Automation and Remote Control, Vol. 64, No. 5, 2003, pp. 673–713. Translated from Avtomatika i Telemekhanika, No. 5, 2003, pp. 3–45.

REVIEWS

Control of Chaos: Methods and Applications. I. Methods[1]

B. R. Andrievskii and A. L. Fradkov

Institute of Mechanical Engineering Problems, Russian Academy of Sciences, St. Petersburg, Russia
Received October 15, 2002

Abstract—The problems and methods of control of chaos, which in the last decade was the subject of intensive studies, were reviewed. The three historically earliest and most actively developing directions of research such as the open-loop control based on periodic system excitation, the method of Poincaré map linearization (OGY method), and the method of time-delayed feedback (Pyragas method) were discussed in detail. The basic results obtained within the framework of the traditional linear, nonlinear, and adaptive control, as well as the neural network systems and fuzzy systems were presented. The open problems concerned mostly with support of the methods were formulated. The second part of the review will be devoted to the most interesting applications.

1. INTRODUCTION

The term *control of chaos* is used mostly to denote the area of studies lying at the interfaces between the control theory and the theory of dynamic systems studying the methods of control of deterministic systems with nonregular, chaotic behavior. In the ancient mythology and philosophy, the word "$\chi\alpha\omega\sigma$" (chaos) meant the disordered state of unformed matter supposed to have existed before the ordered universe. The combination "control of chaos" assumes a paradoxical sense arousing additional interest in the subject.

The problems of control of chaos attract attention of the researchers and engineers since the early 1990's. Several thousand publications have appeared over the recent decade. Statistics of publications in *Science Citation Index* shows that during 1997–2001 only the reviewed journals published annually about four hundred papers. For comparison, search by the key words "*adaptive control*" shows that within this area which is regarded as a field of very intensive research [145] at most three hundred papers appear annually.

It seems that T. Li and J.A. Yorke were the first authors who in 1975 introduced in their paper *Period Three Implies Chaos* [194] the term "chaos" or, more precisely, "deterministic chaos" which is used widely since that. Various mathematical definitions of chaos are known, but all of them express close characteristics of the dynamic systems that are concerned with "supersensitivity" to the initial conditions: even arbitrarily close trajectories diverge with time at a finite distance, that is, long-term forecasts of trajectories are impossible. At that, each trajectory remained bounded, which contradicts the intuitive understanding of instability based on the experience gained with the linear systems.

Nevertheless, the nonlinear deterministic dynamic systems manifesting such characteristics do exist and are not exceptional, "pathological" cases. It also turned out that the methods describing chaotic behavior occur in many areas of science and technology and sometimes are more suitable

[1] This work was supported by the Russian Foundation for Basic Research, project no. 02-01-00765, the Scientific Program 17 of the Presidium of the Russian Academy of Sciences, project no. 3.1.4, and the Federal Program "Integration."

for describing nonregular oscillations and indeterminacy than the stochastic, probabilistic methods. It suffices to note that a wide class of the chaotic systems is represented by the well-known pseudorandom-number generators that appeared much in advance of the term "chaos".

Surprising was the possibility of substantial variation of the characteristics of a chaotic system by a very small variation of its parameters that was discovered in 1990 by the Yorke and his collaborators [228]. By computer-aided modeling of a discrete M. Hénon system, they demonstrated that a sufficiently small variation in a system parameter can transform a chaotic trajectory into a periodic one and *vice versa*. In the aforementioned paper, the parameter was varied with regard for the current system state, that is, by means of the feedback. In the subsequent publications, this effect was confirmed experimentally [104], and the fields of its possible application such as lasers, communication systems, chemical technologies, or medical treatment of arrhythmia and epilepsy were specified. Paradoxicalness of the conclusion that chaos cannot be forecasted, but can be controlled gave rise to an explosive interest of the researchers and an avalanche of publications confirming—usually, by computer-aided modeling—the possibility of substantial variation of the characteristics of various natural and artificial chaotic systems by relatively small variations of their parameters and external actions.

However, despite numerous publications, including several monographs, only few strict facts were established there, and many issues remain open. In view of the wide scope of possible applications, this area is of interest both to the theorists and the control engineers. The present review aims to help them gain an insight into the state-of-art in this vast domain of research and its most interesting applications.

The review consists of two parts. The present, first part presents the necessary information and discusses the basic methods of controlling chaotic systems. The second part will consider applications of the methods of control of chaos in the scientific and engineering fields such as physics, mechanics, chemistry, medicine, economics, telecommunication, control of mechanical and electronic systems, process control, and so on. The review relies on the materials of the review presented at the 15th Triennial World Congress IFAC [123] and lectures [121, 122] and [127].

2. CHAOTIC SYSTEMS

The present section offers preliminary information about the dynamic chaotic processes. The chaotic systems represent a class of indeterminacy models differing from the stochastic models. Whereas with a knowledge of the current system state the deterministic model can predict the future trajectory for an arbitrarily long period and the stochastic model cannot make a precise forecast, generally speaking, even for an arbitrarily short time, the forecast error of the chaotic model grows exponentially and, consequently, a forecast can be made only for a limited time defined by the admissible forecast error. The processes in the chaotic models have the form of nonregular oscillations where both frequency and amplitude vary or "float."

Before the XX century, the linear differential equations were the main mathematical models of oscillations in the mechanical, electrical, and other systems. Yet at the turn of this century it became clear that the linear oscillation models fail to describe adequately the new physical and engineering phenomena and processes. The fundamentals of a new mathematical apparatus, the theory of nonlinear oscillations, were laid by A. Poincaré, B. Van der Pol, A.A. Andronov, N.M. Krylov, and N.N. Bogolyubov. The most important notion of this theory is that of the stable limit cycle.

Even the simplest nonlinear models enable one to describe complex oscillations such as the relaxation, that is, close to rectangular, oscillations, take into account variations in the form of oscillations depending on the initial conditions (systems with several limit cycles), and so on. The linear oscillation models and the nonlinear models with limit cycles satisfied the needs of engineers for several decades. It was believed that they describe all possible types of oscillations of the deterministic systems. This conviction was supported by the mathematical findings. For example, the well-known Poincaré–Bendixson theory asserts that the equilibrium state and the limit cycle are the only possible kinds of limited stable motions in continuous systems of the second order.

However, in the middle of the last century the mathematicians M. Cartwright, J. Littlewood, and S. Smale established that this is not the case already for the systems of the third order: very complex motions such as limited nonperiodic oscillations become possible in the system. In 1963, the physicist E. Lorenz revolutionized the situation by his paper [195] demonstrating that the qualitative nature of atmospheric turbulence which obeys the Navier–Stokes complex partial differential equations is representable by a simple nonlinear model of the third order (Lorenz equation):

$$\begin{cases} \dot{x} = \sigma(y - x) \\ \dot{y} = rx - y - xz \\ \dot{z} = -bz + xy. \end{cases} \tag{1}$$

For some values of the parameters (for example, $\sigma = 10$, $r = 97$, and $b = 8/3$), the solutions of system (1) look like nonregular oscillations. The trajectories in the state (phase) space can approach the limit set (attractor) featuring very sophisticated form. Attention of the physicists and mathematicians, and later engineers, was attracted to these models by the work of D. Ruelle and F. Takens [252] (1971) who called these attractors "strange" and also by the work of Li and Yorke [194] (1975) who introduced the term "chaos" to designate the nonregular phenomena in the deterministic systems. We note that the main result of [194] is a special case of the theorem of a Kievan mathematician A.N. Sharkovrkii that was published in 1964, and the fundamentals of the mathematical apparatus for studying the chaotic phenomena were laid in the 1960's–1970's by the national scientific schools of A.N. Kolmogorov, D.V. Anosov, V.I. Arnol'd, Ya.G. Sinai, V.K. Mel'nikov, Yu.I. Neimark, L.P. Shil'nikov, and their collaborators. From this time on, the chaotic behavior was discovered in numerous systems in mechanics, laser and radio physics, chemistry, biology and medicine, electronic circuits, and so on [37]. The newly developed methods of analytical and numerical study of systems demonstrated that chaos is by no means an exceptional kind of behavior of the nonlinear system. Roughly speaking, chaotic motions arise whenever the system trajectories are globally bounded and locally unstable. In the chaotic system, an arbitrarily small initial divergence of the trajectories does not remain small, but grows exponentially. The frequency spectrum of the chaotic trajectory is continuous. In many cases such nonregular and nonperiodic oscillations better represent the processes in physical systems. It deserves noting that it is practically impossible to distinguish "by eye" the chaotic process from the periodic or quasiperiodic process.

The terminology in the domain of chaotic models has not yet settled, and there are several different definitions of the chaotic systems of which we present one of the simplest. Let us consider the continuous-time dynamic system

$$\dot{x} = F(x), \tag{2}$$

where $x = x(t) \in \mathbf{R}^n$ is the system vector, $0 \le t < \infty$.

Definition 1. The closed set $\Omega \subset \mathbf{R}^n$ is called the attractor of system (2) if (a) there exists an open set $\Omega_0 \supset \Omega$ such that all trajectories $x(t)$ of system (2) beginning in Ω_0 are definite for all $t \ge 0$ and tend to Ω for $t \to \infty$, that is, $\operatorname{dist}(x(t), \Omega) \to 0$ for $t \to \infty$, if $x(0) \in \Omega_0$, where $\operatorname{dist}(x, \Omega) = \inf_{y \in \Omega} \|x - y\|$ is the distance[2] from the point x to the set Ω, and (b) no eigensubset of Ω has this property.

[2] By $\|\cdot\|$ everywhere is meant the Euclidean norm, and by $\|\cdot\|_\infty$, the uniform norm in the spaces of vectors and functions. The Euclidean space of the n-dimensional vectors is denoted by $\mathbf{R}^n$.

Definition 2. An attractor is called chaotic if it is bounded and any trajectory beginning in it is a Lyapunov-unstable trajectory.

Definition 3. A system is called chaotic if it has at least one chaotic attractor.

Lyapunov instability characterizes the main property of chaotic oscillations called the "supersensitivity" or "sensitive dependence" on the initial conditions: any two arbitrarily close trajectories necessarily move away from each other at a finite distance.

The so-called *recurrence* of the trajectories of chaotic processes is essential for the problems of control: with time these trajectories hit an arbitrarily small neighborhood of their position in the past. Let us consider this property in more detail.

Definition 4. The function $x : \mathbf{R}^1 \to \mathbf{R}^n$ is called recurrent if for any $\epsilon > 0$ there exists $T_\epsilon > 0$ such that for any $t \geq 0$ there exists $T(t, \epsilon)$, $0 < T(t, \epsilon) < T_\epsilon$, such that $\|x(t + T(t, \epsilon)) - x(t)\| < \epsilon$.

The recurrent trajectories have two important characteristics described by the lemmas of C.C. Pugh and Anosov.

Lemma 1 (Pugh). *Let $\overline{x}(t)$, $t \geq 0$, be the recurrent trajectory of system* (2) *having smooth right side $F(x)$. Then, for any $\varepsilon > 0$ there exists a smooth function $F_1(x)$ such that $\|F_1(x)\|_\infty + \|DF_1(x)\|_\infty < \varepsilon$ and the solution $x(t)$ of system $\dot{x} = F(x) + F_1(x)$ with the same initial condition $x(0) = \overline{x}(0)$ is periodic.*

Lemma 2 (Anosov). *Let $\overline{x}(t)$, $t \geq 0$, be the recurrent trajectory of system* (2) *having smooth right side $F(x)$. Then, for any $\varepsilon > 0$ there exists x^* such that $\|x^* - \overline{x}(0)\| < \varepsilon$ and the solution $x(t)$ of system* (2) *with the initial condition $x(0) = x^*$ is periodic.*

These lemmas show that the chaotic attractor is the closure of all periodic trajectories that are contained in it. The notion of attractor is also related with the following recurrence criterion formulated by G. Birkhoff in 1927.

Theorem 1 (Birkhoff). *Any trajectory belonging to a compact minimum invariant set is recurrent. Any compact invariant minimum set is the closure of some recurrent trajectory.*

As follows from the theorem, any solution beginning from its ω-limit set is recurrent. If additionally the ω-limit set $\overline{x}(t)$ is an attractor, then any chaotic trajectory beginning in its ω-limit set is recurrent.

There are other definitions of the chaotic attractors and chaos. For example, the definition of the chaotic attractor often includes additional requirements such as existence of trajectories (or a family of periodic trajectories) that are everywhere dense in Ω, topological transitivity, and so on that emphasize that the trajectories are "mixed." The recent results of G.A. Leonov [21] indicate that instead of the lack of Lyapunov stability in the definition of the chaotic attractor it is recommendable to require absence of the so-called *Zhukovskii stability* which admits different speeds of time flow on different system trajectories. The notion of "chaotic attractor" often coincides with that of the "strange attractor" introduced in 1971 by Ruelle and Takens as a "porous" set; in 1977 it was named by B. Mandelbrot the "fractal" set. Strict proof of the system chaoticity is difficult even if the simplest definition is used. For some universally recognized chaotic systems such as the Lorenz system (1) and the Hénon systems for the standard values of parameters, the proofs of chaoticity are awkward, if any, although there are enough numerical and experimental demonstrations of this fact. Therefore, the numerical simulation and estimation of various characteristics remains the main method to study the chaotic systems. We present some examples of the chaotic systems mentioned in what follows.

Example 2.1. Chua system (circuit). Experts in electronic circuits L. Chua and M. Matsumoto proposed in 1984 a simple electronic circuit with one nonlinear element which is capable of generating diverse, including chaotic, oscillations. The mathematical model of the Chua circuit is as follows:

$$\begin{cases} \dot{x} = p(y - f(x)) \\ \dot{y} = x - y + z \\ \dot{z} = -qy, \end{cases} \tag{3}$$

where x, y, z are dimensionless variables and $f(x) = M_1x + 0.5(M_1 - M_0)(|x+1| - |x-1|)$. For $p = 9$, $q = 14.3$, $M_1 = -6/7$, and $M_0 = 5/7$, the trajectories of system (3) manifest chaotic behavior.

Example 2.2. Various chaotic oscillations can be generated by feeding into the nonlinear oscillators a harmonic signal, for example, by substituting the sinusoidal function $z(t) = A\sin\omega_0 t$ for zero in the right sides of the Van der Pol equations $\ddot{y} + \varepsilon(y^2 - 1)\dot{y} + \omega^2 y = 0$, the Duffing equations $\ddot{y} + p\dot{y} - qy + q_0y^3 = 0$, and the autooscillatory system with a relay element $\ddot{y} + p\dot{y} + qy - \operatorname{sgn} y = 0$. For some values of excitation frequency and amplitude, the limit cycle is “smeared” and the oscillations in nonlinear system become chaotic.

For discrete time, the examples of chaotic systems exist for any dimensionality of the system state, even for $n = 1$.

Example 2.3. The discrete system with quadratic right side $x_{k+1} = \lambda x_k(1 - x_k)$, $x_k \in R^1$ constructed using the so-called *logistic map* $F(x) = \lambda x(1 - x)$ is chaotic for $3.57 < \lambda < 4$ [37]. The segment $[0, 1]$ is its attractor.

Example 2.4. The system

$$x_{k+1} = \{Mx_k\}, \tag{4}$$

where $\{A\}$ is the fractional part of the real number A, is chaotic for any $M > 1$. System (4) often is used to generate pseudorandom numbers, which, possibly, is the first use of chaos. It is based on the fact that for any initial condition x_0 incommensurable with M the fraction of points of sequence (4) hitting some interval lying within the segment $[0, 1]$ is proportional to the length of this interval. Therefore, if one regards the frequency of hitting the interval by the points as an estimate of certain probability, then the totality of these probabilities will define a uniform distribution over $[0, 1]$.

Example 2.5. The Hénon system is defined by the difference equations

$$\begin{cases} x_{k+1} = 1 - \alpha x_k^2 + y_k \\ y_{k+1} = \beta x_k. \end{cases} \tag{5}$$

Chaotic behavior of the solutions of (5) is observed, for example, for $\alpha = 1.4$, $\beta = 0.3$.

The “delayed coordinates” and the Poincaré map found extensive use in the studies of chaotic processes and solution of the problems of their control. Let only the scalar output coordinate $y(t) = h(x(t))$ of system (2) be measurable. By the vector of *delayed coordinates* is meant the vector function $X(t) \triangleq \left[y(t), y(t-\tau), \ldots, y(t-(N-1)\tau)\right]^{\mathrm{T}} \in \mathbf{R}^N$. The initial model of system (2) is reduced in this vector to the form $\dot{X} = \overline{F}(X(t))$. As follows from the *embedding theorems*, if $N > 2n$, where n is the order of the initial system (2), then in the general situation there exists a diffeomorphism between the state space of the initial system and the state subspace of the transformed system such that if the initial system has an attractor of certain dimensionality, then the transformed system also will have an attractor of the same dimensionality.

The *Poincaré map* is introduced on the assumption that there exists a T-periodic solution $\overline{x}(t)$ of Eq. (2) beginning at some point x_0, that is, that $\overline{x}(t+T) = x(t)$ is satisfied for all $t \geq t_0$ and $x(t_0) = x_0$. Let S be a smooth *transversal* surface of the trajectory at the point x_0 obeying the equation $s(x) = 0$, where $s : \mathbf{R}^n \to \mathbf{R}^1$ is a smooth scalar function transversally intersecting the trajectory point x_0, that is, $s(x_0) = 0, \nabla s(x_0)^{\mathrm{T}} F(x) \neq 0$ is satisfied. The solution beginning at the point $x \in S = \{x : \ s(x) = 0\}$ near the point x_0 can be shown to intersect the surface $s(x) = 0$ at least once. Let $\tau = \tau(x)$ be the time of the first return and $x(\tau) \in S$ be the point of the first return.

Definition 5. The map $P : x \mapsto x(\tau)$ is called the Poincaré or point map. It is widely used to study the chaotic processes. In Section 4, use of the Poincaré map in control of chaos is considered.

The chaotic models can be used to describe nonperiodic oscillatory processes with nonconstant varying characteristics (frequency and phase, for example). The existing methods enable one to estimate these characteristics from the results of observations. At that, the oscillation frequency becomes "fuzzy" and gives way to the spectrum which is continuous. As was already mentioned above, the local instability, that is, scatter of the initially close trajectories, is the main criterion for chaoticity. Correspondingly, the speed of scatter defined by the so-called *senior Lyapunov index* is the main characteristic of chaoticity.

The Lyapunov indices are determined for the given "reference" trajectory $\overline{x}(t)$ of system (2) with the initial condition $\overline{x}(0) = x_0$ for which purpose an equation in variations (a system linearized near $\overline{x}(t)$) is composed:

$$\frac{d}{dt}\delta x = W(t)\delta x, \tag{6}$$

where $\delta x = x - \overline{x}(t)$, $W(t) = \dfrac{\partial F(x(t))}{\partial x}$ is the Jacobian matrix of system (2), that is, the matrix of partial derivatives of the right sides, which is calculated along the solution $\overline{x}(t)$. It is assumed that there exist partial derivatives of $F(x)$, that is, the right sides of (2) are smooth functions. By defining the initial deviation $z = \delta x(0)$, one can calculate

$$\alpha(x_0, z) = \lim_{t\to\infty} \frac{1}{t} \ln \frac{\|\delta x(t)\|}{\|z\|} \tag{7}$$

which characterizes the rate of exponential growth of the solutions of (6) in the direction z and is called the *characteristic index* (Lyapunov exponent) *in the direction* z [13, 36, 37, 214].

