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Feedback Kalman-Yakubovich Lemma and its Applications to Adaptive Control

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Abstract

In this paper we give a survey of results related to the so called Feedback Kalman-Yakubovich Lemma (FKYL) giving necessary and sufficient solvability conditions for some class of bilinear matrix inequalities or conditions of feedback passivity of linear systems. Applications to adaptive and variable structure control systems are also discussed.

1 Introduction

The concepts of passivity and dissipativity widely used since 60s [5], [21], [29] have become important tools of input-output nonlinear systems analysis. An important role also belongs to Kalman-Yakubovich Lemma [17], [30], explicitly linking these concepts with classical state-space concepts of stability and optimality. Turning to the problem of system synthesis gives rise to an interest in studying feedback counterparts of passivity and dissipativity, meaning the existence of feedback rendering the system passive, or, correspondingly, dissipative. This interest was still strengthened due to recent breakthroughs in nonlinear control theory and extending main "linear" concepts to nonlinear systems [6], [16]. The new metodology of control systems design based on passification was developed [15], [24].

However the investigations of feedback passivity and feedback dissipativity properties are not yet completed even for linear systems. In this paper we give a brief survey of results related to the feedback counterpart of the Kalman-Yakubovich Lemma (FKYL). The existing formulations of FKYL are systematized in the unified framework in Section 2. An extension of the FKYL to some class of feedback dissipativity problems is given in Section 2.4. Some applications of FKYL to nonlinear and adaptive control are discussed in Section 3.

2 Main Algebraic Statements

2.1 Formulation of FKYL

In this section we present one of the first versions of FKYL. It is of special importance for various applications especially for adaptive control and output feedback design.

Consider linear time invariant system

$$\dot{x}(t) = Ax(t) + Bu(t)\ , y(t) = Cx(t). \qquad (1)$$

Here $A,\ B,\ C$ are known constant real matrices of sizes $n \times n,\ n \times m,\ m \times n$ respectively.

Definition 1. System (1) is called:

- *minimum phase* if the polynomial

$$\varphi_0(\lambda) = \det \begin{bmatrix} A - \lambda I_n & B \\ C & 0 \end{bmatrix} \qquad (2)$$

is Hurwitz (its zeros are inside open left half plane);

- *strictly minimum phase*, if it is minimum phase and the matrix CB is nonsingular ($\det\ CB \neq 0$);

- *hyper minimum phase*, if it is minimum phase and the matrix CB is symmetric and positive definite.

Note that in case $m = 1$ the polynomial $\varphi_0(\lambda)$ is just the numerator of transfer function of system (2), i.e. introduced definition of minimum phaseness coincides with the conventional one. Strict minimum phaseness means that additionally degree of $\varphi_0(\lambda)$ is $n-1$ while hyper minimum phase property requires positivity of high-frequency gain CB.

Theorem 2.1. Let $\operatorname{rank} B = m$ (i.e. the matrix B is of full rank). Let G be a symmetric negative semidefinite $n \times n$-matrix. Then the following statements are equivalent.

(A1) There exist a positive definite $n \times n$-matrix H and a $n \times m$-matrix K such that the relations

$$H(A + BK) + (A + BK)^T H < G,\ HB = C^T \qquad (3)$$

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hold.

(B1) The system (1) is hyper minimum phase.

Moreover, if the condition (B1) is satisfied then a matrix K in (3) can be determined in the form $K = -\alpha C$ where α is a sufficiently large positive real number.

Corollary. Conditions (A1) and (B1) are also equivalent to the following one:

(A1') There exist a positive definite $n \times n$-matrix H and a $m \times m$-matrix K' such that the relations

$$H(A+BK'C)+(A+BK'C)^T H < G, \; HB = C^T \quad (4)$$

hold.

Theorem 2.1 has been applied in control theory since 70s. It seems to be first proved in [9] for $m = 1$ and in [10] for $m > 1$. These proofs used the classical Kalman-Yakubovich lemma in a crucial manner. Note that (3) and (4) are special cases of Bilinear Matrix Inequalities which solvability in general case is known to be $\mathcal{NP}$-problem [25]. As it was shown in [2] Theorem 2.1 also gives necessary and sufficient conditions for feedback passivity of system (1) and for "almost strict positive realness" of its transfer function.

