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Control of Passage through a Resonance Area during the Start of a Two-Rotor Vibration Machine

D. A. Tomchin and A. L. Fradkov

St. Petersburg

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Abstract—Control of the passage of a two-rotor vibration machine through a resonance area is studied. A control algorithm based on the speed gradient method is proposed. The possibility of accelerating unbalanced vibration exciters to velocities exceeding critical resonance values for a limited level of the control signal is studied. The control system robustness with respect to variation in the suspension rigidity of the carrying body, the rotation resistance factor, and rotor eccentricity is analyzed.

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Vibration machines with unbalanced rotors are widely used in modern industry. It is well known that the maximum power of motors is required when a vibration machine is started and speeds up. If this power is reduced, the nominal power of electric motors, as well as their mass and dimensions, is reduced and the noise level during machine operation drops. To implement the desirable operation mode of a machine, it is necessary to control rotor velocity in a wide range, including the resonance area and the areas outside resonances [1]. But, if motor power is decreased in systems with several degrees of freedom, the Sommerfeld effect can appear [2–5] and rotor angular velocity capture can occur.

The Sommerfeld effect appears when the rotor rotation frequency becomes close to the resonance frequency of the carrying body. Its influence is especially noticeable for weakly damped objects and prevents passage through the resonance area during acceleration and deceleration of different industrial machines. It is especially important to overcome the Sommerfeld effect in design of vibration machines with rotors having great eccentricity.

One method of suppressing the Sommerfeld effect is controlling the motor during the acceleration stage. In [6] the so-called double start method was suggested. The idea of this method is to introduce into the motor control circuit a time relay enabling the motor to be switched off and on repeatedly at certain moments of time. However, the moments of switching the motor on and off should be calculated in advance. This approach is a procedure of program control characterized by a high complexity of calculations and sensitivity to model errors and noises.

With the development of computing machinery, controllers with feedback are being used ever more widely. Different approaches to the synthesis of control systems for unbalanced rotor acceleration are suggested in [7, 8]. In [7], an optimal control law was proposed that was developed on the basis of the maximum principle of Pontryagin. But practical realization of the optimal law is difficult since it is necessary to solve numerically the task of optimal control of a nonlinear object. This task is solved using successive approximations and requires knowledge of system parameters and initial conditions.

In [1, 9, 10], for synthesis of a control algorithm permitting passage through a resonance, it was suggested to use the speed gradient procedure [11, 12] extended to tasks of control by nonlinear oscillations [13]. However, in [1, 9, 10], only problems where the carrying body performs 1D motion were studied. Control of resonance passage for a rotor with an elastic shaft performing a planar motion was described in [14] using the speed gradient method.

Here, the passage of a two-rotor vibration machine through a resonance area is studied. A control algorithm based on the speed gradient method is suggested. The possibility of accelerating unbalanced vibration exciters up to velocities higher than the critical resonance ones if the control signal is limited is also examined. The system robustness with respect to variation in the suspension rigidity of the carrying body, the rotation resistance factor, and rotor eccentricity is also studied.

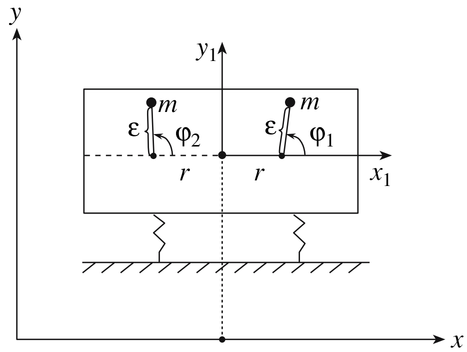

Fig. 1.

Statement of the Problem

Let us examine the task of acceleration and reaching the operation point for a two-rotor vibration machine consisting of two rotors placed on a carrying body connected elastically with fixed base. We will assume that the carrier is orientated horizontally and performs an in-plane parallel motion (Fig. 1). Such an assumption is correct for a high spring force.