Already A.M. Lyapunov proved that under additional minor assumptions the limit in (7) exists, is finite for any $z \in R^n$, and independent of the initial choice of the point x_0 on the trajectory $x(t)$. Moreover, the number of different characteristic indices is finite, they can be numerated in the descending order $\alpha_1 \geq \alpha_2 \geq \ldots \geq \alpha_n$, and there exists a basis $z_i \in R^n$, $i = 1, \ldots, n$, for which $\alpha(x_0, z_i) = \alpha_i$, $i = 1, \ldots, n$.

The senior Lyapunov index α_1 is the most important one. If $\alpha_1 > 0$ along the bounded solution $\overline{x}(t)$ that is dense in the attractor Ω, then this solution is Lyapunov-unstable and the attractor is strange. At that, α_1 characterizes the degree of instability or, stated differently, the degree of exponential sensitivity to the initial data. Obviously, $\alpha_1 = \max_i \operatorname{Re} \lambda_i(A)$ for the linear system with constant matrix $\dot{x} = Ax$ and zero reference solution $\overline{x}(t) = 0$, that is, $|\alpha_1|$ coincides with the usual degree of stability (or instability) of the system.

The senior index α_1 can be calculated approximately without constructing fundamental solutions of the equations in variations

$$\alpha_1 = \frac{1}{t} \ln \frac{\|x(t) - \overline{x}(t)\|}{\epsilon}, \tag{8}$$

where $x(t)$ is the solution of (2) with the initial condition $x(0)$, $\|x(0) - \overline{x}(0)\| = \epsilon$, at that t is sufficiently large and $\epsilon > 0$, sufficiently small. To improve the accuracy of calculations, one can calculate the mean value of the right sides of (8) for different initial conditions x_0 taken along the trajectory $\overline{x}(t)$. Then, there is no need to take a very great t [37].

The Lyapunov indices characterize prediction of the system trajectories. Indeed, if

$$T \leq \frac{1}{\alpha_1} \ln \frac{\Delta}{\epsilon}, \tag{9}$$

where ϵ is the initial error, then after the time T the trajectory $\overline{x}(t)$ is approximated with the error Δ by another trajectory. Therefore, a chaotic trajectory can be predicted with a given accuracy for some time in advance. This is the basic distinction of the chaotic systems as models of indeterminacy from the stochastic systems where, generally speaking, the prediction error can be arbitrarily great even for an arbitrarily small prediction horizon.

3. PROBLEMS OF CONTROL OF CHAOTIC PROCESSES

The mathematical formulation of the well-known problems of controlling the chaotic processes is preceded by presenting the basic models of the chaotic systems that are used in what follows. The most popular mathematical models encountered in the literature on control of chaos are represented by the systems of ordinary differential equations (state equations)

$$\dot{x}(t) = F(x, u), \tag{10}$$

where $x = x(t)$ is the n-dimensional vector of the state variables; $u = u(t)$ is the m-dimensional vector of inputs (controls); $\dot{x} = dx/dt$; and the vector function $F(x, u)$ is usually assumed to be continuous. In the presence of external perturbations, the nonstationary model

$$\dot{x} = F(x, u, t) \tag{11}$$

is used. In many cases, a simpler, *affine control* model

$$\dot{x} = f(x) + g(x)u \tag{12}$$

may be used.

We note that some publications consider the coordinate control, where external actions (forces, moments, intensities of electrical or magnetic fields, and so on) play the part of input variables, and the parametric control, where the input variables are variations of the physical system parameters (for example $u(t) = p - p_0$, where p_0 is the rated value of the physical parameter p), as two basically different problems. But if consideration is given to processes obeying the nonlinear models which comprehend both classes, this difference in fact is not fundamental. G. Chen and Z. Liu [91] noted that for many chaotic systems equivalence of the problems of coordinate control with linear feedback and those of parametric control can be established only by a nonlinear change of coordinates.

The measured system output is denoted by $y(t)$. It can be defined as a function of the current system state:

$$y(t) = h(x(t)). \tag{13}$$

If the output variables are not given explicitly, then we can assume that the entire state vector is observable, that is, $y(t) \equiv x(t)$.

Used also are the discrete models defined by the difference state equations

$$x_{k+1} = F_d(x_k, u_k), \tag{14}$$

where as usual $x_k \in \mathbf{R}^n$, $u_k \in \mathbf{R}^m$, and $y_k \in \mathbf{R}^l$ denote, respectively, the values of the vectors of state, input, and output at the kth step. Such a model is defined by the map F_d. It is assumed that for all $t \geq t_0$ all the aforementioned models have solutions under the given initial conditions; it is usually assumed that $t_0 = 0$.

Now, we proceed immediately to formulating the problems of control of chaotic processes. The problems of *stabilization* of the unstable periodic solution (orbit) arise in suppression of noise and vibrations of various constructions, elimination of harmonics in the communication systems, electronic devices, and so on. These problems are distinguished for the fact that the controlled plant is strongly oscillatory, that is, the eigenvalues of the matrix of the linearized system are close to the imaginary axis. The harmful vibrations can be either regular (quasiperiodic) or chaotic. The problems of suppressing the chaotic oscillations by reducing them to the regular oscillations or suppressing them completely can be formalized as follows.

Let us consider a free (uncontrollable, $u(t) \equiv 0$) motion $x_*(t)$ of system (10) with the initial condition $x_*(0) = x_{*0}$. Let this motion be T-periodic, that is, $x_*(t+T) = x_*(t)$ be satisfied for all $t \geq 0$. We need to stabilize it, that is, reduce the solutions $x(t)$ of system (10) to $x_*(t)$:

$$\lim_{t\to\infty} \big(x(t) - x_*(t)\big) = 0 \tag{15}$$

or drive the system output $y(t)$ to the given function $y_*(t)$:

$$\lim_{t\to\infty} \big(y(t) - y_*(t)\big) = 0 \tag{16}$$

for any solution $x(t)$ of system (10) under the initial state $x(0) = x_0 \in \Omega$, where Ω is the given set of initial conditions.

The problem lies in determining the control function either as the *open-loop control action*

$$u(t) = U(t, x_0), \tag{17}$$

or the *state feedback*

$$u(t) = U\big(x(t)\big), \tag{18}$$

or the *output feedback*

$$u(t) = U\big(y(t)\big) \tag{19}$$

satisfying the control objective (15) or (16).

This formulation of the problem of stabilization of periodic motion is undistinguishable from the conventional control-theoretic *tracking problem*. Nevertheless, there exists a fundamental distinction lying in that to control the chaotic processes, one needs to reach the objective with a sufficiently,—in theoretic terms, arbitrarily—small level of the control action [228]. Solvability of this problem is not evident because of instability of the chaotic trajectories $x_*(t)$.

Stabilization of an unstable equilibrium is a special case. Let the right side of (10) satisfy $F(x_{*0}, 0) = 0$. Then, for $u(t) \equiv 0$ system (10) has the equilibrium state x_{*0} that should be stabilized in the above sense by choosing an appropriate control. This problem is characterized by an additional requirement on "smallness" of control.

The second class includes the control problems of *excitation* or *generation* of chaotic oscillations. These problems are also called the *chaotization* or *anticontrol*. They arise where chaotic motion is the desired behavior of the system. The pseudorandom-number generators and sources of chaotic signals in communication and radar systems are the classical examples. Recent information suggests that chaotization of processes could produce an appreciable effect in the chemical and biological technologies, as well as in handling of the loose materials. These problems are characterized by the fact that the trajectory of the system phase vector is not predetermined, is unknown, or is of no consequence for attaining the objective.

The formal objective of control could be represented as (16), but here the objective trajectory $x_*(t)$ is no more periodic. Moreover, it may be required that instead of motion along the given trajectory the controlled process satisfies some formal chaoticity criterion. For example, the scalar objective function $G(x)$ can be given, and the aim of control can be formulated as attainment of the limit equality

$$\lim_{t\to\infty} G(x(t)) = G_* \qquad (20)$$

or the inequality for the lower bound $G(x(t))$

$$\underline{\lim}_{t\to\infty} G(x(t)) \geq G_*. \qquad (21)$$

For the chaotization problems, the senior Lyapunov index $G = \alpha_1$ is usually taken as the objective function, and $G_* > 0$ is defined. The total energy of mechanical or electrical oscillations is sometimes taken as $G(x)$.

The third important class of the control objectives corresponds to the problems of *synchronization* or, more precisely, *controllable synchronization* as opposite to the *autosynchronization*. Synchronization finds important applications in vibration technology (synchronization of vibrational exciters [7]), communications (synchronization of the receiver and transmitter signals) [24, 45], biology and biotechnology, and so on. Numerous publications on control of synchronization of the chaotic processes and its application in the data transmission systems appeared in the 1990's [5, 14, 101, 148, 231].

In the general case, by the synchronization is meant the coordinated variation of the states of two or more systems or, possibly, coordinated variation of some of their characteristics such as oscillation frequencies [72]. If this requirement must be satisfied only asymptotically, then synchronization is said to be *asymptotic*. If without control (for $u = 0$) synchronization cannot arise in a system, then one may pose the problem of determining a control law under which the closed-loop system becomes synchronized. Therefore, synchronization can be used as the aim of control. Full or partial coincidence of the state vectors such as the equality

$$x_1 = x_2 \qquad (22)$$

can be a formal expression of the synchronous motion of two subsystems with the state vectors $x_1 \in R^n$ and $x_2 \in R^n$. In the united state space of the interacting subsystems, equality (22) specifies a subspace (diagonal). The requirement of *asymptotic synchronization* of the states x_1 and x_2 of the two systems can be represented as

$$\lim_{t\to\infty} (x_1(t) - x_2(t)) = 0 \qquad (23)$$

which implies convergence of $x(t)$ to the diagonal set $\{x : x_1 = x_2\}$ relative to the integral state vector $x = \{x_1, x_2\}$ of the entire system.

The fact that the desired behavior is not uniquely fixed and its characteristics are defined only partially is common to the problems of control of excitation and synchronization of oscillations. In the problem of excitation of oscillations, for example, requirements can be presented only to the oscillation amplitude, their frequency and form being permitted to vary within certain limits. In the synchronization problems, the main requirement often is formulated as coincidence or coordination of the oscillations of all subsystems, whereas the characteristics of each subsystem can vary within wide limits.

Definition of one or more desired numerical indices is a convenient mathematical expression of the aim of control in the problems of this sort. The system energy, for example, can be used as such an index in the problem of oscillation excitation. In the problems of synchronization, asymptotic coincidence of the values of some index $G(x)$ for both systems can be the aim:

$$\lim_{t\to\infty}\Big(G\big(x_1(t)\big) - G\big(x_2(t)\big)\Big) = 0. \tag{24}$$

The aims of control (15), (16), (20), (23), or (24) often can be more conveniently rearranged in terms of the corresponding objective function $Q(x, t)$ in

$$\lim_{t\to\infty} Q\big(x(t), t\big) = 0. \tag{25}$$

For example, in order to rearrange the aim of control (23) in (25), one can take $Q(x) = \|x_1 - x_2\|^2$. For (15), one can use an objective function of the form $Q(x, t) = \big(x - x_*(t)\big)^{\mathrm{T}} \varGamma \big(x - x_*(t)\big)$, where $\varGamma$ is a positive definite symmetrical matrix. One needs to take into account that the choice of an appropriate objective function is an important step in the design of the control algorithm.

We note that the class of the admissible control laws (18), (19) can be extended owing to the dynamic feedbacks that are described by the differential equations or the lag equations. We also note that similar formulations are applicable to other classes of the aforementioned models.

An important type of problems of control of the chaotic processes is represented by the *modification of the attractors*, for example, transformation of the chaotic oscillations into the periodic ones and *vice versa*. Development of the approaches to the problems of this kind was stimulated by new applications in the laser and chemical technologies, in the telecommunications, biology, and medicine [89, 127]. By introducing a weak feedback, for example, in the optical channel, one can restore operability of a laser exhibiting the chaotic (multimode) behavior. As the result, it becomes possible to increase its radiation power while retaining coherence. In the chemical technology, chaoticity of mixing in the reactor, on the contrary, is useful because it accelerates reaction and improves product quality. Consequently, increased chaoticity is the reasonable aim of control in this case. Finally, in medicine it was proposed to treat some cases of arrhythmia by means of feedback pacers that vary the degree of nonregularity of the cardiac rhythm [78, 133] by generating stimulating pulses at appropriate time instants. Since arrhythmia can manifest itself both in increased and reduced chaoticity of the cardiac rhythm as compared with the individual norm of a patient, the aim of control in this case is to support the given degree of nonregularity.

4. METHODS OF CONTROL OF THE CHAOTIC PROCESSES

4.1. Open-Loop Control

The principle of control by perturbation or "control by the program signal," that is, generation of a control signal as some time function disregarding the values of the controlled process, is based on varying behavior of the nonlinear system under the action of predetermined external input $u(t)$ which can be either a certain physical action on the system such as force or field or variation ("modulation") of some parameter of the controlled system. This approach has appeal owing to its simplicity because it does without any measurements or sensors. This is especially important for control of superfast processes occurring, for example, at the molecular or atomic level where the system state cannot be measured (at least in real time).

The possibility of appreciable variation of the system dynamics by periodic excitation is known for a long time. For example, as was demonstrated in the first half of the last century [16, 263], high-frequency excitation can stabilize a pendulum in an unstable state. This discovery laid the foundation of the *vibrational mechanics* [8]. Analysis of the impact of high-frequency excitation on the behavior of the general-form nonlinear systems was based on the Krylov–Bogolyubov *method of averaging* [9]. In the control theory, the high-frequency actions and the parametric modulation were considered within the framework of the vibrational control [69, 208] and the so-called dither control [290]. They were also considered in the recent works of G.A. Leonov on nonstationary stabilization [22, 23]. Yet these works considered only system stabilization either in the given equilibrium state or relative to the given ("objective" or "reference") trajectory.

To modify the characteristics of a system represented in the *Lur'e form*,[3] the recent publications [215, 216] suggested proposed to use the vibrating control with the piecewise-constant stochastic (dither) input, which enables one to influence the form of equivalent nonlinearity, system equilibrium, and so on (similar to harmonic and static linearization, see [39, 40]). In particular, the aforementioned works used the heuristic criterion for chaos [134] to study the possibility of exciting or suppressing chaotic processes in the system.

The effect of middle-frequency excitations, that is, excitations lying within the range of system eigenfrequencies,[4] was discussed in a vast literature. K. Matsumoto and I. Tsyda [204] demonstrated the possibility of suppressing chaos in the Belousov–Zhabotinskii reaction by adding a white noise-like perturbation. V.V. Alekseev and A.Yu. Loskutov [1–3] considered a system of the fourth order describing the dynamics of a water ecosystem that consists of two kinds of phytoplankton and two kinds of zooplankton. They showed that it is possible to transform chaotic oscillations into periodic by means of a weak periodic action upon the system parameter whose value never goes outside the domain of chaoticity. The above results are based on computer-aided modeling.

First attempts of theoretical conceptualization were made in [189, 234] where the *Mel'nikov method* [34] was used to consider the so-called "Duffing–Holmes oscillator"

$$\ddot{\varphi} - c\varphi b\varphi^3 = -a\dot{\varphi} + d\cos(\omega t). \tag{26}$$

The right side of (26) was considered as a small perturbation acting on an unperturbed Hamiltonian system. The Mel'nikov function representing the rate of changing the distance between the stable and unstable manifolds under small perturbations was established analytically. It allowed one to determine the values of parameters for which the system behaves chaotically. An additional perturbation that lies in varying the function $b(1 + \eta\cos\Omega t)$ that replaces the nonlinearity parameter b was then introduced and a new Mel'nikov function was determined. Numerical studies of this function demonstrated that the chaotic behavior can be suppressed if the frequency Ω is taken close to that of the initial excitation ω. This effect was verified experimentally by a setup consisting of two permanent magnets, electromagnetic vibrator, and optical sensor [130]. The results obtained and the formulations of some new problems can be found in [190]. For a wider class of nonlinear oscillators, similar results were obtained in [83, 84]. The results of [1–3] were developed and analytically justified in [26–28].

The recent studies are aimed to improve chaos suppression with simultaneous reduction of the required level of external action and make the system trajectories to converge to the desired periodic orbit (limit cycle). Additionally, studies on control of the discrete-time systems (control

[3] That is, as a linear dynamic subsystem with static feedback nonlinearity.
[4] We note that for the nonlinear systems, their eigenfrequency depends on their amplitude.

of *maps*) were carried out. For example, computer-aided modeling of the Josephson junction, processes in liquid crystals, and experiments with a bistable mechanical system were used in [131] to demonstrate that variations of the phase and frequency of the parametric perturbation can both increase or reduce the threshold of occurrence of chaos.

The influence of quasiperiodic excitation was studied in [15, 68] by reducing it to a periodic action. The Mel'nikov method was used in [62] to analyze the effect of parametric excitation, which is a random process, on the system; and in [269] it was proposed to choose an excitation frequency close to the resonance peak of the spectral density of one of the system variables. An attempt to reach resonance by excitation with the frequency of the desired periodic process was made in [212]. The required amplitude (energy) of excitation can be reduced substantially by an appropriate choice of control because the chaotic attractor includes the trajectories of processes close to periodic processes with different periods. This approach was illustrated in [212] as applied to the Lorenz system (1) and a high-order system of 32 diffusion-connected Lorenz systems. The parameter r of system (1) was excited harmonically. The possibility of stabilizing unstable periodic trajectories under a periodic signal with frequency much lower than the characteristic system frequency was shown in [94, 235]. Suppression of the chaotic processes of ferromagnetic resonance in the yig films was considered in [236].

Some papers relate the choice of excitation function with the form of nonlinearity inherent in the system. Let us consider this method in more detail. Let the plant model be as follows:

$$\dot{x} = f(x) + Bu, \quad x \in \mathbf{R}^n, \quad u \in \mathbf{R}^m. \tag{27}$$

We assume that $m = n$ and $\det B \neq 0$. If $x_*(t)$ is the desired trajectory of the controlled motion, then the choice of excitation (the so-called "Hubler action") in the form

$$u_*(t) = B^{-1}\left(\dot{x}_*(t) - f(x_*(t))\right) \tag{28}$$

is intuitively justified [157] because the function $x_*(t)$ satisfies the equations of motion of the excited system. In this case, the error equation $e = x - x_*(t)$ has the form $\dot{e} = f(e + x_*(t)) - f(x_*(t))$. Therefore, if the linearized system with the matrix $A(t) = \partial f(x_*(t))/\partial x$ is uniformly stable in the sense that $A(t)+A(t)^{\mathrm{T}} \leq -\lambda I$ is satisfied for some λ and all $t \geq 0$,[5] then all solutions of (27) and (28) converge to $x_*(t)$, that is, the aim of control (15) is reached. The monograph [127] gives more general conditions for convergence. If $m < n$ and B is a degenerate matrix, then a similar result can be obtained by satisfying the following *condition for coordination*: the values of the vector function $\dot{x}_*(t) - f(x_*(t))$ must lie within the linear subspace generated by the columns of the matrix B. Then, the corresponding control can be taken in the form $u_*(t) = B^+(\dot{x}_*(t) - f(x_*(t)))$, where B^+ is a matrix pseudoinverse to B. As some papers note, despite the fact that satisfaction of uniform stability rules out the possibility of chaotic, that is, *unstable*, trajectories x_*, if the domains with unstable behavior are not dominating, then local convergence to chaotic trajectories is possible. For some examples, this approach is compared in [245] with other methods. Consideration was given to a system of the second order describing the so-called "Murali–Lakshmanan–Chua electrical circuit" and the FitzHugh–Nagumo equations describing passage of the nerve impulses through the neuron membrane. The results of numerical study of different methods of program excitation of chaos in noise can be found in [247]. Similar results were obtained for the discrete systems in [154, 210].

