / T_{di},$$
$$b_i = Y_{ij}(x_{di} - x'_{di}) / T_{di},$$

where $a_i > 0$, $b_i > 0$, and x_{di}, x'_{di} represent the direct-axis – synchronous and transient – reactances; note that all parameters are positive and $x_{di} > x'_{di}$.

3.2 Problem Formulation

In the transient stability tasks it is required that the differences between rotor speeds ω_i vanish while rotor angles δ_i tend to some constant values. The formal definition of transient stability for power systems was introduced by J.L. Willems [Willems (1974)]:

Definition. Trajectory $x(t_0, x_0)$ of the uncontrolled system (19) is called *transient stable* if its initial point $x_0 = (\delta_0, \omega_0)^T$ belongs to the attracting domain defined by the equations

$$\delta_i - \delta_j = c_{ij}; \ \omega_1 = \omega_2 = \ldots = \omega_N = 0. \quad (20)$$

Some additional conditions are needed in order to prevent the possible loss of synchronism during transition processes [Anderson, Fouad (1977)]: assuming that the initial point $x_0 = (\delta, \omega, E)^T$ belongs to the domain

$$\begin{array}{l} 0 < \delta_i < \overline{\delta} < \pi/2, \ |\omega_i| < \overline{\omega}, \\ E_i = E_{di} = const > 0, \ i = \overline{1, N}, \end{array} \quad (21)$$

the trajectory $x(t_0, x_0)$ stays in (21) for all $t > 0$.

Now we can introduce the control goal: for given initial conditions it is necessary to provide that the trajectories $\delta(t, \delta_0)$, $\omega(t, \omega_0)$, $E(t, E_0)$ tend to the domains (20), (21).

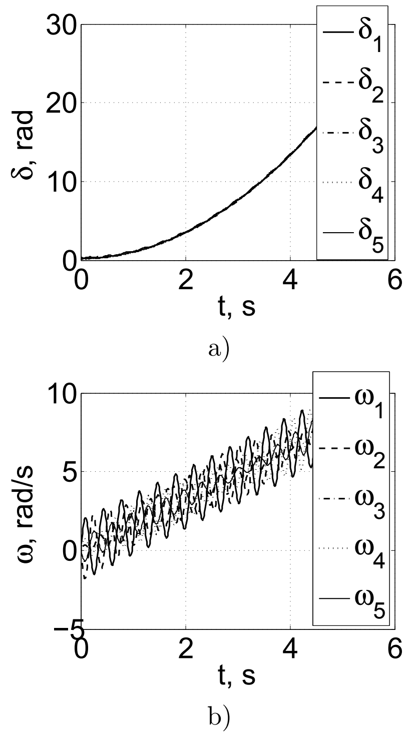

Fig. 1. Uncontrolled power system (19): a) the rotor angles δ, b) the rotor speeds ω.

3.3 Main Result

In order to use results from the previous section, reduce the system (19) to the form (8). Introduce new control inputs in the system (19):

$$u_i = f_i + v_i,\ i = 1, \ldots, N, \tag{22}$$

Then the equations for the quadrature axis internal voltages E_i are

$$\dot{E}_i = u_i,\ i = 1, \ldots, N, \tag{23}$$

and the system (19) is

$$\begin{cases} \dot{\delta}_i = \omega_i, \\ \dot{\omega}_i = -D_i\omega_i + P_{mi} - G_i E_i^2 - \\ -E_i \sum\limits_{j=1, j\neq i}^{N} E_j Y_{ij} \sin\left(\delta_i - \delta_j + \alpha_{ij}\right), \\ \dot{E}_i = u_i.\ i = \overline{1, N}, \end{cases} \tag{24}$$

with $z = (\delta, \omega, E)^T \in R^{3N}$, $g(z) = (0, 0, 1, \ldots, 0, 0, 1)^T \in R^{3N}$ and $f(z) = (f_{\delta_1}, f_{\omega_1}, 0, \ldots, f_{\delta_N}, f_{\omega_N}, 0)^T \in R^{3N}$, where f_{δ_i}, f_{ω_i} are corresponding right parts of (24). As in (8), in the system (24) controls appear in the equations for a part of variables (these are quadrature axis internal voltages E_i).

Following the technique proposed in Section 2, introduce the following goal functional of (24):

$$V = \sum_{i=1}^{N} \left[\frac{1}{2}\left(E_i - E_{di}\right)^2 + \right. \\ \left. + \int\limits_0^t \left(\left(E_i - E_{di} - p_i\right)^2 - p_i u_i \right) d\tau \right], \tag{25}$$

where $p_i = p_i(\delta, \omega)$ are some smooth functions, $z = (\delta, \omega, E)^T \in R^{3N}$, $E_{di} = const > 0$.

According to Proposition 1 the functional V is invariant for the controlled system (19), (22), (23) closed by $u_i = u_{i*}$, where

$$u_{i*} = -\left[E_i - E_{di} + p_i(\delta, \omega)\right],\ i = 1, \ldots, N. \tag{26}$$

Introduce the functional Q for real V_d

$$Q(z) = |V(z) - V_d|. \tag{27}$$

Using the functional (27) as the goal one, design the speed-gradient algorithm:

$$u_i = -\gamma_i sign\left(V - V_d\right)\left[E_i - E_{di} + p_i(\delta, \omega)\right], \tag{28}$$

for $i = 1, \ldots, N$.

