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IMPLICIT MODEL REFERENCE ADAPTIVE CONTROLLER BASED ON FEEDBACK KALMAN-YAKUBOVICH LEMMA

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Abstract

A class of implicit model reference adaptive controllers is described designed via feedback Kalman-Yakubovich lemma. Stability conditions of adaptive system are formulated. An example of application to the adaptive stabilization of distillation column is given. A new simple solution of adaptive stabilization problem for minimum phase plants with arbitrary relative degree is also suggested based on implicit reference model and a parallel feedforward compensator (filter) of order k-1 making plant hyper minimum phase.

1. Introduction

As it is well known, model reference adaptive control (MRAC) offers the potential for high performance control in the presence of uncertain and time-varying parameters. However the most of existing schemes [1-4] require so called matching (adaptability) condition: the main loop controller should be able to provide the equivalence of closed-loop system equation to that of the reference model. This condition imposes a strong restriction on the control problem, especially for multivariable and large-scale systems. Matching condition leads to complicated controller structure and increased amount of tunable parameters. As a consequence the performance of the closed-loop system decreases, especially in presence of disturbances.

The papers proposing alternative, so called simple adaptive controllers can be partitioned into two large groups. The first group of papers study simple adaptive stabilizers using as small a priori information as possible [5-8]. A lot of efforts was put to deal with the case when the sign of high frequency gain of the plant is unknown, in this case the adaptive stabilizers with "switching functions" were proposed and analyzed. Another group treats problems with command signals and reference models [9-13]. Different simple adaptive control laws were proposed. Stability of the basic scheme [9] was justified under so called ASPR (almost strict positive realness) condition which is not both easy to check and applicable to many plants in practice. Some attempts were made to overcome these disabilities using parallel feedforward compensator [10-12]. However the performance of simple adaptive controllers is not well investigated yet.

Meanwhile some related adaptive control schemes were studied in Russian literature since 70s [13-18] based on so called "feedback Kalman-Yakubovich lemma" [13-14]. These schemes form a basis for adaptive controllers of rather simple structure providing given performance of the system. The controller of such kind, so called implicit model reference adaptive controller (IMRAC), will be described below.

2. Adaptive controller with implicit reference model for plants with relative degree one

Consider the linear time invariant plant described by equation

$$A(p)y(t)=B(p)u(t)+\varphi(t),\ t>0, \tag{1}$$

where $u(t)$ is scalar control action, $y(t)$ is scalar controlled variable, $\varphi(t)$ is bounded disturbance: $|\varphi(t)|\le\Delta_\varphi$; $A(p)=p^n+a_{n-1}p^{n-1}+\ldots+a_1p+a_0$, $B(p)=b_mp^m+\ldots+b_1p+b_0$ - are polynomials, $p=d/dt$ is time derivative, $n-m=k>0$ is relative degree of the plant. Coefficients a_i, $i=0,\ldots,n-1$, b_j, $j=0,\ldots,m$ are unknown plant parameters. The control goal is to provide output signal $y(t)$ tracking the command signal $r(t)$:

$$|y(t)-r(t)|\le\Delta_y \text{ when } t\ge t_*. \tag{2}$$

For the disturbance free case the control goal can be qualified:

$$\lim_{t\to\infty}[y(t)-r(t)]=0. \tag{2a}$$

To solve the posed problem introduce the secondary goal (adaptation goal), prescribing the desired tracking error behavior:

$$|\delta(t)|\le\Delta \text{ when } t\ge t_*, \tag{3}$$

where $\delta(t)=G(p)y(t)-D(p)r(t)$ is the adaptation error signal; $G(p)=p^l+\ldots+g_1p+g_0$, $D(p)=d_sp^s+\ldots+d_1p+d_0$ are given polynomials specifying the desired properties of the closed-loop system. $G(p)$ is assumed to be stable (Hurwitz) polynomial. Note that the signal $\delta(t)$ may be interpreted as equation error for the equation

$$G(p)y_*(t)=D(p)r(t), \tag{4}$$

because $\delta(t)=G(p)e(t)$, where $e(t)=y(t)-y_*(t)$. Hence the equation (4) may be interpreted as reference equation representing reference model implicitly.
Take the main control law in the form

$$u(t)=K(t)[D(p)r(t)]+\sum_{i=0}^{l}k_i(t)[p^iy(t)] \tag{5}$$

where $K(t)$, $k_i(t)$, $i=0,\ldots,l$ - are tunable parameters, and the adaptation algorithm take as follows:

$$dk_i/dt=-\gamma\delta(t)p^i y(t)-\alpha k_i(t),\quad i=0,1,\dots,l,$$
$$dK(t)/dt=\gamma\delta(t)D(p)r(t)-\alpha K(t), \tag{6}$$

where $\gamma > 0$ - is the adaptation gain and $\alpha\geq 0$ is the parametric feedback gain. The adaptive controller (5),(6) applicability conditions are as follows:

C1. B(p) is Hurwitz polynomial.

C2. $l=k-1$, where $k=n-m$ is the relative degree of plant equation.

It follows from the results of [13-15, 17], that all the trajectories of the system (1),(5),(6) are bounded and the goals (2) and (3) are achieved under conditions C1, C2, if $\alpha>0$ and the command signal r(t) is bounded together with its (s+1) derivatives. Additionally, for the disturbance free case ($\varphi(t)\equiv 0$) the goal (2a) is achieved, if $\alpha=0$ in (6) and r(t) has vanishing derivatives:

$$\int_0^\infty [r^{(i)}(t)]^2 dt<\infty\ ,\ i=1,\dots,s+1.$$

It can be also shown, that for G(0)=1 the upper bound for Δ_y in (2) has a form $\Delta_y\leq\beta(\Delta_\varphi\gamma)$, where $\beta(.)$ is a smooth function, tending to zero when $\Delta_\varphi\to 0$, $\gamma\to\infty$.