Analytical frequency conditions for global convergence of the solutions of the Lur'e systems to a stable mode under the action of nonperiodic excitations were obtained in [125]. They are based on the earlier results for the periodic input processes [20] and admit presence of instability domains of the controlled system.

[5] Here and in what follows, the matrix inequalities for the symmetrical matrices are understood in the sense of quadratic forms: $A \geq B$ if the matrix $A - B$ is positive semidefinite.

To sum up, one can state that numerous methods of control of chaotic processes in open loop have been developed. Their majority were studied numerically for special cases and model problems. Yet the general problem of the conditions for excitation or suppression of chaotic oscillations by the program still remains unsolved.

4.2. Linear and Nonlinear Control

The possibilities of using the traditional approaches and methods of automatic control to the problems of chaos control are discussed in numerous papers. The desired aim can be reached sometimes even by means of the simple proportional law of control and feedback. As was shown in [163], for example, the method of *combined control*, which is called in the physical papers the open-plus-closed-loop (OPCL) control, for $m = n$ and $\det B \neq 0$ is applicable to the systems of the form (27). The control law was proposed in the form

$$u(t) = B^{-1}\left(\dot{x}_*(t) - f(x_*(t)) - K(x - x_*(t))\right), \tag{29}$$

where K is the square gain matrix. The numerical results of studying this method for the chaotic systems can be found in [49, 162]. Nonlinear variants of the method of combined control were proposed in [270, 283]. Control with the proportional amplitude-pulse modulation was studied in [82, 86, 87]. The feedback in an *extended space* (x, u), that is, the dynamic controller, was studied in [29–32, 292] by considering the configuration of poles. In this case, one obtains space-local results because of imprecise linearization.

The case of $m = n$, $\det B \neq 0$ is trivial from the point of view of the modern control theory. Indeed, for the solution of system (27), (29) to converge to the desired trajectory $x_*(t)$, it suffices that K be chosen as $K = \kappa\mathbf{I}$, where $\kappa > \sup\limits_t \|A(t)\|$, $A(t) \triangleq \partial f(x_*(t))/\partial x$. This choice is always possible if the vector function $x_*(t)$ is bounded, in particular, for the periodic and chaotic trajectories $x_*(t)$.

The theory of nonlinear control developed a diversity of methods to solve more involved problems under incomplete control and measurement. One of the most elaborate methods is the *feedback linearization* [35, 161, 168] that was applied to the chaotic systems in [51, 58, 93, 288]. We explain its concept for the affine-control systems

$$\dot{x} = f(x) + g(x)u, \quad x \in \mathbf{R}^n, \quad u \in \mathbf{R}^m. \tag{30}$$

System (30) is called *feedback linearizable in the domain* $\Omega \subset \mathbf{R}^n$ if there exist a smooth reversible change of coordinates $z = \Phi(x)$, $x \in \Omega$ and a smooth transformation of the feedback

$$u = \alpha(x) + \beta(x)v, \qquad x \in \Omega, \tag{31}$$

where $v \in \mathbf{R}^m$ is the new control if the closed-loop system is linear, that is, for some constant matrices A and B its equation in the new coordinates is as follows:

$$\dot{z} = Az + Bv. \tag{32}$$

For one-input systems ($m = 1$), the criterion for feedback linearizability is simple: system (30) is feedback linearizable in the neighborhood of some point $x_0 \in \mathbf{R}^n$ if and only if there exists a scalar function $h(x)$ such that at the point x_0 the system has the degree n in the output $y = h(x)$. We recall that the relative degree is r if successive differentiation of the output function $y = h(x)$ along the trajectories of system (30) provides an expression containing input precisely at the rth step. More formally,

$$L_g L_f^k h(x) = 0, \quad k = 0, 1, \ldots, r-2, \qquad L_g L_f^{r-1} h(x) \neq 0, \tag{33}$$

where $L_\Psi \Phi(x)$ denotes the *Lie derivative* of the vector function $\Phi(x)$ along the vector field Ψ: $L_\Psi \Phi(x) \triangleq \sum_{i=1}^{n} \frac{\partial \Phi}{\partial x_i} \Psi_i(x)$.

If the linearizability criterion is satisfied, then by means of the transformations

$$\begin{aligned} z &= \Phi(x) = \operatorname{col}\left(h(x), L_f h(x), \ldots, L_f^{n-1} h(x)\right), \\ u &= \frac{1}{L_g L_f^{n-1} h(x)} \left(-L_f^n h(x) + v\right) \end{aligned} \tag{34}$$

the system can be reduced to the so-called *Brunovsky canonical form* (chain of integrators).

Example. Let us consider the Lorenz system with a scalar control in the third equation:

$$\begin{cases} \dot{x}_1 = \sigma(x_2 - x_1) \\ \dot{x}_2 = r x_1 - x_2 - x_1 x_3 \\ \dot{x}_3 = -\beta x_3 + x_1 x_2 + u. \end{cases} \tag{35}$$

We choose $y = x_1$. Then, $L_f y = \dot{y} = \dot{x}_1 = \sigma(x_2 - x_1)$, $L_f^2 y = L_f(L_f y) = \ddot{x}_1 = \sigma\left(\dot{x}_2 - \dot{x}_1\right) = \sigma\left((r+1)\, x_1 - 2x_2 + x_1 x_3\right)$. Obviously, the relative degree $r = 3$ everywhere except for the plane $x_1 = 0$. The change of coordinates can be defined by the relations

$$\begin{aligned} x_1 &= z_1, \\ x_2 &= \frac{1}{\sigma} z_2 + z_1, \\ x_3 &= \frac{1}{z_1} \left(\frac{1}{\sigma} z_3 - (r-1) z_1 - \frac{2}{\sigma} z_2\right), \end{aligned}$$

that is, the system is feedback linearizable for $x_1 \neq 0$. Therefore, in each of the semispaces $\{x_1 < 0\}$, $\{x_1 > 0\}$ system (35) is equivalent to the linear system. Since the Brunovsky linear system is fully controllable, any given dynamics of the closed-loop system can be provided using the methods of the theory of linear systems. The fact that this solution is not global is its disadvantage. Another essential disadvantage lies in that this approach completely disregards the eigendynamics of the system. An arbitrary desired dynamics is obtained at the expense of high control power required for substantial initial conditions and tracking of the rapidly varying program motion. Inapplicability to weak controls is a typical disadvantage of many works using the traditional methods of nonlinear and adaptive control.

The potentialities of the dynamic feedbacks can be better realized by using the observers. This approach provides a systematic groundwork for control under incomplete system measurements. The reader is referred to [224] for a review of the methods of designing the nonlinear observers as applied to the problems of control of chaos. Some special methods are described also in [139, 217]. The paper [186] presents the results of applying the linear high-gain observers to control of systems with nonlinearities satisfying the global Lipschitz condition.

We note that for the chaotic models the global Lipschitz condition is often not satisfied because of the presence of polynomial terms such as $x_1 x_2$, x^2, and so on. Limitedness of the trajectories of chaotic systems that takes place in the proper motion can be violated under the action of control. Consequently, when choosing control one must pay special attention to limitedness of the solutions. Otherwise, in a finite time the solution may "break loose," that is, go to infinity, which makes senseless any discussion of stability and convergence. The possibility that the trajectories of nonlinear systems may break loose is often overlooked in the applied papers.

A number of methods are based of the changes of the current value of some *objective function* $Q(x(t), t)$ whose value may correspond to the distance between the system state at the given time $x(t)$ and the current point $x_*(t)$ on the given trajectory—for example, $Q(x, t) = \|x - x_*(t)\|^2$, where $\|x\|$ is the Euclidean norm of the vector x. The distance from the current position of system $x(t)$ to the given *objective surface* $h(x) = 0$ such as $Q(x) = \|h(x)\|^2$ can be taken as the objective function. For the continuous-time systems, $Q(x)$ is not directly dependent (at the same time instant) on the control u. Therefore, a new immediately appearing objective function $\dot{Q}(x) = (\partial Q/\partial x)\, F(x, u)$ can be used instead of $Q(x)$, that is, the rate of change of this function can be decreased instead of decreasing the values of the initial objective function. This is the main idea of the *method of speed gradient* (SG-method) [35, 43] where the control u is changed in the direction of the antigradient in u of the speed $\dot{Q}(x)$ of the original objective function. This approach to controlling the chaotic systems was first suggested in [44]. The SG-algorithms have some modifications. In the so-called *finite form*, they are generally set down as follows:

$$u = -\Psi(\nabla_u \dot{Q}(x, u)), \tag{36}$$

where $\Psi(z)$ is some vector function whose value is directed at an acute angle to its argument z. For the affine controlled plants $\dot{x} = f(x) + g(x)u$, algorithm (36) can be simplified as

$$u = -\Psi(g(x)^{\mathrm{T}} \nabla Q(x)). \tag{37}$$

The *proportional* SG-algorithm

$$u = -\Gamma \nabla_u \dot{Q}(x, u), \tag{38}$$

where Γ is some positive definite matrix, as well as the *relay* SG-algorithm

$$u = -\Gamma \mathrm{sgn}(\nabla_u \dot{Q}(x, u)) \tag{39}$$

are special cases of (36). The *differential form* of the SG-algorithms

$$\dot{u} = -\Gamma \nabla_u \dot{Q}(x, u). \tag{40}$$

is used in the adaptation problems.

The method of speed gradient is based on the Lyapunov function V decreasing along the trajectories of the closed-loop system. The finite form of the SG-algorithms is obtained if the objective function $V(x) = Q(x)$ is itself taken as the Lyapunov function. The differential form of the SG-algorithms corresponds to choosing $V(x, u) = Q(x) + 0.5(u - u_*)^{\mathrm{T}} \Gamma^{-1}(u - u_*)$, where u_* is the desired ("ideal") value of the control variables.

Example (stabilization of equilibrium of the thermal convection model). Experiments carried out in the 1980's with thermal convection were among the first experiments demonstrating the nonregular oscillatory motions in physical systems [36, 214]. Similar facilities were later used to experiment with control of thermal convection [260]. The mathematical model of the controlled process is as follows:

$$\begin{cases} \dot{x} = \sigma(y - x) \\ \dot{y} = -y - xz \\ \dot{z} = -z + xy - r + u, \end{cases} \tag{41}$$

where x is the convection velocity; y, z are the temperature differences in the horizontal and vertical directions, respectively; σ is the Prandtl number; and r is the Rayleigh number. The deviation of the heating rate from the rated value r serves as the control variable u. System (41) is obtained from the Lorenz system by substituting z for $z-r$ under the assumption that $r = \text{const}$ and $b = 1$. For $u = 0$ and $0 < r < 1$, the system has a unique global attracting equilibrium $(0, 0, -r)$ corresponding to the stationary process of thermal convection. For $r = 1$, two new stable equilibria C_+ and C_- with the coordinates $x = y = \pm\sqrt{r-1}$ and $z = -1$ appear. These equilibria in turn lose stability in the Andronov–Hopf bifurcation for $r = \sigma(\sigma+4)/(\sigma-2)$; and for greater values of the parameter r, the system has no equilibria.

S. Sinha [260] proposed a relay control law for stabilization of the system equilibrium:

$$u = -\gamma \,\mathrm{sgn}\,(z+1). \tag{42}$$

Experiments demonstrated that upon introduction of the control law (42) into the loop, it stabilizes convection in either—clockwise or counterclockwise—direction, which corresponds to stabilization of one of the equilibria: C_+ or C_-.

One can readily see that algorithm (42) is a special case of the speed gradient algorithm (39) for the objective function

$$Q(x, y, z) = (x - \sqrt{r-1})^2/\sigma + (y - \sqrt{r-1})^2 + (z+1)^2.$$

As was proved in [127], any trajectory of the closed-loop system tends to one of the equilibria belonging to the set of points (x, y, z) such that

$$\left\{x = y, \quad \left|(x + \sqrt{r-1})(x - \sqrt{r-1})\right| \le \gamma, \quad z = -1\right\}. \tag{43}$$

Therefore, any solution tends to the neighborhood of either of the equilibria C_+ or C_-, size of the neighborhood vanishing with decrease of the gain γ of the algorithm.

The problem of stabilizing the invariant objective manifold $h(x) = 0$ by a small control is solved in [271] using the *method of macrovariables* proposed by Kolesnikov [18, 19].

Other methods of the modern theory of nonlinear control such as the theory of *center manifold* [129], the *backstepping* procedure and the methods of iterative design [202, 203], the method of *passivity-based design* [35, 127], the method of *variable-structure systems* (VS-system) [115, 178, 287, 289]; the theory of *absolute stability* [265], the H_∞-optimal design [102, 264], and a combination of the direct Lyapunov method and linearization by feedback [196, 225] were used to solve the problems of stabilization about the given state or the objective manifold. We note that the variable-structure algorithms with the switching surface $h(x) = 0$ coincide with the speed gradient algorithms (39) for which the objective function is chosen in the form $Q(x) = \|h(x)\|$.

Existence of a feedback *passifying*, that is, making passive, the closed-loop system is the prerequisite for efficiency (attainment of the objective) of the majority of the above approaches. For the control-affine system (30), this means that there exist function $V(x)$ and feedback (31) such that

$$\dot{V}(x) = \frac{\partial V}{\partial x}\Big(f + g\alpha + g\beta v\Big) \le yv. \tag{44}$$

Omitting some details, one can state that (44) is satisfied if the output y is taken as $y = L_g V\beta$. It is nothing but the speed gradient algorithm for $x \ne 0$; at that, $\dot{V}\big|_{y=0} < 0$. The last condition means that the so-called *zero dynamics* of the system, that is, motion on the manifold $y = 0$, is asymptotically stable. This property is called the property of *minimum phase* [35].

A combination of the frequency approach and the methods of nonlinear control proved to be fruitful (see [66, 136] and their references). In particular, the approximate method of harmonic balance was used together with the precise findings of the theory of absolute stability for estimating and predicting the chaotic processes. An interesting result on using the selective ("*washout*") filter that damps all signals of the frequencies lying outside a narrow range (see also [211]) was obtained within the framework of this area of research. If such a filter is introduced into the feedback of a chaotic system and its basic frequency coincides, conventionally speaking, with the frequency of one of the available unstable periodic solutions, then it is more likely that the system behaves periodically, rather than chaotically. Application of this approach to control of lasers was described in [64, 99].

Therefore, the majority of the methods of nonlinear control as applied to control of chaos can be classified with two main approaches: the Lyapunov approach (SG method, passivity method) and the "compensatory" approach (linearization by feedback, geometrical methods, and so on). The relationship between these two approaches may be illustrated as follows.

Let stabilization of the zero value of some output variable $y = h(x)$ of the affine system $\dot{x} = f(x)+g(x)u$ be the aim of control. The Lyapunov methods, including that of speed gradient, use an objective function of the form $Q(x) = \|h(x)\|^2$ and reduce its derivative $\dot{Q}$ according to the condition $h^{\mathrm{T}}\partial h/\partial x(f + gu) < 0$ by moving along the speed gradient $Q(x)$, that is, the antigradient $\dot{Q}$,

$$u = -\gamma g^{\mathrm{T}}(\nabla h)h.$$

Obviously, the condition for "smallness of control" can be satisfied if the gain $\gamma > 0$ is sufficiently small.

The compensatory approach is based on defining a prescribed (desirable) dynamics either of the entire state of the system or of some function of its state. For example, the macrovariable $\alpha(x) = \dot{y} + \rho y$ is introduced to design a control algorithm, where $\rho > 0$ is a parameter whose value is zeroed by choosing the control

$$u = -\frac{f^{\mathrm{T}}(\nabla h) + \rho h}{g^{\mathrm{T}}(\nabla h)}.$$

We note that $\alpha = 0$ if and only if $\dot{Q} = -2\rho Q$, that is, the compensation is equivalent to defining the decrease rate of $Q(x)$. As the result, any desired "instantaneous" speed of the transient processes is obtained at the expense of flexibility and smallness of control.

We conclude this section by emphasizing once more that the works using the well-developed methods of the modern linear and nonlinear control theory often do not pay sufficient attention to the specific characteristics of the chaotic processes, which manifests itself in the disregarded requirement on smallness of control. On the other hand, the publications taking this requirement in account do not use to the full the powerful arsenal of the modern control theory. Moreover, many publications consider only examples of low-order systems and make unrealistic assumptions. For example, some publications assume that the number of control actions is equal to the dimensionality of the system state vector.

4.3. Adaptive Control

Many publications consider the possibility of applying the methods of adaptation to the control of chaotic processes, which is not surprising because in many physical applications the parameters of the controlled plant are unknown and the information about the model structure (for example, dimensionality of the system equations or the form of the nonlinear characteristics) more often than not is incomplete. The majority of works make use of the methods of direct or indirect (identification-based) adaptive parametric control. The system model is, thus, *parametrized*, that is, comes to

$$\dot{x} = F(x, \theta, u), \quad y = h(x), \tag{45}$$

where θ is the vector of the unknown parameters. According to (45), the control law is also set down in the parametric form

$$u = \mathcal{U}(x, \xi), \tag{46}$$

where $\xi = \varPhi(\theta)$, that is, the vector of controller parameters is defined through the vector of parameters of system (45). The processes obtained by measuring the system state $\{x(t)\}$ or output $\{y(t)\}$ are used either online or offline to establish the estimates $\widehat{\theta}(t)$ of the unknown parameters $\theta(t)$ or to adjust directly the controller parameters $\xi(t)$.

A large arsenal of the existing methods of adaptation such as the methods of *gradient* and *speed gradient*, *least squares*, *maximum likelihood*, and so on can be used to develop algorithms of adaptive control and parametric identification. For the continuous-time systems, various adaptation algorithms can be obtained using the *differential form* (40) of the SG-algorithms. The majority of the existing results were obtained by *linear* parametrization of model (45) or controller (46).

These methods are well known from the literature on the control theory (see, for example, [127, 172]). Their validation is usually based on the Lyapunov functions which either are chosen quadratic from the beginning or are rearranged in the quadratic form by some transformation of the variables. For the problems of control of a typical chaotic systems of the second and third orders (systems of Lorenz, Chua, Duffing, and so on), references [81, 188, 291] and many other publications that were not included in the present review present examples making use of this approach. Similar methods are used for higher-order systems [116, 205, 206, 282, 285]. Controller (46) is usually designed using the *reference model* or the methods of linearization by feedback.