The dynamic equations of the system can be written as

$$\begin{aligned} J\ddot{\varphi}_1 + k_\varphi\dot{\varphi}_1 + m\varepsilon g\cos\varphi_1 &= m\varepsilon(\ddot{x}\sin\varphi_1 - \ddot{y}\cos\varphi_1) + u_1(t), \\ J\ddot{\varphi}_2 + k_\varphi\dot{\varphi}_2 + m\varepsilon g\cos\varphi_2 &= m\varepsilon(\ddot{x}\sin\varphi_2 - \ddot{y}\cos\varphi_2) + u_2(t), \\ (2m+M)\ddot{x} + k_x\dot{x} + c_x x &= m\varepsilon(\ddot{\varphi}_1\sin\varphi_1 + \ddot{\varphi}_2\sin\varphi_2 + \dot{\varphi}_1^2\cos\varphi_1 + \dot{\varphi}_2^2\cos\varphi_2), \\ (2m+M)\ddot{y} + k_y\dot{y} + cy + (2m+M)g & \\ = m\varepsilon(-\ddot{\varphi}_1\cos\varphi_1 - \ddot{\varphi}_2\cos\varphi_2 + \dot{\varphi}_1^2\sin\varphi_1 + \dot{\varphi}_2^2\sin\varphi_2), & \end{aligned} \tag{1}$$

where φ_1 and φ_2 are the rotor angles of rotation; y is the carrier deviation from the equilibrium position; $u_1(t)$ and $u_2(t)$ are the control actions (motor torques); J is the moment of inertia of the unbalanced rotors; m are the rotor masses; M is the mass of the carrier; ε are the eccentricities of the rotor centers of masses; c and c_x are the spring forces along the vertical and horizontal axes, respectively; k_φ is the coefficient of viscous friction in the bearing; and k_x and k_y are dumping coefficients. The system state vector has the form $z = [x, \dot{x}, y, \dot{y}, \varphi_1, \dot{\varphi}_1, \varphi_2, \dot{\varphi}_2]^T$. At small levels of control action $u_i(t) \equiv (-1)^i M_0$, $i = 1, 2$, in the near-resonance area, the rotor's angular velocity is captured, but, if the control moment is increased, the rotor passes the resonance area and accelerates up to the prescribed angular velocity.

Figure 2 (passage through the resonance area with $M_0 = 0.42$ N m, $T_\psi = 0.35$ s (solid lines)) shows the results of simulation of system (1) for the base parameters of the system: $J = 0.014$ kg m^2, $m = 1.5$ kg, $M = 9$ kg, $\varepsilon = 0.04$ m, $k_\varphi = 0.01$ J s, $k_x = k_y = 5$ kg/s, $c = 5300$ N/m, $c_x = 1300$ N/m, and constant control moments $M_0 = 0.65$ N m (internal dotted lines, capture) and $M_0 = 0.66$ N m (external dotted lines, passage). The question arises of whether it is possible to decrease the maximum value of the control moment needed for passage through the resonance area by changing the control moment using feedback according to measured signals coming from the vibration machine.

Based on the aforementioned, we study how to find the control algorithm $u_i = U(z)$, where $z = [x, \dot{x}, y, \dot{y}, \varphi_1, \dot{\varphi}_1, \varphi_2, \dot{\varphi}_2]^T$ is a state vector of the control object guaranteeing acceleration of unbalanced vibration exciters up to velocities higher than the critical resonance ones in the case where the control signal $|u_i(t)| \le M$ is limited ($M < M_{cr}$). It is assumed that all state variables in system (1) are measurable.