Definition 2 The *zero dynamics* of the system (1) are the dynamics of those solutions $x(t)$ of (1) for which a function $u(t)$ exists such that $y(t) \equiv 0$ for all $t \geq 0$.

Zero dynamics of (1) can be described explicitly. Let $\det(CB) \neq 0$. Define $n \times (n-m)$ matrix L_0 in such a way that the columns of L_0 form an orthonormal basis in $\operatorname{Ker} C$: $CL_0 = 0, \quad L_0^T L_0 = I_{n-m}$. Consider the matrix $P_0 = I_n - B(CB)^{-1}C$ which is a projector on $\operatorname{Ker} C$ along $\operatorname{Im} B$. Clearly, to ensure $y(t) \equiv 0$ for $t \geq 0$ we have to choose $u(t)$ so that $CAx + CBu \equiv 0$ for $t > 0$. Hence $u = -(CB)^{-1}CAx$, which coincides with the so called "equivalent control" [27]. So the zero dynamics are described by relations

$$\dot{x} = P_0 Ax, \quad Cx(0) = 0. \quad (5)$$

Change the basis in (5) defining $\xi \overset{\text{def}}{=} \operatorname{col}[\xi_0, \xi_1] = \operatorname{col}[L_0^T x, Cx]$. Then for the zero dynamics we have $\xi_1(t) \equiv 0$ and

$$x = \begin{bmatrix} L_0^T \\ 0 \end{bmatrix}^{-1} \xi = [L_0, \; C^T(CC^T)^{-1}]\xi = L_0\xi_0$$

for $t > 0$. Thus we finally obtain the zero dynamics equation

$$\dot{\xi}_0 = L_0^T P_0 A L_0 \xi_0. \quad (6)$$

Theorem 2.2. Let the conditions of Theorem 2.1 hold. Then the statements (A1), (B1) of Theorem 2.1 are equivalent to the following statement.

(C1) The matrix CB is symmetric positive definite and the zero dynamics of the system (1) are asymptotically stable (equivalently, the matrix $L_0^T P_0 A L_0$ in (6) is Hurwitz).

2.2 FKYL for Weakly Minimum Phase Systems

In [22] another version of FKYL was obtained for the case of weakly minimum phase systems. It is given below in slightly different form.

Definition 3. The system (1) is said to be ***weakly minimum phase*** if its zero dynamics are stable in the sense of Lyapunov.

Note, that in terms of the system (6) Lyapunov stability means that all the eigenvalues of the matrix $L_0^T P_0 A L_0$ are in the closed left half plane and all its pure imaginary eigenvalues, if any, have elementary divisors of the first order. It is easily seen, than in the case when the system of zero dynamics has pure imaginary eigenvalues it is impossible to ensure the strict inequality in (3).

Theorem 2.3. Let the conditions of Theorem 2.1 hold and $\operatorname{Ker} C \subset \operatorname{Ker} G$. Then the following statements are equivalent.

(A2) There exist a positive definite $n \times n$-matrix H and a $n \times m$-matrix K such that the matrix $A + BK$ is Hurwitz and

$$H(A+BK)+(A+BK)^T H < G, \quad HB = C^T . \quad (7)$$

(B2) The matrix CB is symmetric positive definite, the pair (A, B) is stabilizable and all the zeros of the polynomial (2) lie in the closed left half plane and all pure imaginary eigenvalues $i\omega$ of the matrix pencil

$$R(\lambda) = \begin{bmatrix} A & B \\ C & 0 \end{bmatrix} - \lambda \begin{bmatrix} I_n & 0 \\ 0 & 0 \end{bmatrix} , \quad (8)$$

if any, have only linear elementary divisors $\lambda - i\omega$.

(C2) The matrix CB is symmetric positive definite, the pair (A, B) stabilizable and the system (1) is weakly minimum phase.