We note that different treatments of the adaptive approach can be encountered in the literature on control of the chaotic processes. For example, [156, 246, 261] apply the term adaptive to the simple linear integral law of control

$$\dot{\xi} = \gamma(y_* - y), \tag{47}$$

where y_* is the desired value of the output variable y and γ is the gain ("*stiffness*"). We explain the situation by discussing system (45), (46). Substitution of (46) into (45) provides the following equations of the adjustable-parameter system:

$$\dot{x} = F(x, \varPhi(\xi), \mathcal{U}(x, \xi)), \quad y = h(x). \tag{48}$$

It is easy to demonstrate that for the affine closed-loop system $\dot{x} = f(x) + g(x)\xi$ and the quadratic objective function $Q = (y - y_*)^2$ algorithm (47) is a special case of SG-algorithm (40) where ξ is substituted for u. The sign of the parameter γ must coincide with that of $\operatorname{sgn}\mu(x)$, where $\mu(x) = \nabla h(x)^{\mathrm{T}} g(x)$. Additionally, $\operatorname{sgn}\mu(x) = \text{const}$ is the necessary condition for efficiency of algorithm (47). In this case, the general form of SG-algorithm (40) is as follows:

$$\dot{\xi} = \gamma(y_* - y)\mu(x). \tag{49}$$

Its convergence follows from the general conditions for stability of the SG-algorithms (see [127]). The so-called *Huberman–Lumer law* generalizing (47) can be easily substantiated in a similar way.

A number of publications suggest control algorithms based on adjusting only one parameter. Already the classical publication of E. Lorenz [195] proposed to analyze systems by using the so-called *return map* $y_k \mapsto y_{k+1}$, where $y_k = y(t_k)$ is the value of some scalar variable $y(t)$ at the time t_k of reaching the current local maximum. It is believed [36, 214] that in the strong-dissipation systems one can confine the memory depth with sufficient precision to unity and consider the characteristics of the original system by analyzing the function $y_{k+1} = \mathcal{L}(y_k)$. For example, the transformation of chaotic motion into periodic corresponds to stabilization of an unstable fixed point of the map $\mathcal{L}(\cdot)$ which usually can be reached by varying one control parameter.

The publications [55, 74] propose to adjust adaptively the time intervals between the successive process maxima. This method is applicable to laser control. The papers [95–97] proposed a discrete algorithm of one-parameter adaptation that is close to the lms method. Its application was illustrated by the models of treating cardiac arrhythmia and controlling the Belousov–Zhabotinskii chemical reaction. Another method of one-parameter adaptive control based on the concept of the "universal adaptive controller" was proposed in [273].

4.4. Linearization of the Poincaré Map (OGY-Method)

The possibility of transforming the chaotic motion into periodic by external action on the system was discovered by Matsumoto and Tsyda [204] and Alekseev and Loskutov [1–3] as early as in the mid-1980's. However, only the 1990's witnessed an explosive growth of interest to the control of chaotic processes, which is largely due to the paper of E. Ott, C. Grebogi, and Yorke [228] where they formulated the following two key ideas: (1) designing controller by the *discrete* system model based on linearization of the Poincaré map and (2) using the property of *recurrence* of the chaotic trajectories and applying the control action only at the instants when the trajectory returns to some neighborhood of the desired state or given orbit.

The original paper described this method for the second-order discrete systems and the third-order continuous systems. Its realization needs real-time (keeping pace with the controlled process) calculation of the eigenvectors and eigenvalues of the Jacobian matrix for the Poincaré map. Publication of this method, which is now called the "OGY-method," was followed by numerous extensions and treatments. The concept of the OGY-method as represented in the recent papers [73, 141–144] is as follows.

Let the controlled process obey the following state equations:

$$\dot{x} = F(x, u), \tag{50}$$

where $x \in \mathbf{R}^n$, $u \in \mathbf{R}^1$. By the variable u is meant [228] the changeable system parameter, rather than the standard "input" control variable. But since the plant is nonlinear, this difference is insignificant from the control standpoint. Let the desired (objective) trajectory $x_*(t)$ be a solution of (50) for $u(t) \equiv 0$. This trajectory can be either periodic or chaotic, but in both cases it is recurrent. We construct the surface (so-called *Poincaré section*)

$$S = \{x : s(x) = 0\} \tag{51}$$

passing through the given point $x_0 = x_*(0)$ transversally to the trajectory $x_*(t)$ and consider the map $x \mapsto P(x, u)$ where $P(x, u)$ is the point of first return to the surface S of the solution of (50) that begins at the point x and was obtained for the constant input u. The map $x \mapsto P(x, u)$ is called the controllable Poincaré map. Owing to the recurrence of $x_*(t)$, this map is defined at least for some neighborhood of the point x_0. (Strict definition of the controllable Poincaré map involves some technicalities [12, 127]). By considering a sequence of such maps, we get the discrete system

$$x_{k+1} = P(x_k, u_k), \tag{52}$$

where $x_k = x(t_k)$, t_k is the time instant of the kth intersection of the surface S and u_k is the value of control $u(t)$ over the interval between t_k and t_{k+1}.

The next step in designing the control law lies in replacing the original system (50) by the linearized discrete system

$$\widetilde{x}_{k+1} = A\widetilde{x}_k + Bu_k, \tag{53}$$

where $\widetilde{x}_k = x_k - x_0$. Stabilizing control is determined for the resulting system as, for example, the linear state feedback $u_k = Cx_k$. The final form of the proposed control law is as follows:

$$u_k = \begin{cases} C\widetilde{x}_k & \text{if } \|\widetilde{x}_k\| \le \Delta \\ 0, & \text{otherwise,} \end{cases} \tag{54}$$

where $\Delta > 0$ is a sufficiently small parameter. The fact that control acts only in some neighborhood of the objective trajectory, that is, an "external" deadzone is introduced into the piecewise-constant control law, is the key characteristic of the method. This provides smallness of the control action which, according to (54), does not exceed in norm $|C\widetilde{x}_k| \le \|C\|\Delta$.

To guarantee efficiency of the method, the controller parameters (matrix C) should be chosen so that in the linear closed-loop system the error norm $\|\tilde{x}_k\|$ $\|(A+BC)x\| \le \rho\|x\|$ decreased, where $\rho < 1$. Another suitable quadratic norm can be used instead of the Euclidean norm, if necessary. Then, having once entered the Δ-neighborhood of the objective, the trajectory of the closed-loop system will not leave it. On the other hand, the trajectory certainly will enter any Δ-neighborhood of the objective trajectory owing to the property of recurrence.

Different authors present the results of numerical studies corroborating efficiency of this approach. They often mention low rate of process convergence, which is the cost of global stabilization of the trajectories of the nonlinear system by a small control.

In order to use the OGY-method, one has to overcome two serious obstacles: inaccuracy of the system model and incompleteness of the current state of the process. To eliminate the latter, it was proposed to use instead of the original state vector x the so-called *delayed coordinates vector* $X(t) = \left[y(t), y(t-\tau), \ldots, y(t-(N-1)\tau)\right]^{\mathrm{T}} \in \mathbf{R}^n$, where $y = h(x)$ is the measured output (for example, one of the system coordinates) and $\tau > 0$ is the delay time (parameter). The control law (54) then becomes

$$u_k = \begin{cases} u'_k, & \text{if } |y_{k,i} - y^*_{k,i}| \le \Delta_y, \ i = 1, \ldots, N-1 \\ 0, & \text{otherwise,} \end{cases} \tag{55}$$

where $y_{k,i} = y(t_k - i\tau)$, $y^*_{k,i} = h(x_*(t_k - i\tau))$, Δ_y is the maximum desired difference between $y_{k,i}$ and $y^*_{k,i}$, $u'_k = \mathcal{U}(y_k, y_{k,1}, \ldots, y_{k,N-1})$ and $\mathcal{U}$ is the function defining the form of the controller.

Design of the control law based on the structure of the plant model is theoretically better substantiated. For example, if the linearized model of the controlled plant in terms of the "input–output" variable is as follows:

$$y_k + a_1 y_{k,1} + \ldots + a_{N-1} y_{k,N-1} = b_0 u_k + \ldots + b_{N-1} u_{k-N-1}, \tag{56}$$

then one can resort to the standard technique of designing the linear controller from the given (reference) equation of the closed-loop system

$$\begin{aligned} u'_k = b_0^{-1}\big((a_1 - g_1) y_{k,1} + \ldots + (a_{N-1} - g_{N-1}) y_{k,N-1} - b_1 u_{k-1} \\ \ldots - b_{N-1} u_{k-N+1} + g_1 y^*_{k,1} + \ldots + g_{N-1} y^*_{k,N-1}\big), \end{aligned} \tag{57}$$

where g_i, $i = 1, 2, \ldots, N-1$, are the coefficients of the *reference equation* that should be chosen so that the polynomial $G(\lambda) = \lambda^{N-1} + g_1\lambda^{N-2} + \ldots + g_{N-1}$ be stable (all roots are less than 1). In a more compact form, (57) is as follows:

$$u'_k = \vartheta^{\mathrm{T}} w_k, \tag{58}$$

where $\vartheta \in \mathbf{R}^{2N-1}$ is the vector of parameters of controller (57) and $w_k = \{y_{k,1}, \ldots, y_{k,N-1}, u_{k-1}, \ldots, u_{k-N+1}, g_1 y^*_{k,1} + \ldots + g_{N-1} y^*_{k,N-1}\}$ is the vector of delayed coordinates and controls that is often called the regressor.

A special case of algorithm (55), the "occasional proportional feedback" (OFP) algorithm) which is used to stabilize the amplitude of the limit cycle was proposed in [159]. It is based on measuring the local maxima (or minima) of the output $y(t)$, that is, its Poincaré section is determined according to (51), where $s(x) = \partial h/\partial x F(x, 0)$, which amounts to satisfying the condition $\dot{y} = 0$ for the free system. If y_k is the value of the kth local maximum, then the OPF-method leads to a simple algorithm of control

$$u_k = \begin{cases} K\widetilde{y}_k & \text{if} \quad |\widetilde{y}_k| \le \Delta \\ 0, & \text{otherwise,} \end{cases} \tag{59}$$

where $\widetilde{y}_k = y_k - y_*$ and $y_* = h(x_0)$ is the required y_k defining the necessary oscillation amplitude.

We note that no full justification of algorithms (57) and (59) appeared until now. The main difficulty lies in estimating the accuracy of the linearized Poincaré map in the delayed coordinates (56).

To overcome the first of the aforementioned obstacles which is caused by indeterminacy of the linearized plant model, [228] and subsequent publications [54, 73, 141, 142] suggest to estimate the model parameters in the state Eqs. (53). However, the above papers did not present in detail a method to estimate the parameters of model (53) from the measurements of the output process. This problem is well known in the identification theory. It is not simple because identification in a closed loop can provide a "bad" estimate for a "good" control.

The algorithm of the OGY-method was modified and substantiated in [4, 12, 120, 124, 127] for the special case of $y_{k,i} = y_{k-i}$, $i = 1, \ldots, n$ where the outputs are measured and the control is changed only at the instants of intersection with the section surface $y^*_{k,i} = y^* = h(x_0)$. The controller was designed using the input–output model (56) that has less coefficients than model (53). The parameters were estimated using the V.A. Yakubovich method of *recurrent objective inequalities* which enables one to solve the problem of identification in the closed loop. It was suggested to introduce an inner deadzone into the law of control. The control algorithm obeys conditions (55) and the relations

$$\begin{aligned}
\mu_{k+1} &= \begin{cases} 1 \text{ if } |y_{k+1} - y_*| > \Delta_y \ \text{ and } \ |y_{k-i} - \overline{y}(t_{k-i})| < \overline{\Delta}, \quad i = 0, \ldots, N-1 \\ 0, \text{ otherwise,} \end{cases} \\
\vartheta'_{k+1} &= \begin{cases} \vartheta_k - \gamma \operatorname{sgn} b_0 (y_{k+1} - y_*) w_k / \|w_k\|^2 & \text{if} \quad \mu_{k+1} = 1 \\ \vartheta_k, \ \text{otherwise,} \end{cases} \\
u'_{k+1} &= \vartheta^{\mathrm{T}}_{k+1} w_{k+1}, \\
\vartheta_{k+1} &= \begin{cases} \vartheta'_{k+1} & \text{if} \quad |u'_{k+1}| \le \overline{u} \quad \text{and} \quad \mu_{k+1} = 1 \\ \vartheta'_{k+1} - (u'_{k+1} - \overline{u})/\|w_k\|^2 & \text{if} \quad u'_{k+1} > \overline{u} \quad \text{and} \quad \mu_{k+1} = 1 \\ \vartheta'_{k+1} - (u'_{k+1} + \overline{u})/\|w_k\|^2 & \text{if} \quad u'_{k+1} < -\overline{u} \quad \text{and} \quad \mu_{k+1} = 1 \\ \vartheta_k & \text{if} \quad \mu_{k+1} = 0, \end{cases}
\end{aligned} \tag{60}$$

where $\gamma > 0$ is the adaptation gain; $\overline{u}$ is the maximum magnitude of control; and $\overline{\Delta}$ is related to the size of the "tube" in the state space about the basic trajectory $\overline{x}(t)$, where the input–output model (56) is defined. Combination of this inner deadzone with an outer one which is inherent in the OGY-method makes the identification-based control robust both to model inaccuracy and measurement errors.

Other modifications and generalizations of the OGY-method followed. For example, [112] suggested to use only the data pertinent to one period of oscillations; and [249] proposed a "quasi-continuous" variant of the OGY-method. A multistep modification of the algorithm was discussed in [153]. Instead of the piecewise-constant control over the intervals between the instants of return, [111, 113] proposed to use the nonstationary action $u(t) = c(t)\overline{u}$ where $c(t)$ is chosen from the condition for the minimum control energy. The works [56, 57] proposed an iterative procedure of updating the controller that extends the domain of attraction and reduces the duration of transients. A study of the attraction domain for estimation of the initial state and the parameters can be found in [85], and the system behavior in the transient mode was studied also in [153]. New results provided by the computer-aided modeling and demonstrating the efficiency of the OGY-method are described in [48] for the *Kopel map*, in [59] for the *Bloch wall*, and in [227] for the magnetic domain-wall system. Additionally, efficiency of the method was demonstrated by physical experiments with a bronze band [257], glow discharge [80], and controllable RL-diod electrical circuit [6]. The OPF-method was used in [258] to stabilize the frequency of infrared stripe laser diod and realized as an electronic unit for chaos control. A modification of the OPF-method was considered in [117].

4.5. Time-Delayed Feedback (Pyragas Method)

The recent years witnessed increase of interest to the method of *time-delayed feedback* proposed in 1992 by a Lithuanian physicist K. Pyragas [241] who considered the problem of stabilizing an unstable τ-periodic orbit of a nonlinear system (10) by a simple feedback law

$$u(t) = K\big(x(t) - x(t-\tau)\big), \tag{61}$$

where K is the transmission coefficient and τ is the time of delay. If τ is equal to the period of the existing periodic solution $\overline{x}(t)$ of Eq. (10) for $u = 0$ and the solution $x(t)$ of the equation of the closed-loop system (10), (61) begins on the orbit $\varGamma = \{\overline{x}(t)\}$, then it remains in $\varGamma$ for all $t \geq 0$. Interestingly, $x(t)$ can converge to $\varGamma$ even if $x(0)\overline{\in}\varGamma$.

The feedback law (61) is also used to stabilize the periodic excited process in system (10) with T-periodic right side. Then, τ must be taken equal to T. For the discrete systems, the algorithm of this method is constructed in an obvious manner.

An extended variant of the Pyragas method was proposed in [262]. Here, the control is as follows:

$$u(t) = K\sum_{k=0}^{M} r_k\big(y(t-k\tau) - y(t-(k+1)\tau)\big), \tag{62}$$

where $y(t) = h(x(t)) \in \mathbf{R}^1$ is the measured output and r_k, $k = 1, \ldots, M$, are the parameters of the algorithm. For $r_k = r^k$, $|r| < 1$, and $M \to \infty$, algorithm (62) assumes the form

$$u(t) = K\big(y(t) - y(t-\tau)\big) + Kru(t-\tau). \tag{63}$$

Despite the simple form of algorithms (61)–(63), analytical study of the closed-loop system is a challenge. Only numerical and experimental results pertaining to the properties and area of application of the Pyragas method were known until now.

The works [63, 65] considered stability of the excited T-periodic solution of the Lur'e system with the "generalized Pyragas controller"

$$u(t) = G(p)\big(y(t) - y(t-\tau)\big), \tag{64}$$

where $G(p)$ $(p = d/dt)$ is the filter transfer function. These works used the methods of the absolute stability theory [183] to obtain the sufficient conditions to be satisfied by the transfer function of the linear part of the controlled system, as well as the conditions for the slope of the nonlinear characteristic that must be satisfied for $G(p)$ to be a stabilizing filter. A procedure of designing an "optimal" controller maximizing the stability domain was proposed in [65] and extended in [66] to the systems with a nonlinear nominal part.

A simple necessary condition for stabilizability by the Pyragas algorithm (61) was obtained in [274] for one class of discrete systems. The fundamental matrix $\Phi(t)$ of a system linearized in the given τ-periodic solution is introduced. This condition lies in that the number of the real eigenvalues of the matrix $\Phi(t)$ that are greater than unity needs not to be odd. For a more general case and for continuous-time system, the proofs were obtained independently in [165, 220]. For the extended control law (62), the results can be found in [179, 221]. These works apply the Floquet theory to systems linearized in the given periodic solution. Obviously, the eigenvalues of the matrix $\Phi(t)$ (multipliers) μ_i, $i = 1, 2, \dots, n$, are related by $\rho_i = \tau^{-1} \ln |\lambda_i|$ with the Lyapunov indices ρ_i of the τ-periodic solution. By means of a similar approach, [166] carried out a more detailed analysis and roughly estimated the boundaries of the feedback coefficient K that make the periodic solution stable. In particular, the domain [165] of values of K supporting stabilization includes arbitrarily small values of K for a small degree of instability $\max \rho_i$ and vanishes for a sufficiently large $\max \rho_i$. Some boundaries of the parameter K for the Lorenz system were obtained in [259] using the *Poincaré–Lindstedt method of small parameter*.

As was noted in [253] for the discrete system $y_{k+1} = f(y_k, u_k)$, the inequality $\lambda < 1$, where $\lambda = \partial f / \partial y(0, 0)$, is the necessary existence condition for the discrete variant of the stabilizing feedback (62). This result follows from the Giona theorem [135]. It was shown that the constraint $\lambda < 1$ can be overcome by periodic modulation of the parameter K.

The Pyragas method was extended to the connected (open flow) systems in [174, 175, 177] and modified in [221] for the systems with symmetry. Additionally, [180] proposed to generalize the method by introducing an observer estimating the difference between the system state and the desired unstable trajectory (or the given point).

If in (63) one takes $|r| > 1$, then the resulting algorithm still can be applied, although the resulting controller becomes unstable, which allows one to relax substantially the constraints on the plant matrix $\Phi(t)$ and, in particular, remove the "odd number" constraint [240].

We note that the problem of determining the sufficient conditions guaranteeing applicability of the original algorithm (61) remains still unsolved despite an appreciable recent information about the Pyragas method.

The literature mentions use of this method in stabilization of the laser coherent modes [25, 70, 222, 223] and magnetoelastic systems [147, 149], control of the heart conductivity model [79], step oscillations [108], traffic model [176, 179], voltage transformer with pulse-width modulation [67] excited by an oscillator described by the popular physiological FitzHugh–Nagumo model [71], or catalytic reactions in the bubbling gas-solid fluidized bed reactors [167]. Comparison of the method of control with time delay in the feedback and the methods of open-loop control of lasers can be found in [137].