Synthesis of the Control Algorithm

The main idea of the suggested approach is to separate the motions into quick and slow components appearing near the resonance zone. According to [7, 15, 16], slow "pendulum" motion can appear near the resonance zone if the relative eccentricity of the rotating subsystem is small. The results presented in the mentioned works show that the frequency of slow oscillating motions tends to zero if the angular velocity of rotation ω tends to the resonance frequency of the carrying subsystem ω^*. This conclusion is correct for constant control torque $u = u_0$, which means that the static performance of the electric motor is horizontal. It also holds for a small negative slope of the electric motor's static characteristics, which is equivalent to an additional small damping present in the initial dynamic equations of the system. The only condition of slow motion appearing near the resonance zone (i.e., under $\omega \to \omega^*$) is the condition that the control torque $u = u_0$ is not enough for passage of the resonance (the capture phenomenon).

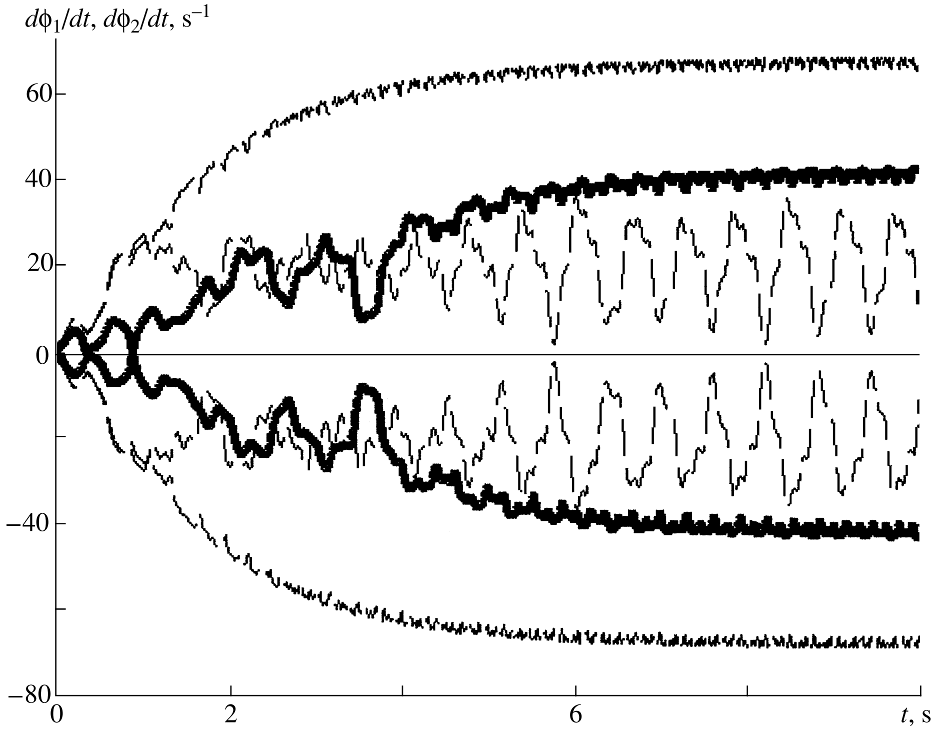

Fig. 2.

In order to synthesize the control algorithm, let us use the speed gradient method. Application of the method starts with setting the criterion functional $Q(z)$; if this value decreases, it means that the goal of the control has been reached. At the synthesis stage, let us assume that the object of control is a conservative system; i.e., there is no friction in the system. Then the goal of the control is fulfilled if the prescribed total energy level $H(x, \dot{x}, y, \dot{y}, \varphi_1, \dot{\varphi}_1, \varphi_2, \dot{\varphi}_2) = H^*$ is reached and the square deviation of the total energy of the system from the prescribed value H^* can be chosen as the criterion functional; i.e.,

$$Q(z) = 1/2(H(z) - H^*)^2,$$

where $z = [x, \dot{x}, y, \dot{y}, \varphi_1, \dot{\varphi}_1, \varphi_2, \dot{\varphi}_2]^T$ is the total state vector of the system. Then it is necessary to write the gradient of $Q(z)$ according to Eq. (1). We will assume that the object is represented in Hamiltonian form