The equivalence of (A2) and (C2) was proved in [22]. The proof was based on the concept of "special coordinate basis" [23]. Some related constructions were used by V.I.Utkin (see e.g. [28]). Observe that in the case of Theorem 2.3 we can not state that K can be found in the form $K = -\alpha C$ (in other words, the system can be stabilized by state feedback, rather than by output feedback). If we omit the condition that $A + BK$ is Hurwitz in (A2) then we can omit in (B2),(C2) stabilizability of the pair (A, B). In this case we can state only that all the eigenvalues of the matrix $A + BK$ are in the closed left half plane. Note, that in Theorem 2.1 Hurwitz property of the matrix $A+BK$ follows directly from the strict inequality (3) and the positive definiteness of H. It can be shown that the first inequality in (7) can be reformulated as follows: there exists a positive real number ε such that

$$H(A+BK)+(A+BK)^T H \leq G-\varepsilon C^T C\ . \quad (9)$$

2.3 FKYL for the Nonproper Systems (Case of Input-Dependent Observations)

Consider the system in more general form than (1)

$$\dot{x}=Ax+Bu,\quad y=Cx+Du\ . \quad (10)$$

Here A,B,C,D are constant real matrices of sizes $n\times n$, $n\times m$, $m\times n$, $m\times m$ respectively, the matrix D is symmetric positive semidefinite. The version of FKYL for such a system was obtained in [20]. As in [20], we suppose without loss of generality that

$$D=\begin{bmatrix} D_1 & 0\\ 0 & 0\end{bmatrix}$$

where the matrix D_1 is positive definite and the matrices B,C are written in the block form:

$$B=[B_1,\ B_2],\qquad C=\begin{bmatrix} C_1\\ C_2\end{bmatrix} \quad (11)$$

Matrices D_1,B_1,B_2,C_1,C_2 are of sizes $m_1\times m_1$, $n\times m_1$, $n\times m_2$, $m_1\times n$, $m_2\times n$ respectively ($m_1+m_2=m$).

Definition 4. System (10) is called ***hyper minimum phase*** if the polynomial

$$\varphi(\lambda)=\det\begin{bmatrix} A-\lambda I_n & B\\ C & D\end{bmatrix} \quad (12)$$

is Hurwitz and the matrix C_2B_2 is symmetric positive definite.

Let us obtain the system of zero dynamics for (10). Using the block representation of coefficient matrices write (10) in the form

$$\begin{aligned}\dot{x} &= Ax+B_1u_1+B_2u_2\ ,\\ y_1 &= C_1x+D_1u_1,\quad y_2=C_2x\ .\end{aligned}$$

Suppose that $\det(C_2B_2)\neq 0$. Define $n\times(n-m_2)$ matrix L such that its columns form an orthonormal basis in $\operatorname{Ker} C$. Introduce also the matrix $P=I_n-B_2(C_2B_2)^{-1}C$. Clearly for the zero dynamics we have $u_1=-D_1^{-1}C_1x$. So, as in the section 2.1 we obtain the zero dynamics equation

$$\dot{\xi}_0=L^TP(A-B_1D_1^{-1}C_1)L\xi_0\ , \quad (13)$$

where $\xi_0=L^Tx$.

Theorem 2.4. Let $\operatorname{rank} B_2=m_2$. Then the following statements are equivalent. (A3) There exist a positive definite $n\times n$ matrix H, a $n\times m$ matrix K and a positive real number ε such that the inequality

$$\begin{bmatrix} HA_K+A_K^TH+\varepsilon I_n & HB-C^T-K^TD\\ B^TH-C-DK & -2D\end{bmatrix}\leq 0, \quad (14)$$

where $A_K=A+BK$, holds. (B3) The system (10) is hyper minimum phase. (C3) The matrix C_2B_2 is symmetric positive definite and the zero dynamics of the system (10) is asymptotically stable. Note, that as in the previous section we can not guarantee the output feedback stabilization of the system.

Theorem 2.4 was in fact proved in [20]. The work [20] also contains a detailed study of the situation where only the positive semidefinite property of H is required. This situation is of interest for the passivity theory. In this case the conditions of Theorem 2.4 can be substantially weakened. It is achieved by the possibility to put those components of the matrix H, which correspond to unstable modes of the zero dynamics, equal to zero. We give an example of such a result (see [20] for the detailed analysis).