Sensitivity to the parameter, especially to the delay time τ, is a disadvantage of the control law (61). Obviously, if the system is T-periodic and the aim of control is to stabilize the forced T-periodic solution, then one necessarily has to choose $\tau = T$. An alternative heuristic technique lies in modeling the proper processes in the system for the initial conditions $x(0)$ until the current state $x(t)$ approaches $x(s)$ for some $s < t$, that is, until the condition $\|x(t) - x(s)\| < \varepsilon$ is satisfied. Then, the choice of $\tau = t - s$ provides a reasonable estimate of the period, and the vector $x(t)$ is the initial state from which control of the process begins. However, this approach often results in excessive values of the period. Since the chaotic attractors have periodic solutions with different periods, it is important to determine the least-period motion and stabilize it by a small control. This problem is still open. Finally, the adaptive estimate of τ [171] is not always helpful, and study of the adaptive algorithm [171] is extremely difficult. No analytical results on the adaptive algorithms with time-delayed feedback were obtained by now.

4.6. Discrete Systems

Some of the discrete-time algorithms were described in Section 4.4 discussing the Poincaré map-based methods of control and in Section 4.5 discussing the methods of control with time-delayed feedback. They may be regarded as varieties of the pulse laws of control. Many general results on stability of the pulse feedback systems were obtained thus far. Analysis of their stability in the context of systems with chaotic dynamics can be found in [286].

Despite the fact that many authors make use of the term "optimal control," they mostly propose only *local-optimal* solutions based on minimizing the current value of the loss function $Q(F_d(x_k, u), u)$ in the control u, where F_d is the function in the right side of the plant model (14) and $Q(x, u)$ is the given objective function. For example, in [46, 47] the function $Q(x, u) = ||x - x_*||^2 + \kappa||u||^2$ is used. The requirement on smallness of the control action can be satisfied by choosing a great weight coefficient $\kappa > 0$. We note that for great κ the local-optimal control is close to the control by the gradient $u_{k+1} = -\gamma\nabla_u Q(F_d(x_k, u), u)$ with small $\gamma > 0$ [127].

The majority of publications on discrete-time control of chaos consider systems of lower orders. As follows from the Poincaré–Bendixson theorem, the set of examples of discrete chaotic systems seems to be even wider than for the continuous-time systems owing to the systems of the first and second orders that have no continuous equivalents. Popular examples are furnished by the systems described by the *logistic map* $x_{k+1} = ax_k(1 - x_k)$. They were discussed, in particular, in [100, 114, 207, 209]. Consideration is also given to the Hénon system (5), see [146]; the *tent map* ($x_{k+1} = rx_k$, $0 \le x_k < 0.5$; $x_{k+1} = r(1 - x_k)$, $0.5 \le x_k \le 1$), see [237]; and the *standard (Chirikov) map* ($v_{k+1} = v_k + K \sin\varphi_k$, $\varphi_{k+1} = \varphi_k + v_k$), see [182].

Behavior of the linear OGY-like controller with an outer deadzone of width ε and one-dimensional quadratic map $x_{k+1} = 1 - 2(x_k + u_k)^2$ in the neighborhood of the unstable equilibrium of the free system $x = 0.5$ was considered in [191]. Prior to closing the system by the controller, the plant is identified in the open loop under the action of a random sequence u_k, $(k = 1, 2, \dots, N)$. The paper gives recommendations on the choice of the parameters of ε and N from the requirements on system stability and robustness.

Only a few results on higher-order discrete systems are known. They are based on the gradient methods [46, 47, 127]; variable-structure systems [187]; and the generalized predicting control [229].

4.7. Neural Network-Based Control

There are several approaches to using the neural networks for control of chaotic processes. First, many studies rely on the universal ability of the neural networks to control and predict behavior of the nonlinear systems. Since the chaotic systems are basically nonlinear, the potentiality of their neural control is not surprising. Some universal neural-like learning networks for control of nonlinear systems were suggested in [150, 151, 239]. The structures of neural networks for control and prediction of the processes in nonlinear chaotic systems can be found in [61, 107, 145]. The papers [169, 245] compare the neural approach with other methods of control. It was proposed in [192, 284] to use the *genetic algorithms* for learning the neural network. A numerical example is considered, and the results of modeling are presented for it.

Second, many papers describe identification of the controlled plants by the neural networks combined with a standard method of control of chaotic systems such as the OGY-method [132], proportional feedback controller [170], and so on. Identification often is carried out in the closed loop in the course of normal system operation, which leads to *adaptive* or *learning* controllers. It was noted that the chaotic nature of the system processes speeds up identification and learning owing to diversification of the learning sample [92, 226, 251]. Some papers study new, nontraditional algorithms of identification and control. For example, [110] proposes a new approach to learning (adjustment of weights and modification of structure) of a serial neural network which lies in using controller, chaotic neural filter, and associative memory.

The third path of research, which is concerned with the neural networks as sources of chaos regarded as the controlled plant, is related with the psychological school of American Pragmatists—J. Dewey, in particular,—and also some European philosophers such as Heidegger and Piaget who considered the brain as a basically unstable and self-exciting (recreating) structure. Reasonable behavior is then understood in connection with the chaotic images produced as the result of neural activity. Therefore, analysis of the chaotic dynamics of neural networks and its control are of profound interest to the psychologists and physiologists (see, for example, [128]). The neural networks operating in the chaotic mode are used to model storage of information and pattern recognition by the brain [173, 218, 219, 250, 267]. These networks may consist not only of artificial neurons, but also of other nonlinear systems with controllable chaotic behavior such as chemical oscillators [152, 160].

The main problem discussed within the framework of this field of study lies in clarifying how an assembly of neurons behaving each chaotically and disconnectedly can generate functional chains manifesting stable and correct behavior. These issues were studied both theoretically and experimentally in [242–244]. Some other papers consider the possibility of control of chaos for some types of neural networks [213] and establish the conditions for origination of chaos in small neural networks [103].

4.8. Fuzzy Systems

Description of system indeterminacy in terms of fuzzy models provides specific versions of the control algorithms. The so-called *Takagi–Sugeno fuzzy systems* (*T–S-fuzzy systems*) obeying the set of fuzzy rules

$$\begin{array}{c} \text{IF} \quad z_1(t) \in F_{1i} \quad \text{AND} \ldots \text{AND} \quad z_p(t) \in F_{1p} \quad \text{THEN} \\ \dot{x} = A_i x + B_i u, \quad y = C_i x + D_i u, \quad i = 1, 2, \ldots, r \end{array} \tag{65}$$

are most convenient for design of control. Here, $x(t) \in \mathbf{R}^n$, $u(t) \in \mathbf{R}^m$, $y(t) \in \mathbf{R}^m$ are, respectively, the vectors of state, input, and output of the system; $z_j(t)$ are the variable antecedents that are the functions of the system state, its input variables $u(t)$, and, possibly, time; and F_{ji} are the fuzzy sets defined by the membership functions F_{ji}. The matrices A_i, B_i can depend on the variables $z_j(t)$, which enables one to describe the nonlinear systems in the form (65). The system output is determined by means of the so-called *defuzzying* by the method of center of gravity

$$y = \sum_{i=1}^{r} h_i(z) C_i x, \tag{66}$$

where $h_i(z) = \dfrac{\omega_i(z_i)}{\sum\limits_{i=1}^{r} \omega_i(z_i)}$, $\omega_i(z) = \prod\limits_{j=1}^{n} F_{ji}(z_j)$, $z = (z_1, z_2, \ldots, z_n)$. With this representation, the nonlinearity "hides" in the rule of defuzzying (66), which allows one to construct fuzzy models for a wide class of dynamic, including chaotic, systems. For example, the Lorenz system

$$\begin{cases} \dot{x}_1 = \sigma(x_2 - x_1) \\ \dot{x}_2 = \rho x_1 - x_2 - x_1 x_3 \\ \dot{x}_3 = \beta x_3 + x_1 x_2, \end{cases} \tag{67}$$

is representable as (65) if one takes $z_1 = z_2 = x_1$, $F_1(x_1) = 0.5(1 + x_1/d)$, $F_2(x_1) = 0.5(1 + x_1/d)$, where $d > 0$ is the estimate of the system limit set:

$$|x_1| \leq d; \quad A_1 = \begin{bmatrix} -\sigma & \sigma & 0 \\ \rho & -1 & -d \\ 0 & -d & -\beta \end{bmatrix}; \quad A_2 = \begin{bmatrix} -\sigma & \sigma & 0 \\ \rho & -1 & -d \\ 0 & d & -\beta \end{bmatrix}; \quad b_1 = b_2 = 0.$$

It is only natural to construct the algorithms to control the T–S-fuzzy systems as the fuzzy rules

$$\begin{gathered} \text{IF} \quad z_1(t) \in F_{1i} \quad \text{AND} \ldots \text{ AND} \quad z_p(t) \in F_{1p} \quad \text{THEN} \\ u = -K_i y, \quad i = 1, 2, \ldots, r, \end{gathered} \tag{68}$$

where K_i are the matrices of the coefficients of the linear fuzzy controller. The rules of defuzzying (66) allow one to represent the closed-loop fuzzy system as

$$\dot{x} = \sum_{i=1}^{r} \sum_{j=1}^{r} \sum_{k=1}^{r} h_i(z) h_j(z) h_k(z) \, (A_i - B_i K_j C_k) x. \tag{69}$$

One can readily see that for system (69) a diversity of the design problems can be posed by means of the quadratic Lyapunov function $V(x) = x^{\mathrm{T}} P x$ as the problems of simultaneous robust stabilization and reduced to the linear matrix inequalities (*LMI*). This approach was proposed in [266] for stabilization and synchronization of the chaotic systems, extended to the problems of *observer-based synchronization*, and applied to the information transmission systems [184, 185].

Another approach to designing the fuzzy models of chaotic systems relies on identification of their parameters in combination with the standard methods of designing the nonlinear systems [90]. On the contrary, [268] makes use of the direct (without identification) adaptive method. A two-frequency scheme of discrete control of the continuous fuzzy systems was proposed in [164]. In some papers, the fuzzy models of nonlinear systems are combined with the neuron-like network structure of controllers [98, 198].

We note that many works use the specific characteristics of the chaotic systems incompletely. They just exemplify unstable nonlinear systems demonstrating the potentialities of the control algorithms that can operate with a much wider class of plants. Therefore, the requirement on smallness of control usually is disregarded.

4.9. Other Problems and Methods

In this section we give just a brush treatment to other paths of research on the control of chaotic processes. We first note that important areas of research such as synchronization of the chaotic systems and control of chaos in distributed (space–time) systems were left out of the scope of this review for reasons of space. They are discussed in an extensive literature including some reviews such as [75, 76, 105, 106, 155, 232]. The following problems and methods of control of chaos deserve mentioning here.

Controllability. Although controllability of the nonlinear system is well studied, only few results were obtained on the reachability of the aim of control by small control actions, see [50, 77, 88, 120, 275]. The general concept that the more "unstable" (chaotic, turbulent) the system, the "simpler" or "cheaper" its precise or approximate controllability, was illustrated in [193].

Chaotization. The problem of system chaotization by feedback—also called the problem of "chaos synthesis," "chaos generation," or "anticontrol of chaos"—lies in constructing a control algorithm providing chaotic behavior of the system trajectories. At that, additional requirements can be imposed on the system characteristics. This problem which occurs in the broadband communication systems, computer applications, and so on, is in essence that of generating pseudorandom numbers and processes. Studies of the methods of generation of the chaotic signals can cast light on the mechanisms of biological systems such as the mechanisms of heart and brain activity.

This problem was first formulated in 1994 by A. Vanecek and S. Celikovsky [276] who proposed a scenario of chaotization for the Lur'e systems with monotone odd feedback nonlinearity. The Vanecek–Celikovsky scenario lies in choosing the poles $s_1, \ldots, s_n$ and zeros $z_1, \ldots, z_{n-1}$ of the transfer function of the linear part so that for all k, $0 < k < \infty$, the linear system embraced by the feedback $u = ky$ features the following: partial instability (presence of the poles both with positive and negative real parts); hyperbolicity (lack of poles on the imaginary axis); dissipation (negative sum of the real parts of the poles); and nonpotentiality (presence of poles with the zero imaginary part). Presence of chaos is established using the Shil'nikov theorem. The approximate chaoticity criterion [134] based on the method of harmonic balance can be used for chaotization.

In 1999, X.F. Wang and G. Chen [281] advanced a method based on the Marotto theorem [201] which is the multidimensional counterpart of the Sharkovrkii–Li–Yorke chaoticity criterion. For the discrete systems of the form $x_{k+1} = f(x_k) + u_k$, a feedback of the form $u_k = \varepsilon g(\sigma x_k)$, $\varepsilon > 0$, $\sigma > 0$, where the function $g(x)$ has a sawtooth or sinusoidal form, is constructed. By choosing a rather great σ, the stable equilibrium $x = 0$ is made unstable while retaining—as is required by the Marotto theorem—convergence at least of one trajectory to the point $x = 0$ in a finite number of steps. At that, the maximum value of $|u_k|$ can be made arbitrarily small by an appropriate choice of ε. The problem of chaotization of various discrete and continuous systems was studied by G. Chen and his collaborators [278–280] and other authors [17, 41, 181].

Other aims of control that were discussed in the literature include provision of process characteristics such as the mean period of oscillations [118]; fractal dimension [248]; invariant measure [53, 77, 138]; and Kolmogorov entropy [230]. A method of solving the problems of the so-called *tracking chaos*, that is, tracking of a nonstationary unstable orbit, was proposed in [256]. It is based on the method of continuation for solution of nonlinear equations [254]. The latest results in this area are generalized in [255].

Identification of chaotic systems is discussed in a number of papers. Their majority makes use of the traditional methods of identification. Presence of chaos was shown to be helpful in parameter estimation and improvement of convergence of this process [112, 158, 205, 233, 239, 272].

Chaos in the control systems. The problems of *control of chaos* are distinct from the problems of *studying chaos in the control systems*. Publications on this issue appeared as early as in the 1970's. They considered the possibility of chaotic behavior in the traditional linear [197], nonlinear [60], and adaptive [199] feedback control systems. Among the latest results, the conditions for origination of chaotic modes in the nonsmooth systems of the second order [52], higher-order hysteresis systems [41], pulse systems with pulse-width modulation [17], and some mechanical feedback systems [109, 140] deserve mentioning. The work [277] demonstrated that chaos can be conductive to better control.

CONCLUSIONS

Control of chaos remains an area of intensive research. The three pioneering and most powerful domains of study such as open-loop control (vibrational) control, linearization of the Poincaré map, and time-delayed feedback are actively explored. Nevertheless, they face numerous unsolved problems concerned mostly with substantiation of methods. The well-developed methods of nonlinear and adaptive control should be used carefully because of smallness of the control action. Disregard of this requirements may produce an impression of simplicity of the problem. From the point of view of "small control," the methods based on passivity offer an advantage because they allow one to attain the goal independently of the gain.

Interestingly, even now, a decade after the appearance of this area the majority of publications on control of chaos appear in physical journals. On the contrary, the number of papers in the journals devoted to automation and control is small. For example, of more than 1,700 papers presented at the 15th Triennial World Congress of IFAC (Barcelona, 2002) only ten had the word "chaos" in their titles.

Yet the number of applied physical and engineering problems where the methods of control of chaos can be used is steadily increasing, which allows us to predict a growth of interest to the methods of control of chaos and their further development. The second part of the present review will be devoted to some most interesting applications.

REFERENCES

1. Alekseev, V.V. and Loskutov, A.Yu., Destocastization of a System with Strange Attractor by Parametric Action, *Vestn. MGU*, 1985, vol. 26, no. 3, pp. 40–44.
2. Alekseev, V.V. and Loskutov, A.Yu., On the Possibility of Controlling a System with a Strange Attractor, in *Problemy ekologicheskogo monitoringa i modelirovaniya ekosistem* (Problems in Ecological Monitoring and Modeling of Ecosystems), Leningrad: Gidrometeoizdat, 1985, vol. VIII, pp. 175–189.
3. Alekseev, V.V. and Loskutov, A.Yu., Controlling a System with a Strange Attractor by Periodic Parametric Action, *Dokl. Akad. Nauk SSSR*, 1987, vol. 293, no. 6, pp. 1346–1348.
4. Andrievskii, B.R. and Fradkov, A.L., *Izbrannye glavy teorii avtomaticheskogo upravleniya, gl. 13* (Selected Chapters of the Automatic Control Theory, ch. 13), St. Petersburg: Nauka, 1999.
5. Andrievskii, B.R. and Fradkov, A.L., *Elementy matematicheskogo modelirovaniya v programmnykh sredakh MATLAB 5 and Scilab* (Elements of Mathematical Modeling in the MATLAB 5 and Scilab Programming Environments), St. Petersburg: Nauka, 2001.
6. Bezruchko, B.P., Ivanov, R.N., and Ponomarenko, V.I., Two-level Control of Chaos in Nonlinear Oscillators, *Lett. Zh. Teor. Fiz.*, 1999, vol. 25, no. 4, pp. 61–67.
7. Blekhman, I.I., *Sinkhronizatsiya dinamicheskikh sistem* (Synchronization of Dynamic Systems), Moscow: Nauka, 1971.
8. Blekhman, I.I., *Vibratsionnaya mekhanika* (Vibrational Mechanics), Moscow: Nauka, 1994. Translated under the title *Vibrational Mechanics*, Singapore: World Scientific, 2000.
9. Bogolyubov, N.N. and Mitropol'skii, Yu.A., *Asimptoticheskie metody v teorii nelineinykh kolebanii* (Asymptotic Methods in the Theory of Nonlinear Oscillations), Moscow: Fizmatgiz, 1958. Translated under the title *Asymptotic Methods in the Theory of Nonlinear Oscillations*, New York: Gordon-Breach, 1961.
10. Vorotnikov, V.I., *Ustoichivost' dinamicheskikh sistem po otnosheniyu k chasti peremennykh* (Stability of Dynamic Systems Relative to Part of Variables), Moscow: Nauka, 1991. Translated under the title *Partial stability and Control*, Boston: Birkhaüser, 1998.
11. Vorotnikov, V.I., Problems and Methods of Studying Motion Stability and Stabilization Relative to Part of Variables: Lines of Research, Results, Distinctions, *Avtom. Telemekh.*, 1993, no. 3, pp. 3–62.
12. Guzenko, P.Yu., Discrete Control of Continuous Chaotic Systems, in *Analiz i upravlenie nelineinymi kolebatel'nymi sistemami* (Analysis and Control of Nonlinear Oscillatory Systems), Leonov, G.A. and Fradkov, A.L., Eds., St. Petersburg: Nauka, 1998, pp. 53–84.
13. Demidovich, B.P., *Lektsii po matematicheskoi teorii ustoichivosti* (Lectures of the Mathematical Theory of Stability), Moscow: Mosk. Gos. Univ., 1998.
14. Dmitriev, A.S., Panas, A.I., and Starkov, S.O., Dynamic Chaos as a Paradigm of the Modern Communication Systems, *Zarub. Radioelektron.*, 1997, no. 10, pp. 4–26.
15. Zhalnin, A.Yu., Control of Chaos in Nonautonomous Systems with Quasiperiodic Excitation, *Lett. Zh. Teor. Fiz.*, 1999, vol. 25, no. 16, pp. 63–70.