According to Lemma from Section 2 for $\gamma_i > 1$, $i = 1, \ldots, N$ we have $V(t) \to V_d$, or $p_i(\delta, \omega) \to const$ in the closed loop system (19), (22), (23), (28).

Choosing $p_i = \mu_i\omega_i + \delta_i$, where $\mu_i > 0$ we obtain $\delta_i \to const$ and $\omega_i \to 0$. Thus, rotor speeds ω_i vanish while rotor angles δ_i tend to some constant values. So the designed control provides that the trajectories $\delta(t, \delta_0)$, $\omega(t, \omega_0)$, $E(t, E_0)$ tend to the domain (20), while about the domain (21) Lemma says nothing. However, choosing μ_i, E_{di}, V_d we can obtain (21) and achieve the control goal.

4. SIMULATION RESULTS

In this section simulation results for the system consisting of five synchronous machines are presented. The parameter values of the system were taken from [Ortega et al. (2005)]. These parameter values correspond to the case after the fault: the fault has been cleared, but the trajectory of the system has left its stability domain. Fig. 1 shows this case: the rotor angles and speeds are unbounded and there is no transient stabilization in the system.

Fig. 2 and Fig. 3 show the closed loop system: it is seen that the proposed technique provides transient stabilization of the system. Moreover, it is possible to control the value of the auxiliary invariant functional V. Finally, in Fig. 4 the proposed control inputs (26) are seen. It is important that all control signals tend to zero.

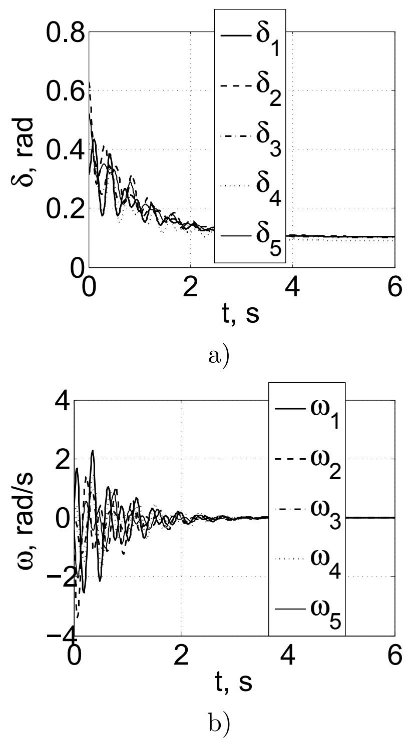

Fig. 2. Controlled power system: a) the rotor angles δ, b) the rotor speeds ω.

5. CONCLUSION

We considered the transient stabilization problem for the model of the multimachine power system. The invariant functional for the system is defined and the combined speed gradient algorithm based on controlling of this invariant functional is proposed. Convergence of the system to the synchronous regime is proven analytically and its performance is studied by numerical simulation for the example of the system consisting of five generators.

The proposed algorithm was designed under assumption that all system variables can be measured. However, in practice due to lack of appropriate measurement devices necessary signals can not always be measured. In addition, even a small change in any of the unaccounted parameters can lead to the loss of stability of the system. This problem is particularly important in emergency conditions, when

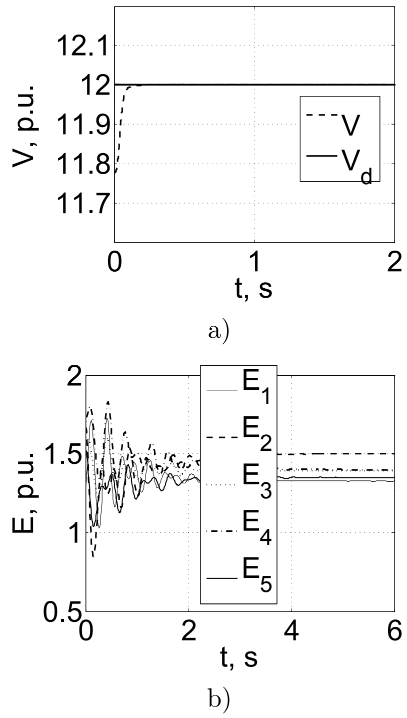

Fig. 3. Controlled power system: a) the functional V, b) the internal voltages E.

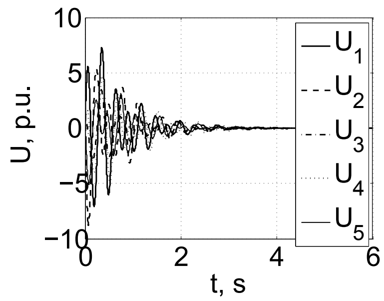

Fig. 4. Controlled power system: control signals.

the parameters and variables of the system or of its individual parts can vary significantly during a short period of time. Alternative approaches may be adaptive and robust control methods, such as in [Fradkov, Furtat (2012)]. Therefore, a possible further development of this study could be to design an adaptive control algorithm.

Besides, here as in most existing studies, aggregate models of power systems have been used; these aggregated models erase the identity of the network components and impose an unrealistic treatment of the loads. So the next stage of this study also could be to abandon the aggregated n port view of the network and consider the more natural and widely popular structure preserving models, first proposed in [Bergan (1986)]. Thus, the obtained results can be useful in further control design of multimachine power systems.

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