$$\dot{q} = \partial H/\partial p, \ \dot{p} = -(\partial H/\partial q) + Bu,$$

where q and p are the generalized coordinates and momenta, respectively, $q = [\varphi_1, \varphi_2, x, y]^T$, $B = \begin{bmatrix} 1, 0, 0, 0 \\ 0, 1, 0, 0 \end{bmatrix}^T$. The rate of change in $\dot{Q}(z)$ is then $\dot{Q}(z) = (H - H^*)(\dot{\varphi}_1 u_1 + \dot{\varphi}_2 u_2)$.

Then the control algorithm is synthesized, in the right part of which we write a function whose sign is opposite that of the value $\partial \dot{Q}/\partial u_i$. One of the most widely used forms of the speed gradient algorithm is the relay algorithm

$$u_i = (-1)^i M_0 \operatorname{sgn}[(H - H^*)\dot{\varphi}_i]. \quad (2)$$

However, algorithm (2) is good only for systems with one degree of freedom. In our case, the control according to Eq. (2) includes interfering rapidly oscillating components appearing due to motion interconnection and also due to the Sommerfeld effect.

The principle of operation of the method is the following: the slow motion is separated and “swung” in order to increase the energy of the rotating subsystem. To separate the slow motion, we add a low-pass filter to the control algorithm. During small damping, the slow motion also dies out slowly. Thus, the algorithm

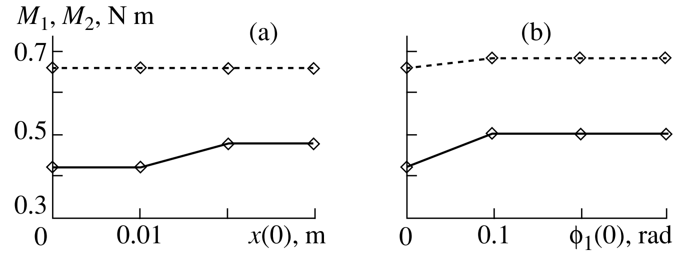

Fig. 3. Dependence of the minimal control action under which the system passes through the resonance area on (a) the initial displacement $x(0)$ and (b) $\varphi_1(0)$ under a constant torque (dotted line) and for control according to algorithm (3) (solid line).

is able to create a situation for passage of the resonance area, and after that it is possible to switch off the "swinging" and to use constant moment control.

It is suggested to introduce into the algorithm an additional filter of lower frequencies suppressing undesirable oscillations of the measured angular coordinate. Modified control algorithm (2) will have the form

$$u_i = \begin{cases} (-1)^i M_0, & \text{if} \quad (H - H^*)(\dot{\varphi}_i - \psi_i) > 0, \\ 0, & \text{else}, \end{cases}$$

$$T_\psi \dot{\psi}_i = -\psi_i + \dot{\varphi}_i, \quad i = 1, 2,$$

where $\psi_i(t)$ are the filter variables and $T_\psi > 0$, $T_\psi = \text{const}$, and H^* are algorithm parameters. The filter time constant T_ψ should be higher than the period of resonance oscillations. However, if this value is too large, the average power of the signal decreases and the algorithm works more slowly.

At the final stage of starting (when the resonance frequencies have already been passed), the algorithm is modified in the following way. A variable $\gamma(t)$ is introduced defined as

$$\gamma(t) = \max_{0 \le \tau < t} \operatorname{sgn}(H(\tau) - H^*) ,$$

where $\operatorname{sgn}[z] = 1$ under $z > 0$, $\operatorname{sgn}[z] = 0$ under $z \le 0$.