16. Kapitsa, P.L., Dynamic Stability of Pendulum with Oscillating Suspension Point, *Zh. Teor. Fiz.*, 1951, vol. 21, no. 5,

17. Kipsnis, M.M., Chaotic Phenomena in the Deterministic One-dimensional Pulse-width Control System, *Izv. Ross. Akad. Nauk, Tekh. Kibern.*, 1992, no. 1, pp. 108–112.

18. Kolesnikov, A.A., Analytical Design of the Nonlinear Aggregated Controllers by the Given Assembly of Invariant Manifolds, *Izv. Vuzov, Elektromekh.*, 1987, part I, no. 3, pp. 58–66.

19. Kolesnikov, A.A., *Osnovy teorii sinergeticheskogo upravleniya* (Fundamentals of the Theory of Synergetic Control), Moscow: Ispo-Servis, 2000.

20. Leonov, G.A., Frequency Criterion for Stabilization by External Harmonic Action, *Avtom. Telemekh.*, 1986, no. 1, pp. 169–174.

21. Leonov, G.A., Burkin, I.M., and Shepelyavyi, A.I., *Chastotnye metody v teorii kolebanii* (Frequency Methods in the Oscillation Theory), St. Petersburg: S.-Peterburg. Gos. Univ., 1992.

22. Leonov, G.A., Algorithms of Linear Nonstationary Stabilization and the Brockett Problem, *Prikl. Mat. Mekh.*, 2001, vol. 65, no. 5, pp. 801–808.

23. Leonov, G.A. and Shumafov, M.M., *Problemy stabilizatsii lineinykh upravlyaemykh sistem* (Problems of Stabilizing Linear Controllable Systems), St. Petersburg: S.-Peterburg. Gos. Univ., 2002.

24. Lindsey, W., *Synchronization Systems in Communications and Control*, Englewood Cliffs: Prentice Hall, 1972. Translated under the title *Sistemy sinkhronizatsii v svyazi i upravlenii*, Moscow: Mir, 1978.

25. Loiko, N.A., Naumenko, A.V., and Turovets, S.I., Effect of the Pyragas Feedback on the Dynamics of Laser with Modulation of Losses, *Zh. Eksp. Teor. Fiz.*, 1997, vol. 112, no. 4 (10), pp. 1516–1530.

26. Loskutov, A.Yu., Rybalko, S.D., and Akinshin, L.G., Control of Dynamic Systems and Suppression of Chaos, *Diff. Uravn.*, 1998, no. 8, pp. 1143–1144.

27. Loskutov, A.Yu., Problems of Nonlinear Dynamics. II. Suppression of Chaos and Control of Dynamic Systems, *Vestn. MGU*, 2001, no. 2, pp. 3–21.

28. Loskutov, A.Yu., Chaos and Control of Dynamic Systems, in *Nelineinaya dinamika i upravlenie* (Nonlinear Dynamics and Control), Emel'yanov, S.V. and Korovina, S.K., Eds., Moscow: Fizmatlit, 2001, pp. 163–216.

29. Magnitskii, N.A., On Stabilization of Fixed Points of Chaotic Maps, *Dokl. Ross. Akad. Nauk*, 1996, vol. 351, no. 2, pp. 175–177.

30. Magnitskii, N.A., On Stabilization of Fixed Points of Chaotic Dynamic Systems, *Dokl. Ross. Akad. Nauk*, 1997, vol. 352, no. 5, pp. 610–612.

31. Magnitskii, N.A., On Stabilization of Unstable Cycles of Chaotic Maps, *Dokl. Ross. Akad. Nauk*, 1997, vol. 355, no. 6, pp. 747–749.

32. Magnitskii, N.A. and Sidorov, S.V., Control of Chaos in Nonlinear Dynamic Systems, *Diff. Uravn.*, 1998, no. 11, pp. 1501–1509.

33. Magnitskii, N.A. and Sidorov, S.V., Stabilization of Unstable Periodic Solutions in Equations with Delays, *Diff. Uravn.*, 2000, no. 11, pp. 1488–1492.

34. Mel'nikov, V.K., Stability of the Center under Time-periodic Perturbations, *Tr. Mosk. Mat. Ob.*, 1963, vol. 12, pp. 3–52.

35. Miroshnik, I.V., Nikiforov, V.O., and Fradkov, A.L., *Nelineinoe i adaptivnoe upravlenie slozhnymi dinamicheskimi sistemami* (Nonlinear and Adaptive Control of Complex Dynamic Systems), St. Petersburg: Nauka, 2000.

36. Moon, F.C., *Chaotic Vibrations: An Introduction for Applied Scientists and Engineers*, New York: Wiley, 1987. Translated under the title *Khaoticheskie kolebaniya*, Moscow: Mir, 1990.

37. Neimark, Yu.I. and Landa, P.S., *Stokhasticheskie i khaoticheskie kolebaniya* (Stochastic and Chaotic Oscillations), Moscow: Nauka, 1987.

38. *Osnovy matematicheskogo modelirovaniya* (Fundamentals of Mathematical Modeling), Fradkov, A.L., Ed., St. Petersburg: BGTU, 1996.

39. Pervozvanskii, A.A., *Kurs teorii avtomaticheskogo upravleniya* (Course of the Automatic Control Theory), Moscow: Nauka, 1986.

40. Popov, E.P., *Teoriya nelineinykh sistem avtomaticheskogo regulirovaniya i upravleniya* (Theory of Nonlinear Systems of Automatic Regulation and Control), Moscow: Nauka, 1979.

41. Postnikov, N.S., Stochasticity of the Relay Hysteresis Systems, *Avtom. Telemekh.*, 1998, pp. 349–358.

42. *Upravlenie mekhatronnymi vibratsionnymi ustanovkami* (Control of Mechanotronic Vibrational Installations), Blekhman, I.I. and Fradkov, A.L., Eds., St. Petersburg: Nauka, 2001.

43. Fradkov, A.L., Scheme of Speed Gradient and Its Application to Adaptive Control, *Avtom. Telemekh.*, 1979, no. 9, pp. 90–101.

44. Fradkov, A.L., Adaptive Control of Nonlinear Oscillations, in *Algoritmicheskoe obespechenie protsessov upravleniya v mekhanike i mashinostroenii: tez. dokl.* (Algorithmic Support of Control Processes in Mechanics and Machine-building: Abstracts of papers), Moscow, 1994, pp. 29–30.

45. Shakhgil'dyan, V.V. and Lyakhovkin, A.A., *Sistemy fazovoi avtopodstroiki chastoty* (Systems of Automatic Phase Frequency Control), Moscow: Radio i Svyaz', 1978.

46. Abarbanel, H.D.I., Korzinov, L., Mees A.I., *et al.*, Optimal Control of Nonlinear Systems to Giver Orbits, *Syst. Control Lett.*, 1997, vol. 31, pp. 263–276.

47. Abarbanel, H.D.I., Korzinov, L., Mees, A.I., *et al.*, Small Force Control of Nonlinear Systems to Given Orbits, *IEEE Trans. Circ. Syst. I*, 1997, vol. 44, pp. 1018–1023.

48. Agiza, H.N., On the Analysis of Stability, Bifurcation, Chaos and Chaos Control of Kopel Map, *Chaos, Solutions, Fractals*, 1999, vol. 10, pp. 1909–1916.

49. Aguirre, L.A. and Torres, L.A.B., Control of Nonlinear Dynamics: Where do Models Fit in?, *Int. J. Bifurcat. Chaos*, 2000, vol. 10, pp. 667–681.

50. Alleyne, A., Reachability of Chaotic Dynamic Systems, *Phys. Rev. Lett.*, 1998, vol. 80, pp. 3751–3754.

51. Alvarez-Gallegos, J., Nonlinear Regulation of a Lorenz System by Feedback Linearization Technique, *J. Dynam. Control*, 1994, no. 4, pp. 277–298.

52. Alvarez, J., Curiel, E., and Verduzco, F., Complex Dynamics in Classical Control Systems, *Syst. Control Lett.*, 1997, vol. 31, pp. 277–285.

53. Antoniou, I. and Bosco, F., Probabilistic Control of Chaos Through Small Perturbations, *Chaos, Solutions, Fractals*, 2000, vol. 11, no. 1–3, pp. 359–371.

54. Arecchi, F.T., Boccaletti, S., Ciofini, M., *et al.*, The Control of Chaos: Theoretical Schemes and Experimental Realizations, *Int. J. Bifurcat. Chaos*, 1998, vol. 8, pp. 1643–1655.

55. Arecchi, F.T. and Boccaletti, S., Adaptive Strategies for Recognition, Noise Filtering, Control, Synchronization and Targeting of Chaos, *Chaos*, 1997, vol. 7, pp. 621–634.

56. Aston, P.J. and Bird, C.M., Using Control of Chaos to Refine Approximations to Periodic Points, *Int. J. Bifurcat. Chaos*, 2000, vol. 10, pp. 227–235.

57. Aston, P.J. and Bird, C.M., Analysis of the Control of Chaos—Extending the Basin of Attraction, *Chaos, Solutions, Fractals*, 1997, vol. 8, pp. 1413–1429.

58. Babloyantz, A., Krishchenko, A.P., and Nosov A., Analysis and Stabilization of Nonlinear Chaotic Systems, *Comput. Math. Appl.*, 1997, vol. 34, pp. 355–368.

59. Badescu, C.S., Ignat, M., and Oprisan, S., On the Chaotic Oscillations of Bloch Walls and Their Control, *Chaos, Solutions, Fractals*, 1997, vol. 8, pp. 33–43.

60. Baillieul, J., Brockett, R.W., and Washburn, R.B., Chaotic Motion in Nonlinear Feedback Systems, *IEEE Trans. Circ. Syst. I*, 1980, vol. 27, pp. 990–997.

61. Bakker, R., De Korte, R.J., Schouten, J.C., *et al.*, Neural Networks for Prediction and Control of Chaotic Fluidized Bed Hydrodynamics: A First Step, *Fractals*, 1997, vol. 5, pp. 523–530.

62. Basios, V., Bountis, T., and Nicolis, G., Controlling the Onset of Homoclinic Chaos Due to Parametric Noise, *Phys. Let. A*, 1999, vol. 251, pp. 250–258.

63. Basso, M., Genesio, R., and Tesi, A., Stabilizing Periodic Orbits of Forced Systems via Generalized Pyragas Controllers, *IEEE Trans. Circ. Syst. I*, 1997, vol. 44, pp. 1023–1027.

64. Basso, M., Genesio, R., and Tesi, A., Controller Design for Extending Periodic Dynamics of a Chaotic CO2 Laser, *Syst. Control Lett.*, 1997, vol. 31, pp. 287–297.

65. Basso, M., Genesio, R., Giovanardi, L., *et al.*, On Optimal Stabilization of Periodic Orbits via Time Delayed Feedback Control, *Int. J. Bifurcat. Chaos*, 1998, vol. 8, pp. 1699–1706.

66. Basso, M., Genesio, R., Giovanardi, L., and Tesi, A., Frequency Domain Methods for Chaos Control, in *Controlling Chaos and Bifurcations in Engin. Systems*, Chen, G., Ed., CRC Press, 1999, pp. 179–204.

67. Batlle, C., Fossas, E., and Olivar, G., Stabilization of Periodic Orbits of the Buck Converter by Time-Delayed Feedback, *Int. J. Circ. Theory Appl.*, 1999, vol. 27, pp. 617–631.

68. Belhaq, M. and Houssni, M., Quasi-Periodic Oscillations, Chaos and Suppression of Chaos in a Nonlinear Oscillator Driven by Parametric and External Excitations, *Nonlin. Dynamics*, 1999, vol. 18, pp. 1–24.

69. Bellman, R., Bentsman, J., and Meerkov, S., Vibrational Control of Nonlinear Systems, *IEEE Trans. Aut. Control*, 1986, vol. AC–31, no. 8, pp. 710–724.

70. Bleich, M.E., Hochheiser, D., Moloney, J.V., *et al.*, Controlling Extended Systems with Spatially Filtered, Time-Delayed Feedback, *Phys. Rev. E*, 1997, vol. 55, pp. 2119–2126.

71. Bleich, M.E. and Socolar, J.E.S., Delayed Feedback Control of a Paced Excitable Oscillator, *Int. J. Bifurcat. Chaos*, 2000, vol. 10, pp. 603–609.

72. Blekhman, I.I., Fradkov, A.L., Nijmeijer, H., *et al.*, On Self-Synchronization and Controlled Synchronization, *Syst. Control Lett.*, 1997, vol. 31, pp. 299–305.

73. Boccaletti, S., Grebogi, C., Lai, Y.C., *et al.*, The Control of Chaos: Theory and Applications, *Phys. Reports*, 2000, vol. 329, pp. 103–197.

74. Boccaletti, S., Farini, A., and Arecchi, F.T., Adaptive Strategies for Recognition, Control, and Synchronization of Chaos, *Chaos, Solutions, Fractals*, 1997, vol. 8, pp. 1431–1448.

75. Boccaletti, S., Bragard, J., and Arecchi, F.T., Controlling and Synchronizing Space Time Chaos, *Phys. Rev. E*, 1999, vol. 59, pp. 6574–6578.

76. Boccaletti, S., Kurths, J., Osipov, G., *et al.*, The Synchronization of Chaotic Systems, *Phys. Reports*, 2002, vol. 366, pp. 1–101.

77. Bollt, E.M., Controlling Chaos and the Inverse Frobenius–Perron Problem: Global Stabilization of Arbitrary Invariant Measures, *Int. J. Bifurcat. Chaos*, 2000, vol. 10, no. 5, pp. 1033–1050.

78. Brandt, M. and Chen, G., Feedback Control of Pathological Rhythms in Two Models of Cardiac Activity, *Proc. 1St Int. Conf. "Control of Oscillations and Chaos" (COC'97)*, St. Petersburg, 1997, vol. 2, pp. 219–223.

79. Brandt, M.E., Shih, H.T., and Chen, G., Linear Time-delay Feedback Control of a Pathological Rhythm in a Cardiac Conduction Model, *Phys. Rev. E*, 1997, vol. 56, pp. 1334–1337.

80. Braun, T., Suppression and Excitation of Chaos: The Example of the Glow Discharge, *Int. J. Bifurcat. Chaos*, 1998, vol. 8, pp. 1739–1742.

81. Cao, Y.J., A Nonlinear Adaptive Approach to Controlling Chaotic Oscillators, *Phys. Lett. A*, 2000, vol. 270, pp. 171–176.

82. Casas, F. and Grebogi, C., Control of Chaotic Impacts, *Int. J. Bifurcat. Chaos*, 1997, vol. 7, pp. 951–955.

83. Chacon, R., Maintenance and Suppression of Chaos by Weak Harmonic Perturbations: A Unified View, *Phys. Rev. Lett.*, 2001, vol. 86, pp. 1737–1740.

84. Chancon, R., *Control of Homoclinic Chaos by Weak Periodic Perturbations*, Singapore: World Scientific, 2002.

85. Chanfreau, P. and Lyyjynen, H., Viewing the Efficiency of Chaos Control, *J. Nonlin. Math. Phys.*, 1999, vol. 6, pp. 314–331.

86. Chau, N.P., Controlling Chaos by Periodic Proportional Pulses, *Phys. Lett. A*, 1997, vol. 234, pp. 193–197.

87. Chau, N.P., Controlling Continuous Chaotic Dynamics by Periodic Proportional Pulses, *Phys. Rev. E*, 1998, vol. 57, pp. 378–380.

88. Chen, G., On Some Controllability Conditions for Chaotic Dynamics Control, *Chaos, Solutions, Fractals*, 1997, vol. 8, pp. 1461–1470.

89. Chen, G. and Dong, X., *From Chaos to Order: Perspectives, Methodologies and Applications*, Singapore: World Scientific, 1998.

90. Chen, L., Chen, G., and Lee, Y.W., Fuzzy Modeling and Adaptive Control of Uncertain Chaotic Systems, *Inf. Sci.*, 1999, vol. 121, pp. 27–37.

91. Chen, G. and Liu, Z., On the Relationship Between Parametric Variation and State Feedback in Chaos Control, *Int. J. Bifurcat. Chaos*, 2002, vol. 12, no. 6, pp. 1411–1415.

92. Chen, G. and Dong, X., *From Chaos to Order: Perspectives, Methodologies and Applications*, Singapore: World Scientific, 1998.

93. Chen, L.Q. and Liu, Y.Z., A Modified Exact Linearization Control for Chaotic Oscillators, *Nonlin. Dynam.*, 1999, vol. 20, pp. 309–317.

94. Chizhevsky, V.N., Vilaseca, R., and Corbalan, R., Experimental Switchings in Bistability Domains Induced by Resonant Perturbations, *Int. J. Bifurcat. Chaos*, 1998, vol. 8, pp. 1777–1782.

95. Christini, D.J., In, V., Spano, M.L., Ditto, W.L., *et al.*, Real-time Experimental Control of a System in its Chaotic and Nonchaotic Regimes, *Phys. Rev. E*, 1997, vol. 56, pp. 3749–3752.

96. Christini, D.J. and Collins, J.J., Real-Time, Adaptive, Model-independent Control of Lowdimensional Chaotic and Nonchaotic Dynamical Systems, *IEEE Trans. Circ. Syst. I*, 1997, vol. 44, pp. 1027–1030.

97. Christini, D.J. and Kaplan, D.T., Adaptive Estimation and Control Method for Unstable Periodic Dynamics in Spike Trains, *Phys. Rev. E*, 2000, vol. 61, pp. 5149–5153.

98. Chung, I.F., Lin, C.J., and Lin, C.T., A GA-based Fuzzy Adaptive Learning Control Network, *Fuzzy Sets Syst.*, 2000, vol. 112, pp. 65–84.

99. Ciofini, M., Labate, A., Meucci, R., *et al.*, Stabilization of Unstable Fixed Points in the Dynamics of a Laser with Feedback, *Phys. Rev. E*, 1999, vol. 60, pp. 398–402.

100. Codreanu, S. and Danca, M., Suppression of Chaos in a One-dimensional Mapping, *J. Biol. Phys.*, 1997, vol. 23, pp. 1–9.

101. Cuomo, K., Oppenheim, A., and Strogatz, S., Robustness and Signal Recovery in a Synchronized Chaotic System, *Int. J. Bifurcat. Chaos*, 1993, vol. 3, pp. 1629–1638.

102. Curran, P.F., Suykens, J.A.K., and Chua, L.O., Absolute Stability Theory and Master-Slave Synchronization, *Int. J. Bifurcat. Chaos*, 1997, vol. 7, pp. 2891–2900.

103. Das, A., Roy, A.B., and Das, P., Chaos in a Three Dimensional Neural Network, *Appl. Math. Model.*, 2000, vol. 24, pp. 511–522.

104. Ditto, W.L., Rauseo, S.N., and Spano, M.L., Experimental Control of Chaos, *Phys. Rev. Lett.*, 1990, vol. 65, pp. 3211–3214.

105. Ding, M.Z., Ding, E.J., Ditto, W.L., *et al.*, Control and Synchronization of Chaos in High Dimensional Systems: Review of Some Recent Results, *Chaos*, 1997, vol. 7, pp. 644–652.