Theorem 2.5. Let the matrix C_2B_2 be symmetric positive definite. Then there exist a positive semidefinite $n\times n$ matrix H and a $m\times n$ matrix K such that the inequality

$$\begin{bmatrix} HA_K+A_K^TH & HB-C^T-K^TD\\ B^TH-C-DK & -2D\end{bmatrix}\leq 0 \quad (15)$$

holds.

2.4 Generalized Version of FKYL

Describe a natural generalization of the situation considered in Theorem 2.1. Consider a system

$$\dot{x}=Ax+Bu \quad (16)$$

and a quadratic form

$$F(x,u)=x^TGx+2u^TCx+u^TDu. \quad (17)$$

Here the matrices $A,\ B,\ G,\ C,\ D$ are of sizes $n\times n$, $n\times m$, $n\times n$, $m\times n$ and $m\times m$ respectively, the matrices $G,\ D$ are symmetric. We consider the case when the matrix D is positive semidefinite. Then, without loss of generality, we can suppose that

$$D=\begin{bmatrix} D_1 & 0\\ 0 & 0\end{bmatrix}$$

with matrix D_1 being positive definite. So we can write (11) where the matrices $D_1,\ B_1,\ B_2,\ C_1,C_2$ are of sizes $m_1\times m_1$, $n\times m_1$, $n\times m_2$, $m_1\times n$ and $m_2\times n$ respectively ($m_1+m_2=m$).

Theorem 2.6. Let $\operatorname{rank} B_2=m_2$ and there exists a $m\times n$ matrix K_0 such that

$$F(x,K_0x)\leq 0 \quad (18)$$

for all $x \in \mathcal{R}^n$. Then the following statements are equivalent (A4) There exist a positive definite $n \times n$ matrix H, a $n \times m$ matrix K and a positive real number ϵ such that the matrix $A + BK$ is Hurwitz and the inequality

$$2x^T H\left[(A+BK)x + Bu\right] - F(x,u) \leq -\epsilon x^T x \quad (19)$$

holds for all $x \in \mathcal{R}^n,\ u \in \mathcal{R}^m$.

(B4) The matrix C_2B_2 is symmetric positive definite and the equality

$$\operatorname{rank}\begin{bmatrix} A-\lambda I_n & B_1 & B_2 \\ C_2 & 0 & 0 \end{bmatrix} = n + m_2 \quad (20)$$

holds for all $\lambda \in \mathcal{C},\ \operatorname{Re}\lambda \geq 0$.

Corollary. If D is positive definite, i.e. $m_1 = m$, then the statement (B4) of Theorem 2.6 reduces to stabilizability of the pair (A, B).

3 Applications of FKYL to Adaptive Control Problem

The purpose of this section is to demonstrate possible applications of Feedback Kalman-Yakubovich Lemma to adaptive and variable-structure systems (VSS) design. For simplicity the disturbance free case is considered (most results can be extended in a standard way to systems with bounded disturbances, see e.g. [1], [3]. Other applications of FKYL employ its ability to replace Almost Strict Positive Realness (ASPR) condition (see [18]) by Hyper Minimum Phase condition [2] as well to establish necessary and sufficient conditions of quadratic Lypunov function existence.

3.1 Model Reference Adaptive Control with the Implicit Model

Consider the linear time invariant plant described by equation

$$A(p)y(t) = B(p)u(t), \quad t > 0, \quad (21)$$

where $u(t)$ is scalar control action, $y(t)$ is scalar controlled variable, $A(p) = p^n + a_{n-1}p^{n-1} + \ldots + a_1 p + a_0$, $B(p) = b_m p^m + \ldots + b_1 p + b_0$ are polynomials, $p = d/dt$ is time derivative, $n - m = k > 0$ is relative degree of the plant. Coefficients $a_i, i = 0, \ldots, n-1$, $b_j, j = 0, \ldots, m$ are unknown plant parameters. The control goal is to provide tracking output signal $y(t)$ to the command signal $r(t)$:

$$\lim_{t\to\infty} [y(t) - r(t)] = 0. \quad (22)$$

To solve the posed problem introduce an auxillary goal (adaptation goal), specifying the desired tracking error behavior:

$$\lim_{t\to\infty} \delta(t) = 0. \quad (23)$$

where $\delta(t) = G(p)y(t) - D(p)r(t)$ is the adaptation error signal; $G(p) = p^l + \cdots + g_1 p + g_0$, $D(p) = d_s p^s + \cdots + d_1 p + d_0$ are given polynomials specifying the desired properties of the closed-loop system. $G(p)$ is assumed to be stable (Hurwitz) polynomial. Note that the signal $\delta(t)$ may be interpreted as equation error for the equation

$$G(p)y_*(t) = D(p)r(t), \quad (24)$$

because $\delta(t) = G(p)e(t)$, where $e(t) = y(t) - y_*(t)$. Hence the equation (24) may be interpreted as reference equation describing reference model implicitly.

Take the main loop control law in the form

$$u(t) = K(t)\left[D(p)r(t)\right] + \sum_{i=0}^{l} k_i(t)\left[p^i y(t)\right] \quad (25)$$

where $K(t),\ k_i(t),\quad i = 0, \ldots, l$ - are tunable parameters, and the adaptation algorithm take as follows:

$$\begin{aligned} \dot{k}_i(t) &= -\gamma\delta(t)p^i y(t), \quad i = 0, 1, \ldots, l, \\ \dot{K}(t) &= \gamma\delta(t)D(p)r(t), \end{aligned} \quad (26)$$

where $\gamma > 0$ is the adaptation gain. Similarly to Section 3.1 it can be derived from Theorem 2.1 that all the trajectories of the system (21), (22), (26) are bounded and the goals (22) and (23) are achieved if $B(p)$ is Hurwitz polynomial, $l = k - 1$, and the command signal $r(t)$ has vanishing derivatives: $\int_0^\infty \left[r^{(i)}(t)\right]^2 dt < \infty$, $i = 1, \ldots, s+1$. Morover the above conditions are necessary and sufficient for existence Lyapunov function of type "quadratic form of plant state plus quadratic form of parametric error".

Note that neither degree s of polynomial $D(p)$ nor its coefficients appear in above conditions. The degree of $D(p)$ is determined by the amount of the available derivatives of $r(t)$. Note also that matching condition in the form used for the model reference systems is not necessary for proposed implicit model reference systems. The order of reference equation (24) is equal to l and can be significantly less then the plant order n. Moreover, the true plant order need not be known for system design.

The above approach can be extended to the case of MIMO plants of form

$$\dot{x}(t) = Ax(t) + Bu(t), \quad y(t) = Cx(t), \quad (27)$$

where x is n-dimensional plant state vector, $u(t)$ is m-dimensional control action, $y(t)$ is l-dimensional plant output vector. In this case we can choose the following adaptive control law

$$\begin{aligned} u(t) &= K(t)y(t), \\ \dot{k}_j &= -\left[g_j^T y(t)\right]\Gamma_j y(t), \quad j = 1, \ldots, m, \end{aligned} \quad (28)$$

where $k_j(t)$ are the columns of tunable gain $(m \times n)$-matrix $K(t)$, g_j are l-dimensional vectors, $\Gamma = \Gamma^T > 0$ are $(l \times l)$ adaptive gain matrices.

Adaptive systems with an implicit reference model and their stability conditions were also extended to stochastic [8], distributed [4] and delayed plants [26].

3.2 FKYL in the Variable-Structure Systems Design

The synthesis of VSS with sliding modes is usually carried out in two stages [27], [28]. At the first stage we choose sliding surface $s(x) = 0$ such that the motion on this surface has the desired dynamics (compare with mentioned above zero-dynamics). At the second stage, we have to ensure that the motion reaches the sliding surface in a finite time from any point in the phase space. Note that the first stage of VSS synthesis is similar to the synthesis of the adaptation algorithm. The two-stage approach to system design corresponds to supplementing the main control objective $\lim_{t\to\infty} x(t) = 0$, where $x(t)$ is the plant state vector, with the auxillary objective $\lim_{t\to\infty} s(x(t)) = 0$, where $s(x)$ is the "error" vector function evaluating the deviation of the trajectory from the sliding surface. If the representing point reaches the sliding surface in the finite time t_*, then for $t \geq t_*$ the sliding mode on the surface $s(x) = 0$ is realized.