Finally, the algorithm of passage of resonance frequencies is written as follows:

$$u_i = \begin{cases} (-1)^i M_0, & \text{if} \quad \gamma = 1, \\ (-1)^i M_0, & \text{if} \quad \gamma = 0 \;\&\; (H - H^*)(\dot{\varphi}_i - \psi_i) > 0, \\ 0, & \text{else}, \end{cases} \tag{3}$$

$$T_\psi \dot{\psi}_i = -\psi_i + \dot{\varphi}_i, \quad i = 1, 2.$$

Investigation of the Algorithm Efficiency and Robustness

The efficiency of the suggested control method was investigated using the MATLAB software package by the second-order Runge–Kutta method with a fixed step equal to 0.000125 s in such a way as to ensure that the relative simulation error did not exceed 5%.

The calculations were carried out for the following base parameters of the system: $J = 0.014$ kg m^2, $m = 1.5$ kg, $M = 9$ kg, $\varepsilon = 0.04$ m, $k_\varphi = 0.01$ J s, $k_x = k_y = 5$ kg/s; $c = 5300$ N/m, $c_x = 1300$ N/m. For the given parameters, the minimal constant control action guaranteeing resonance area passage is equal to 0.66 N m.

During simulation, the value of the control torque M_0 guaranteeing resonance area passage for the suggested algorithm but prohibiting resonance area passage at any $M_1 < M_0$ was calculated.

The results of simulation for basis parameters showed that $M_0 = 0.42$ N m with an accuracy of 0.01 N m (Fig. 2, solid lines). In comparison with constant control action, the proposed algorithm enables a 1.5-fold decrease in the control action level.

The behavior of the system was studied under asymmetric initial conditions. In each series of experiments, the following parameters were determined: the value of motor constant torque M_1 guaranteeing resonance area passage under $u(t) \equiv M_1$ but prohibiting resonance area passage under $u(t) \equiv M_0 < M_1$ and the value of motor torque M_2 guaranteeing resonance area passage under relay control algorithm (3) for $M_0 =$

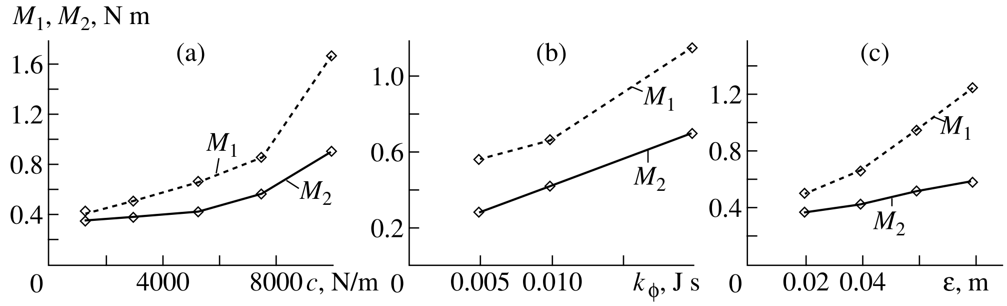

Fig. 4. Dependence of the minimal control action under which the system passes through the resonance area on the spring force c, the viscous friction in bearings k_φ, and the eccentricity ε under a constant torque (dotted line) and for control according to algorithm (3) (solid line).

M_2 but prohibiting resonance area passage for $M_0 < M_2$. Figure 3 depicts the relationships of the necessary control moments under varying initial conditions $x(0)$ and $\varphi_1(0)$. One can see that the asymmetry of the initial conditions weakly affects the process dynamics.

The influence of the spring force c, the viscous friction in bearings k_φ, and the eccentricities of rotor centers of masses ε on the system dynamics at nominal values of other object parameters was examined. The results of simulation are presented in Fig. 4. It is possible to see that the efficiency of the given algorithm increases with increasing c, decreasing k_φ, and increasing ε. Application of the algorithm developed enables a significant reduction in the control torque needed for passage of the resonance area in all experiments.

The algorithm has only two setting variables and it is easy to apply in spite of complicated behavior of the system. A closed control system has weak sensitivity to asymmetry of the initial conditions.

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