106. Dmitriev, A.S., Kassian, G., and Khilinsky, A., Chaotic Synchronization. Information Viewpoint, *Int. J. Bifurcat. Chaos*, 2000, vol. 10, no. 4, pp. 749–761.

107. Dracopoulos, D.C. and Jones, A.J., Adaptive Neuro-Genetic Control of Chaos Applied to the Attitude Control Problem, *Neural Comput. Appl.*, 1997, vol. 6, pp. 102–115.

108. Elmer, F.J., Controlling Friction, *Phys. Rev. E*, 1998, vol. 57, pp. 4903–4906.

109. Enikov, E. and Stepan, G., Microchaotic Motion of Digitally Controlled Machines, *J. Vibr. Control*, 1998, vol. 4, pp. 427–443.

110. Eom, T.D., Sugisaka, M., and Lee, J.J., New Skill Learning Paradigm Using Various Kinds of Neurons, *Appl. Math. Comput.*, 1998, vol. 91, pp. 9–22.

111. Epureanu, B.I. and Dowell, E.H., Optimal Multi-Dimensional OGY Controller, *Physica D*, 2000, vol. 139, pp. 87–96.

112. Epureanu, B.I. and Dowell, E.H., System Identification for the Ott–Grebogi–Yorke Controller Design, *Phys. Rev. E*, 1997, vol. 56, pp. 5327–5331.

113. Epureanu, B.I. and Dowell, E.H., On the Optimality of the Ott–Grebogi–Yorke Control Scheme, *Physica D*, 1998, vol. 116, pp. 1–7.

114. Escalona, J. and Parmananda, P., Noise-Aided Control of Chaotic Dynamics in a Logistic Map, *Phys. Rev. E*, 2000, vol. 61, pp. 5987–5989.

115. Fang, J.Q., Hong, Y.G., Qin, H.S., and Chen, G., Nonlinear Control of Chaotic Systems: A Switching Manifold Approach, *Discret. Dynam. Natur. Soc.*, 2000, vol. 4, pp. 257–267.

116. Femat, R., Alvarez-Ramirez, J., and Fernandez-Anaya, G., Adaptive Synchronization of High-order Chaotic Systems: A Feedback with Low-order Parametrization, *Physica D*, 2000, vol. 139, pp. 231–246.

117. Flynn, C. and Wilson, N., A Simple Method for Controlling Chaos, *Am. J. Phys.*, 1998, vol. 66, pp. 730–735.

118. Fouladi, A. and Valdivia, J.A., Period Control of Chaotic Systems by Optimization, *Phys. Rev. E*, 1997, vol. 55, pp. 1315–1320.

119. Fradkov, A.L., Miroshnik, I.V., and Nikiforov, V.O., *Nonlinear and Adaptive Control of Complex Systems*, Dordrecht: Kluwer, 1999.

120. Fradkov, A., Guzenko, P., and Pavlov, A., Adaptive Control of Recurrent Trajectories Based on Linearization of Poincaré Map, *Int. J. Bifurcat. Chaos*, 2000, vol. 10, pp. 621–637.

121. Fradkov, A.L., Control of Chaos: Methods and Applications, *21th IASTED Conf. "Modeling, Identification and Control" (MIC 2002)*, Innsbruck, 2002, http://ccslab.nm.ru.

122. Fradkov, A.L. and Evans, R.J., Control of Chaos: Some Open Problems, *Proc. 40th IEEE Conf. Dec. Control*, Orlando, 2001, pp. 698–703.

123. Fradkov, A.L. and Evans, R.J., Control of Chaos: Survey 1997–2000, *Preprints of 15Th Triennial World Congress IFAC, Plenary Papers, Survey Papers, Milestones*, Barcelona, 2002, pp. 143–154.

124. Fradkov, A.L. and Guzenko, P.Yu., Adaptive Control of Oscillatory and Chaotic Systems Based on Linearization of Poincaré Map, *Proc. 4th Eur. Control Conf.*, Brussels, 1997.

125. Fradkov, A.L., Leonov, G.A., and Nijmeier, H., Frequency-domain Conditions for Global Synchronization of Nonlinear Systems Driven by a Multiharmonic External Signal, *Proc. 5th Europ. Control Conf.*, Karlsruhe, 1999.

126. Fradkov, A.L. and Pogromsky, A.Yu., Speed Gradient Control of Chaotic Continuous-time Systems, *IEEE Trans. Circ. Syst. I*, 1996, vol. 43, pp. 907–913.

127. Fradkov, A.L. and Pogromsky, A.Yu., *Introduction to Control of Oscillations and Chaos*, Singapore: World Scientific, 1998.

128. Freeman, W.J., Chang, H.J., Burke, B.C., *et al.*, Taming Chaos: Stabilization of Aperiodic Attractors by Noise, *IEEE Trans. Circ. Syst. I*, 1997, vol. 44, pp. 989–996.

129. Friedel, H., Grauer, R., and Marliani, C., Center Manifold Approach to Controlling Chaos, *Phys. Lett. A*, 1997, vol. 236, pp. 45–52.

130. Fronzoni, L., Giocondo, M., and Pettini, M., Experimental Evidence of Suppression of Chaos by Resonant Parametric Perturbations, *Phys. Rev. A*, 1991, vol. 43, pp. 6483–6487.

131. Fronzoni, L. and Giocondo, M., Controlling Chaos with Parametric Perturbations, *Int. J. Bifurcat. Chaos*, 1998, vol. 8, pp. 1693–1698.

132. Funke, M., Herrmann, M., and Der, R., Controlling Low-dimensional Chaos: Determination and Stabilization of Unstable Periodic Orbits by Kohonen Neural Nets, *Int. J. Adaptive Control Signal Proc.*, 1997, vol. 11, pp. 489–499.

133. Garfinkel, A., Spano, M., Ditto, W., *et al.*, Controlling Cardiac Chaos, *Science*, 1992, vol. 257, pp. 1230–1235.

134. Genesio, R. and Tesi, A., Harmonic Balance Methods for the Analysis of Chaotic Dynamics in Nonlinear Systems, *Automatica*, 1992, vol. 28, no. 3, pp. 531–548.

135. Giona, M., Functional Reconstruction of Oscillating Reaction: Prediction and Control of Chaotic Kinetics, *Chem. Engr. Sci.*, 1992, vol. 47, pp. 2469–2474.

136. Giovanardi, L., Basso, M., Genesio, R., *et al.*, Lower Bounds for the Stability Degree of Periodic Solutions in Forced Nonlinear Systems, *Int. J. Bifurcat. Chaos*, 2000, vol. 10, pp. 639–653.

137. Glorieux, P., Control of Chaos in Lasers by Feedback and Nonfeedback Methods, *Int. J. Bifurcat. Chaos*, 1998, vol. 8, pp. 1749–1758.

138. Gora, P. and Boyarsky, A., A New Approach to Controlling Chaotic Systems, *Physica D*, 1998, vol. 111, no. 1–4, pp. 1–15.

139. Grassi, G. and Mascolo, S., Nonlinear Observer Design to Synchronize Hyperchaotic Systems via a Scalar Signal, *IEEE Trans. Circ. Syst. I*, 1997, vol. 44, pp. 1011–1014.

140. Gray, G.L., Mazzoleni, A.P., and Campbell, D.R., Analytical Criterion for Chaotic Dynamics in Flexible Satellites with Nonlinear Controller Damping, *J. Guid. Control Dynam.*, 1998, vol. 21, pp. 558–565.

141. Grebogi, C., Lai, Y.C., and Hayes, S., Control and Applications of Chaos, *Int. J. Bifurcat. Chaos*, 1997, vol. 7, pp. 2175–2197.

142. Grebogi, C., Lai, Y.C., and Hayes, S., Control and Applications of Chaos, *J. Franklin Inst.*, 1997, vol. 334B, pp. 1115–1146.

143. Grebogi, C. and Lai, Y.C., Controlling Chaos in High Dimensions, *IEEE Trans. Circ. Syst. I*, 1997, vol. 44, pp. 971–975.

144. Grebogi, C. and Lai, Y.C., Controlling Chaotic Dynamical Systems, *Syst. Control Lett.*, 1997, vol. 31, no. 3, pp. 307–312.

145. Greenwood, D.P.A. and Carrasco, R.A., Neural Networks for the Adaptive Control of Disruptive Nonlinear Network Traffic, *IEE Proc. Commun.*, 2000, vol. 147, pp. 285–291.

146. Guzenko, P.Yu. and Fradkov, A.L., Gradient Control of Hénon Map Dynamics, *Int. J. Bifurcat. Chaos*, 1997, vol. 7, no. 3, pp. 701–705.

147. Hai, W.H., Duan, Y.W., and Pan, L.X., An Analytical Study for Controlling Unstable Periodic Motion in Magneto-elastic Chaos, *Phys. Lett. A*, 1997, vol. 234, pp. 198–204.

148. Hasler, M., Synchronization of Chaotic Systems and Transmission of Information, *Int. J. Bifurcat. Chaos*, 1998, vol. 8, no. 4, pp. 647–660.

149. Hikihara, T., Touno, M., and Kawagoshi, T., Experimental Stabilization of Unstable Periodic Orbit in Magneto-elastic Chaos by Delayed Feedback Control, *Int. J. Bifurcat. Chaos*, 1997, vol. 7, pp. 2837–2846.

150. Hirasawa, K., Murata, J., Hu, J.L., *et al.*, Chaos Control on Universal Learning Networks, *IEEE Trans. Syst. Man Cybern. C—Appl. Rev.*, 2000, vol. 30, pp. 95–104.

151. Hirasawa, K., Wang, X.F., Murata, J., *et al.*, Universal Learning Network and Its Application to Chaos Control, *Neural Netw.*, 2000, vol. 13, pp. 239–253.

152. Hohmann, W., Kraus, M., and Schneider, F.W., Recognition in Excitable Chemical Reactor Networks. Experiments and Model-Simulations, *J. Phys. Chem. A*, 1997, vol. 101, pp. 7364–7370.

153. Holzhuter, T. and Klinker, T., Control of a Chaotic Relay System Using the OGY Method, *Zeitschrift Naturforschung Section Ama J. Phys. Sci.*, 1998, vol. 53, pp. 1029–1036.

154. Hsu, R.R., Su, H.T., Chern, J.L., *et al.*, Conditions to Control Chaotic Dynamics by Weak Periodic Perturbation, *Phys. Rev. Lett.*, 1997, vol. 78, pp. 2936–2939.

155. Hu, G., Xiao, J.H., Yang, J.Z., *et al.*, Synchronization of Spatiotemporal Chaos and its Applications, *Phys. Rev. E*, 1997, vol. 56, pp. 2738–2746.

156. Hu, H.Y., An Adaptive Control Scheme for Recovering Periodic Motion of Chaotic Systems, *J. Sound Vibr.*, 1997, vol. 199, pp. 269–274.

157. Hubler, A. and Lusher, E., Resonant Stimulation and Control of Nonlinear Oscillators, *Naturwissenschaft*, 1989, vol. 76, pp. 67–72.

158. Huijberts, H., Nijmeijer, H., and Willems, R., System Identification in Communication with Chaotic Systems, *IEEE Trans. Circ. Syst. I*, 2000, vol. 47, no. 6, pp. 800–808.

159. Hunt, E.R., Stabilizing High-Period Orbits in a Chaotic System—The Diode Resonator, *Phys. Rev. Lett.*, 1991, vol. 67, pp. 1953–1955.

160. Ishii, S. and Sato, M., Associative Memory Based on Parametrically Coupled Chaotic Elements, *Physica D*, 1998, vol. 121, pp. 344–366.

161. Isidori, A., Bars, R., Dion, J.-M., *et al.*, IFAC 2002 Milestone Report on Design Methods, *Prep. 15th Triennial World Congress IFAC, Plenary Papers, Survey Papers, Milestones*, Barcelona, 2002, pp. 207–212.

162. Jackson, E.A., The OPCL Control Method for Entrainment, Model-Resonance, and Migration Actions on Multiple-Attractor Systems, *Chaos*, 1997, vol. 7, pp. 550–559.

163. Jackson, E.A. and Grosu, I., An OPCL Control of Complex Dynamic Systems, *Physica D*, 1995, vol. 85, pp. 1–9.

164. Joo, Y.H., Shieh, L.S., and Chen, G., Hybrid State-Space Fuzzy Model-Based Controller with Dualrate Sampling Digital Control of Chaotic Systems, *IEEE Trans. Fuzzy Syst.*, 1999, vol. 7, pp. 394–408.

165. Just, W., Bernard, T., Ostheimer, M., *et al.*, Mechanism of Time-Delayed Feedback Control, *Phys. Rev. Lett.*, 1997, vol. 78, pp. 203–206.

166. Just, W., Reibold, E., Benner, H., *et al.*, Limits of Time-Delayed Feedback Control, *Phys. Lett. A*, 1999, vol. 254. pp. 158–164.

167. Kaart, S., Schouten, J.C., and Van Den Bleek, C.M., Improving Conversion and Selectivity of Catalytic Reactions in Bubbling Gas-Solid Fluidized Bed Reactors by Control of the Nonlinear Bubble Dynamics, *Catalysis Today*, 1999, vol. 48, pp. 185–194.

168. Khalil, H., *Nonlinear Systems*, Englewood Cliffs: Prentice Hall, 1996.

169. Kim, S.H. and Lee, C.M., Nonlinear Prediction of Manufacturing Systems Through Explicit and Implicit Data Mining, *Comput. Industr. Engin.*, 1997, vol. 33, pp. 461–464.

170. Kiss, I.Z. and Gaspar, V., Controlling Chaos with Artificial Neural Network: Numerical Studies and Experiments, *J. Phys. Chem. A*, 2000, vol. 104, pp. 8033–8037.

171. Kittel, A., Parisi, J., and Pyragas, K., Delayed Feedback Control of Chaos by Self-Adapted Delay Time, *Phys. Lett. A*, 1995, vol. 198, pp. 433.

172. Krstic, M., Kanellakopoulos, I., and Kokotovich, P., *Nonlinear and Adaptive Control Design*, New York: Wiley, 1995.

173. Klotz, A. and Brauer, K., A Small-Size Neural Network for Computing with Strange Attractors, *Neural Netw.*, 1999, vol. 12. pp. 601–607.

174. Konishi, K., Kokame, H., and Hirata, K., Decentralized Delayed-Feedback Control of a Coupled Ring Map Lattice, *IEEE Trans. Circ. Syst. I*, 2000, vol. 47, pp. 1100–1102.

175. Konishi, K., Kokame, H., and Hirata, K., Delayed-Feedback Control of Spatial Bifurcations and Chaos in Open-Flow Models, *Phys. Rev. E*, 2000, vol. 62, pp. 384–388.

176. Konishi, K., Kokame, H., and Hirata, K., Decentralized Delayed-Feedback Control of an Optimal Velocity Traffic Model, *Europ. Phys. J. B*, 2000, vol. 15, pp. 715–722.

177. Konishi, K., Hirai, M., and Kokame, H., Decentralized Delayed-Feedback Control of a Coupled Map Model for Open Flow, *Phys. Rev. E*, 1998, vol. 58, pp. 3055–3059.

178. Konishi, K., Hirai, M., and Kokame, H., Sliding Mode Control for a Class of Chaotic Systems, *Phys. Lett. A*, 1998, vol. 245, pp. 511–517.

179. Konishi, K., Kokame, H., and Hirata, K., Coupled Map Car-Following Model and Its Delayed-Feedback Control, *Phys. Rev. E*, 1999, vol. 60, pp. 4000–4007.

180. Konishi, K. and Kokame, H., Observer-based Delayed-Feedback Control for Discrete-Time Chaotic Systems, *Phys. Lett. A*, 1998, vol. 248, pp. 359–368.

181. Kousaka, T., Ueta, T., and Kawakami, H., A Method for Generating a Chaotic Attractor by Destabilization, *Electron. Commun. Jpn.*, 1997, vol. 80, pp. 73–81.

182. Kwon, O.J., Targeting and Stabilizing Chaotic Trajectories in the Standard Map, *Phys. Lett. A*, 1999, vol. 258, pp. 229–236.

183. Leonov, G.A., Ponomarenko, D.V., and Smirnova, V.B., *Frequency Methods for Nonlinear Analysis. Theory and Applications*, Singapore: World Scientific, 1996.

184. Lian, K.-Y., Chiu, Ch.-S., Chiang, T.-Sh., *et al.*, LMI-Based Fuzzy Chaotic Synchronization and Communications, *IEEE Trans. Fuzzy Syst.*, 2001, vol. 9, no. 4, pp. 539–553.

185. Lian, K.-Y., Chiu, Ch.-S., Chiang, T.-Sh., *et al.*, Secure Communications of Chaotic Systems with Robust Performance via Fuzzy Observer-based Design, *IEEE Trans. Fuzzy Syst.*, 2001, vol. 9, no. 1, pp. 212–220.

186. Liao, T.L., Observer-based Approach for Controlling Chaotic Systems, *Phys. Rev. E*, 1998, vol. 57, pp. 1604–1610.

187. Liao, T.L. and Huang, N.S., Control and Synchronization of Discrete-Time Chaotic Systems via Variable Structure Control Technique, *Phys. Lett. A*, 1997, vol. 234, pp. 262–268.

188. Liao, T.L. and Lin, S.H., Adaptive Control and Synchronization of Lorenz Systems, *J. Franklin Inst. Eng. Appl. Math.*, 1999, vol. 336, pp. 925–937.

189. Lima, R. and Pettini, M., Suppression of Chaos by Resonant Parametric Perturbations, *Phys. Rev. A*, 1990, vol. 41, pp. 726–733.

190. Lima, R. and Pettini, M., Parametric Resonant Control of Chaos, *Int. J. Bifurcat. Chaos*, 1998, vol. 8, pp. 1675–1684.

191. Lim, E. and Mareels, I.M.Y., Identification of a 1-dimensional Chaotic System: Expectations and Limitations for Control, *Proc. 39Th IEEE Conf. Dec. Control.*, 2000, pp. 4823–4824.

192. Lin, C.T. and Jou, C.P., Controlling Chaos by GA-Based Reinforcement Learning Neural Network, *IEEE Trans. Neural Netw.*, 1999, vol. 10, pp. 846–859.

193. Lions, J.L., On the Controllability of Distributed Systems, *Proc. Nat. Acad. Sci. USA*, 1997, vol. 94, no. 10, pp. 4828–4835.

194. Li, T. and Yorke, J.A., Period Three Implies Chaos, *Am. Math. Monthly.*, 1975, vol. 82, pp. 985–992.

195. Lorenz, E.N., Deterministic Nonperiodic Flow, *J. Atmosferic Sci.*, 1963, vol. 20, no. 2, pp. 130–141.

196. Loria, A., Panteley, E., and Nijmeijer, H., Control of the Chaotic Duffing Equation with Uncertainty in All Parameters, *IEEE Trans. Circ. Syst. I*, 1998, vol. 45, pp. 1252–1255.

197. Mackey, M.C. and Glass, L., Oscillation and Chaos in Physiological Control Systems, *Science*, 1977, vol. 197, pp. 287–280.

198. Maguire, L.P., Roche, B., Mcginnity, T.M., *et al.*, Predicting a Chaotic Time Series Using a Fuzzy Neural Network, *Inf. Sci.*, 1998, vol. 112, pp. 125–136.