Let the plant be described by the equation

$$\dot{x} = Ax + Bu, \quad y = L^T x, \tag{29}$$

where $x(t) \in \mathcal{R}^n$ is plant state vector, $u(t) \in \mathcal{R}^1$ is scalar control action, $y(t) \in \mathcal{R}^l$ is measurable output vector. Let the control objective be given as $\lim_{t\to\infty} x(t) = 0$, and the subsidiary objective is taken as maintaining the sliding mode on plane $s = C^T y = 0$, where C^T is a $l \times n$ matrix. Choose the objective Lyapunov function in the form $V_1(x) = \frac{1}{2} x^T P x$, where $P = P^T > 0$ is some positive-definite $n \times n$ matrix. Evaluating derivative of $V(x)$ with respect to (29) we obtain $\dot{V}(t) = x^T P(Ax + Bu)$. Let us take control action in the form

$$u = -\gamma \operatorname{sign} s, \quad s = C^T y \tag{30}$$

where $\gamma > 0$ is some chosen parameter. As it shown in [11], [1], the above goal is achieved in the system (29), (30) if there exist matrix $P = P^T > 0$ and vector K_* such that

$$PA_* + A_*^T P < 0, \quad PB = LC, \quad A_* = A + BK_*^T L^T.$$

As it is clear from Theorem 2.1, mentioned condition is fulfilled iff the function $C^T W(p)$ is strictly minimum phase, where $W(p) = L^T (pI_n - A)^{-1} B$, and the sign of high frequency gain $C^T L^T B$ is known. In that case for sufficiently large γ holds $\lim_{t\to\infty} x(t) = 0$. To eliminate dependence of system stability from initial conditions and plant parameters it was suggested to use instead of (30) the following adaptive control law ([1])

$$u = -K^T(t) y(t) - \gamma \operatorname{sign} s, \quad s(y) = C^T y \tag{31}$$
$$\dot{K}(t) = -s(y)\Gamma y(t),$$

where $\Gamma = \Gamma^T > 0, \ \gamma > 0$.

It should be noted that the convergence $s(t)$ to zero at the finite time t_* is essential for VSS systems. It can be shown (see e.g. [12]) that this is valid for any bounded region of initial conditions for the system (29), (31).

3.3 Adaptive Stabilization of Plants with Arbitrary Relative Degree

Note that conditions of Theorem 3.1 are valid only for the case $r = 1$ restricting their practical applications. However, the design and analysis for general case $r > 1$ involve well known difficulties. Standard solutions based on explicit reference models [7], [19] provide adaptive controllers of high order which are both difficult to implement and sensitive to noise. It was shown in [13] that FKYL allows to design simplified adaptive controler based on so called "shunt" – feedforward parallel compensator. As it follows from Theorem 2.1 applicability conditions of the algorithm [13] are necessary and sufficient for existence of quadratic Lyapunov function in state space of augmented plant (including shunt). The solution is based on the following statement (see [13]).

Theorem 3.2. Assume the plant with transfer function $G^T W(\lambda)$ is minimum phase with scalar relative degree $r > 1$ for some $l \times m$ matrix G, the matrix $-G^T C A^{r-1} B$ being Hurwitz. Let $P(\lambda)$, $Q(\lambda)$ be Hurwitz polynomials of degrees $r-2$, $r-1$, correspondingly and all three polynomials $P(\lambda)$, $Q(\lambda)$, $\varphi(\lambda) = \delta(\lambda) \det G^T W(\lambda)$ have the same signs of coefficients. Denote

$$W_{\kappa\varepsilon}(\lambda) = G^T W(\lambda) + \kappa\varepsilon(\varepsilon\lambda)/Q(\lambda) I_m \tag{32}$$

Then there exist scalar $\kappa_0 > 0$ and function $\varepsilon(\kappa) > 0$ such, that matrix $W(\lambda)$ is hyper minimum phase as $\kappa > \kappa_0$, $0 < \varepsilon < \varepsilon_0(\kappa)$.

4 Conclusions

It has been shown that various control design problems can get simple solution by means of using feedback counterparts of the Kalman-Yakubovich lemma.

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