199. Mareels, I.M.Y. and Bitmead, R.R., Non-linear Dynamics in Adaptive Control: Chaotic and Periodic Stabilization, *Autom.*, I: 1986, vol. 22, pp. 641–655; II: 1988, vol. 24, pp. 485–497.

200. Markov, A.Yu. and Fradkov, A.L., Adaptive Synchronization of Chaotic Systems Based on Speed Gradient Method and Passification, *IEEE Trans. Circ. Syst.*, 1997, no. 11, pp. 905–912.

201. Marotto, F.R., Snap-back Repellers Imply Chaos in $\mathbf{R}^N$, *J. Math. Anal. Appl.*, 1978, vol. 63, pp. 199–223.

202. Mascolo, S. and Grassi, G., Controlling Chaos via Backstepping Design, *Phys. Rev. E*, 1997, vol. 56, pp. 6166–6169.

203. Mascolo, S. and Grassi, G., Controlling Chaotic Dynamics Using Backstepping Design with Application to the Lorenz System and Chua's Circuit, *Int. J. Bifurcat. Chaos*, 1999, vol. 9, pp. 1425–1434.

204. Matsumoto, K. and Tsyda, I., Noise-induced Order, *J. Stat. Phys.*, 1983, vol. 31, pp. 87–106.

205. Maybhate, A. and Amritkar, R.E., Dynamic Algorithm for Parameter Estimation and its Applications, *Phys. Rev. E*, 2000, vol. 61, pp. 6461–6470.

206. Maybhate, A. and Amritkar, R.E., Use of Synchronization and Adaptive Control in Parameter Estimation From a Time Series, *Phys. Rev. E*, 1999, vol. 59, pp. 284–293.

207. Mcguire, J., Batchelor, M.T., and Davies, B., Linear and Optimal Non-linear Control of One-dimensional Maps, *Phys. Lett. A*, 1997, vol. 233, pp. 361–364.

208. Meerkov, S.M., Principle of Vibrational Control: Theory and Applications, *IEEE Trans. Automat. Control*, 1980, vol. AC-25, pp. 755–762.

209. Melby, P., Kaidel, J., Weber, N., *et al.*, Adaptation to the Edge of Chaos in the Self-adjusting Logistic Map, *Phys. Rev. Lett.*, 2000, vol. 84, pp. 5991–5993.

210. Mettin, R., Control of Chaotic Maps by Optimized Periodic Inputs, *Int. J. Bifurcat. Chaos*, 1998, vol. 8, pp. 1707–1711.

211. Meucci, R., Labate, A., and Ciofini, M., Controlling Chaos by Negative Feedback of Subharmonic Components, *Phys. Rev. E*, 1997, vol. 56, pp. 2829–2834.

212. Mirus, K.A. and Sprott, J.C., Controlling Chaos in a High Dimensional System with Periodic Parametric Perturbations, *Phys. Lett. A*, 1999, vol. 254, pp. 275–278.

213. Mizutani, S., Sano, T., Uchiyama, T., *et al.*, Controlling Chaos in Chaotic Neural Networks, *Electron. Commun. Jpn.*, 1998, vol. 81, pp. 73–82.

214. Moon, F.C., *Chaotic Vibrations: An Introduction for Applied Scientists and Engineers*, New York: Wiley, 1987.

215. Morgul, O., On the Control of Chaotic Systems in Lur'e Form by Using Dither, *IEEE Trans. Circ. Syst. I*, 1999, vol. 46, pp. 1301–1305.

216. Morgul, O., On the Control of Some Chaotic Systems by Using Dither, *Phys. Lett. A*, 1999, vol. 262, pp. 144–151.

217. Morgul, O. and Solak, E., On the Synchronization of Chaotic Systems by Using State Observers, *Int. J. Bifurcat. Chaos*, 1997, vol. 7, pp. 1307–1322.

218. Nakagawa, M., Chaos Associative Memory with a Periodic Activation Function, *J. Phys. Soc. Jpn.*, 1998, vol. 67, pp. 2281–2293.

219. Nakagawa, M., A Chaos Associative Model with a Sinusoidal Activation Function, *Chaos, Solutions, Fractals*, 1999, vol. 10, pp. 1437–1452.

220. Nakajima, H., On Analytical Properties of Delayed Feedback Control of Chaos, *Phys. Lett. A*, 1997, vol. 232, pp. 207–210.

221. Nakajima, H. and Ueda, Y., Half-period Delayed Feedback Control for Dynamical Systems with Symmetries, *Phys. Rev. E*, 1998, vol. 58, pp. 1757–1763.

222. Naumenko, A.V., Loiko, N.A., Turovets, S.I., *et al.*, Bias Current Impulsive Feedback Control of Nonlinear Dynamics in External Cavity Laser Diodes, *Electron. Lett.*, 1998, vol. 34, pp. 181–182.

223. Naumenko, A.V., Loiko, N.A., Turovets, S.I., *et al.*, Chaos Control in External Cavity Laser Diodes Using Electronic Impulsive Delayed Feedback, *Int. J. Bifurcat. Chaos*, 1998, vol. 8, pp. 1791–1799.

224. Nijmeijer, H. and Mareels, I.M.Y., An Observer Looks at Synchronization, *IEEE Trans. Circ. Syst. I*, 1997, vol. 44, pp. 882–890.

225. Obradovic, D. and Lenz, H., When Is OGY Control More Than Just Pole Placement, *Int. J. Bifurcat. Chaos*, 1997, vol. 7, pp. 691–699.

226. Okada, T. and Nakagawa, M., A Study of Back Propagation Learning with Periodic Chaos Neurons, *Chaos, Solutions, Fractals*, 1999, vol. 10, pp. 77–97.

227. Okuno, H., Sakata, T., and Takeda, H., Controlling Chaos of Nonlinear Domain-Wall Motion, *J. Appl. Phys.*, 1999, vol. 85. pp. 5083–5085.

228. Ott, E., Grebogi, C., and Yorke, J., Controlling Chaos, *Phys. Rev. Lett.*, 1990, vol. 64 (11), pp. 1196–1199.

229. Park, K.S., Park, J.B., Choi, Y.H., *et al.*, Generalized Predictive Control of Discrete-Time Chaotic Systems, *Int. J. Bifurcat. Chaos*, 1998, vol. 8, pp. 1591–1597.

230. Park, J., Kim, J., Cho, S.H., Han, K.H., *et al.*, Development of Sorbent Manufacturing Technology by Agitation Fluidized Bed Granulator, *Korean J. Chem. Eng.*, 1999, vol. 16, no. 5, pp. 659–663.

231. Pecora, L. and Caroll, T., Synchronization in Chaotic Systems, *Phys. Rev. Lett.*, 1990, vol. 64, pp. 821–824.

232. Pecora, L.M., Carroll, T.L., Johnson, G.A., *et al.*, Fundamentals of Synchronization in Chaotic Systems, Concepts and Applications, *Chaos*, 1997, vol. 7, no. 4, pp. 520–543.

233. Petrick, M.H. and Wigdorowitz, B., A Priori Nonlinear Model Structure Selection for System Identification, *Control Eng. Practice*, 1997, vol. 5, pp. 1053–1062.

234. Pettini, M., Controlling Chaos Through Parametric Excitations, in *Dynamics and Stochastic Processes*, Lima, R., Streit, L., and Vilela–Mendes, R.V., Eds., New York: Springer, 1988, pp. 242–250.

235. Pisarchik, A.N. and Corbalan, R., Parametric Nonfeedback Resonance in Period Doubling Systems, *Phys. Rev. E*, 1999, vol. 59, pp. 1669–1674.

236. Piskun, N.Y. and Wigen, P.E., Bifurcation to Chaos in Auto-oscillations in Circular Yttrium-Iongarnet Films, *J. Appl. Phys.*, 1999, vol. 85, pp. 4521–4523.

237. Place, C.M. and Arrowsmith, D.K., Control of Transient Chaos in Tent Maps Near Crisis, *Phys. Rev. E*, 2000, vol. 61, I: pp. 1357–1368; II: pp. 1369–1381.

238. Pogromsky, A.Yu., Passivity Based Design of Synchronizing Systems, *Int. J. Bifurcat. Chaos*, 1998, vol. 8, no. 2, pp. 295–320.

239. Poznyak, A.S., Yu, W., and Sanchez, E.N., Identification and Control of Unknown Chaotic Systems via Dynamic Neural Networks, *IEEE Trans. Circ. Syst. I*, 1999, vol. 46, pp. 1491–1495.

240. Pyragas, K., Control of Chaos via an Unstable Delayed Feedback Controler, *Phys. Rev. Lett.*, 2001, vol. 86, pp. 2265–2268.

241. Pyragas, K., Continuous Control of Chaos by Self-controlling Feedback, *Phys. Lett. A*, 1992, vol. 170. pp. 421–428.

242. Rabinovich, M.I., Varona, P., and Abarbanel, H.D.I., Nonlinear Cooperative Dynamics of Living Neurons, *Int. J. Bifurcat. Chaos*, 2000, vol. 10, pp. 913–933.

243. Rabinovich, M.I., Abarbanel, H.D.I., *et al.*, Self-regularization of Chaos in Neural Systems: Experimental and theoretical Results, *IEEE Trans. Circ. Syst. I*, 1997, vol. 44, pp. 997–1005.

244. Rabinovich, M.I., Varona, P., Torres, J.J., *et al.*, Slow Dynamics and Regularization Phenomena in Ensembles of Chaotic Neurons, *Physica. A*, 1999, vol. 263, pp. 405–414.

245. Rajasekar, S., Characterization and Control of Chaotic Dynamics in a Nerve Conduction Model Equation, *Pramana J. Phys.*, 1997, vol. 48, pp. 249–258.

246. Ramaswamy, R., Sinha, S., and Gupte, N., Targeting Chaos Through Adaptive Control, *Phys. Rev. E*, 1998, vol. 57, pp. 2507–2510.

247. Ramesh, M. and Narayanan, S., Chaos Control by Nonfeedback Methods in the Presence of Noise, *Chaos, Solutions, Fractals*, 1999, vol. 10, pp. 1473–1489.

248. Ravindra, B. and Hagedorn, P., Invariants of Chaotic Attractor in a Nonlinearly Damped System, *J. Appl. Mechanics—Trans. ASME*, 1998, vol. 65, pp. 875–879.

249. Ritz, T., Schweinsberg, A.S.Z., Dressler, U., *et al.*, Chaos Control with Adjustable Control Times, *Chaos, Solutions, Fractals*, 1997, vol. 8, pp. 1559–1576.

250. Rouabhi, S., Storage of Information in One-dimensional Piecewise-continuous Maps, *Int. J. Bifurcat. Chaos*, 2000, vol. 10, pp. 1127–1137.

251. Roy, A., Govil, S., and Miranda, R., A Neural-Network Learning theory and a Polynomial Time RBF Algorithm, *IEEE Trans. Neural Netw.*, 1997, vol. 8, pp. 1301–1313.

252. Ruelle, D. and Takens, F., On the Nature of Turbulence, *Comm. Math. Phys.*, 1971, vol. 20, no. 2, pp. 167–192.

253. Schuster, H.G. and Stemmler, M.B., Control of Chaos by Oscillating Feedback, *Phys. Rev. E*, 1997, vol. 56, pp. 6410–6417.

254. Schwartz, I.B., Carr, T.W., and Triandaf, I., Tracking Controlled Chaos: Theoretical Foundations and Applications, *Chaos*, 1997, vol. 7, pp. 664–679.

255. Schwartz, I.B. and Triandaf, I., Tracking Sustained Chaos, *Int. J. Bifurcat. Chaos*, 2000, vol. 10, pp. 571–578.

256. Schwartz, I.B. and Triandaf, I., Tracking Unstable Orbits in Experiments: A New Continuation Method, *Phys. Rev. A*, 1992, vol. 46, pp. 7439–7444.

257. Schweinsberg, A.S.Z., Ritz, T., Dressler, U., *et al.*, Quasicontinuous Control of a Bronze Ribbon Experiment Using Time-delay Coordinates, *Phys. Rev. E*, 1997, vol. 55, pp. 2145–2158.

258. Senesac, L.R., Blass, W.E., Chin, G., *et al.*, Controlling Chaotic Systems with Occasional Proportional Feedback, *Rev. Sci. Instr.*, 1999, vol. 70, pp. 1719–1724.

259. Simmendinger, C., Hess, O., and Wunderlin, A., Analytical Treatment of Delayed Feedback Control, *Phys. Lett. A*, 1998, vol. 245, pp. 253–258.

260. Singer, J., Wang, Y.Z., Bau, H.H., Controlling a Chaotic System, *Phys. Rev. Lett.*, 1991, vol. 66, pp. 1123–1125.

261. Sinha, S., Controlling Chaos in Biology, *Current Sci.*, 1997, vol. 73, pp. 977–983.

262. Socolar, J.E.S., Sukow, D.W., and Gauthier, D.J., Stabilizing Unstable Periodic Orbits in Fast Dynamical Systems, *Phys. Rev. E*, 1994, vol. 50, pp. 3245–3248.

263. Stephenson, A., On a New Type of Dynamical Stability, *Mem. Proc. Manch. Lit. Phil. Soc. 52, 1–10; On Onduced Stability, Phil. Mag.*, 1908, no. 15, pp. 233–236.

264. Suykens, J.A.K., Vandewalle, J., Chua, L.O., Nonlinear H-infinity Synchronization of Chaotic Lur'e Systems, *Int. J. Bifurcat. Chaos*, 1997, vol. 7, pp. 1323–1335.

265. Suykens, J.A.K., Yang, T., and Chua, L.O., Impulsive Synchronization of Chaotic Lur'e Systems by Measurement Feedback, *Int. J. Bifurcat. Chaos*, 1998, vol. 8, pp. 1371–1381.

266. Tanaka, K., Ikeda, T., Wang, H.O., A Unified Approach to Controlling Chaos via Arm LMI-based Fuzzy Control System Design, *IEEE Trans. Circ. Syst. I*, 1998, vol. 45, pp. 1021–1040.

267. Tanaka, T. and Nakagawa, M., A Chaos Association Model with a Time-dependent Periodic Activation Function, *Electron. Commun. Jpn.*, 1998, vol. 81, pp. 52–70.

268. Tang, Y.Z., Zhang, N.Y., and Li, Y.D., Stable Fuzzy Adaptive Control for a Class of Nonlinear Systems, *Fuzzy Sets Syst.*, 1999, vol. 104, pp. 279–288.

269. Tereshko, V. and Shchekinova, E., Resonant Control of the Rössler System, *Phys. Rev. E*, 1998, vol. 58(1), pp. 423–426.

270. Tian, Y.C., Tade, M.O., and Tang, J.Y., Nonlinear Open-Plus-Closed-Loop (NOPCL) Control of Dynamic Systems, *Chaos, Solutions, Fractals*, 2000, vol. 11, pp. 1029–1035.

271. Tian, Y.P., Controlling Chaos Using Invariant Manifolds, *Int. J. Control*, 1999, vol. 72, pp. 258–266.

272. Tian, Y.C. and Gao, F.R., Adaptive Control of Chaotic Continuous-time Systems with Delay, *Physica D*, 1998, vol. 117, pp. 1–12.

273. Townley, S., Ilchmann, A., Weiss, M.G., *et al.*, Existence and Learning of Oscillations in Recurrent Neural Networks, *IEEE Trans. Neural Netw.*, 2000, vol. 11, pp. 205–214.

274. Ushio, T., Limitation of Delayed Feedback Control in Nonlinear Discrete-time Systems, *IEEE Trans. Circ. Syst. I*, 1996, vol. 43, pp. 815–816.

275. Van De Vorst, E.L.B., Kant, A.R., Van De Molengraft, M.J.G., *et al.*, Stabilization of Periodic Solutions of Nonlinear Mechanical Systems: Controllability and Stability, *J. Vibrat. Control*, 1998, vol. 4, pp. 277–296.

276. Vanecek, A and Celikovsky, S., Chaos Synthesis via Root Locus, *IEEE Trans. Circ. Syst. I*, 1994, vol. 41, pp. 59–60.

277. Vincent, T.L., Control Using Chaos, *IEEE Contr. Syst. Mag.*, 1997, vol. 17, pp. 65-76.

278. Wang, X.F. and Chen, G., Chaotifying a Stable LTI System by Tiny Feedback Control, *IEEE Trans. Circ. Syst. I*, 2000, vol. 47, pp. 410–415.

279. Wang, X.F. and Chen, G., Chaotifying a Stable Map via Smooth Small-amplitude High-frequency Feedback Control, *Int. J. Circuit Theory Appl.*, 2000, vol. 28.

280. Wang, X.F. and Chen, G., Chaotification via Arbitrarily Small Feedback Controls: Theory, Methods, and Applications, *Int. J. Bifurcat. Chaos*, 2000, vol. 10, pp. 549–570.

281. Wang, X.F. and Chen, G., On Feedback Anticontrol of Discrete Chaos, *Int. J. Bifurcat. Chaos*, 1999, vol. 9, pp. 1435–1441.

282. Wang, J. and Wang, X., Parametric Adaptive Control in Nonlinear Dynamical Systems, *Int. J. Bifurcat. Chaos*, 1998, vol. 8, pp. 2215–2223.

283. Wang, J. and Wang, X.H., A Global Control of Polynomial Chaotic Systems, *Int. J. Control*, 1999, vol. 72, no. 10, pp. 911–918.

284. Weeks, E.R. and Burgess, J.M., Evolving Artificial Neural Networks to Control Chaotic Systems, *Phys. Rev. E*, 1997, vol. 56, pp. 1531–1540.

285. Yang, T., Yang, C.M., and Yang, L.B., A Detailed Study of Adaptive Control of Chaotic Systems with Unknown Parameters, *Dynam. Control*, 1998, vol. 8, pp. 255–267.

286. Yang, T. and Chua, L.O., Control of Chaos Using Sampled-Data Feedback Control, *Int. J. Bifurcat. Chaos*, 1998, vol. 8, pp. 2433–2438.

287. Yau, H.T., Chen, C.K., and Chen, C.L., Sliding Mode Control of Chaotic Systems with Uncertainties, *Int. J. Bifurcat. Chaos*, 2000, vol. 10, pp. 1139–1147.

288. Yu, X.H., Controlling Chaos Using Input-Output Linearization Approach, *Int. J. Bifurcat. Chaos*, 1997, vol. 7, pp. 1659–1664.

289. Yu, X.H., Variable Structure Control Approach for Controlling Chaos, *Chaos, Solutions, Fractals*, 1997, vol. 8, pp. 1577–1586.

290. Zames, G. and Shneydor, N.A., Dither in Nonlinear Systems, *IEEE Trans. Automat. Control*, 1976, vol. 21, pp. 660–667.

291. Zeng, Y.M. and Singh, S.N., Adaptive Control of Chaos in Lorenz System, *Dynam. Control*, 1997, vol. 7, pp. 143–154.

292. Zhao, H., Wang, Y.H., and Zhang, Z.B., Extended Pole Placement Technique and its Applications for Targeting Unstable Periodic Orbit, *Phys. Rev. E*, 1998, vol. 57, pp. 5358–5365.

This paper was recommended for publication by B.T. Polyak, a member of the Editorial Board