Suppose $\delta(t)=0$; then adaptation process (6) stops, while the controlled variable y(t) satisfies the reference equation (4). So the plant output will have the desired behavior without explicit employing reference model output in the controller. Therefore the reference model may be called implicit.

Note that neither degree s of polynomial D(p) nor its coefficients appear in above conditions. The degree of D(p) is determined by the amount of the available derivatives of r(t).

Note also that matching condition in the form used for the model reference systems is not necessary for proposed implicit model reference systems. The order of reference equation (4) is equal to l and can be significantly less then the plant order n. Moreover, the true plant order need not be known for system design.

This approach can be extended to the case of MIMO plant using the results of [14]. Consider the plant equation of form

$$dx/dt=Ax(t)+Bu(t),\quad y(t)=Cx(t), \tag{7}$$

where x is n-dimensional plant state vector, u(t) is m-dimensional control action, y(t) is l-dimensional plant output vector. In this case we can choose the following adaptive control law

$$u(t)=K(t)y(t),$$
$$dk_j/dt=-[g_j^T y(t)]\ \Gamma_j y(t)-\alpha k_j(t)\ ,\ j=1,\dots,m \tag{8}$$

where $k_j(t)$ are the columns of tunable gain ($m\times n$)-matrix K(t), g_j are l-dimensional vectors, $\Gamma=\Gamma^T>0$ are ($l\times l$) adaptive gain matrices, $\alpha\geq 0$ is parametric feedback gain.

3. Feedback Kalman-Yakubovich lemma

The conditions C1, C2 mean that the transfer function W(p)=B(p)G(p)/A(p) is minimum-phase and has the lowest possible relative degree, equal to one. Transfer functions with these properties were called *hyper-minimum-phase (HMP)* in [13].

It was proved in [13] that the HMP property of transfer function $W(p)=C(pI-A)^{-1}B$, where A,B,C are matrices of size $n\times n$, $n\times l$, $l\times n$, correspondingly, is necessary and sufficient for existence of $n\times n$ symmetric positive-definite matrix P and l-vector θ such that the following relations are valid:

$$PA(\theta)+A(\theta)^T P<0,\ PB=C,\ A(\theta)=A+B\theta C^T. \tag{9}$$

This statement called "feedback Kalman-Yakubovich lemma" was extended to MIMO plants in [14]. It allows to prove that the conditions C1, C2 are necessary and sufficient for existence quadratic Lyapunov function for system of plant (7) and dynamic controller

$$u=\theta^T y,\quad \dot\theta=\theta(y,\theta), \tag{10}$$

having the form:

$$V(x,\theta)=x^T Px+(\theta-\theta)^T P_1(\theta-\theta_*), \tag{11}$$

such, that

$$V(x,\theta)>0 \text{ as } x\neq 0,\ \theta\neq\theta_*,\ \dot V(x,\theta)<0 \text{ as } x\neq 0. \tag{12}$$

Hence, adaptation algorithm (6) encompasses all the algorithms which can be designed using Lyapunov function (11) with property (12).

Another consequence of the feedback Kalman-Yakubovich lemma is that the HMP property is necessary and sufficient for ASPR condition.

4. Adaptive controller with implicit reference model for arbitrary relative degree

The solution for plants with arbitrary relative degree proposed below is based on augmenting the plant (1) by auxiliary parallel filter to make the augmented system HMP. The following result shows the possibility of such augmentation.

Theorem 1. Let the transfer function $g^T W(p)$ be minimum phase with relative degree k>1 and $g^T W(0)>0$. Then there exist $\varkappa_0>0$ and function $\varepsilon_0(\varkappa)>0$ such that function $g^T W(p)+W_{\varkappa\varepsilon}(p)$ is HMP for $\varkappa>\varkappa_0$, $0<\varepsilon<\varepsilon_0(\varkappa)$, where

$$W_{\varkappa\varepsilon}(p)=\varkappa\varepsilon Q(\varepsilon p)/R(p), \tag{13}$$

Q(p), R(p) are Hurwitz polynomials of degrees k-2, k-1, respectively.

Theorem 2. Let the transfer function $g^T W(p)$ be minimum phase with relative degree k.
Then there exist $\varkappa_0>0$ and function $\varepsilon_0(\varkappa)>0$ such that the algorithm

$$u=\theta^T y_a, \tag{14}$$

$$\dot\theta=-\Gamma(g^T y_a)y_a \tag{15}$$

where $y_a=y+y_{\varkappa\varepsilon}$, $y_{\varkappa\varepsilon}$ is the output of the auxiliary system

$$R(p)y_{\varkappa\varepsilon}=\varkappa\varepsilon\ Q(\varepsilon p)u, \tag{16}$$

solves the problem of adaptive stabilization of the plant (1) for all $\varkappa>\varkappa_0$, $0<\varepsilon<\varepsilon_0(\varkappa)$.

The proof of the theorems follows from some more general statement proved in [20]. The Lyapunov function of system (1), (14)-(16) has, in accordance with [13], the form

$$V(x_a,\theta)=x_a^T Px_a+(\theta+\varkappa g)^T\Gamma^{-1}(\theta+\varkappa g) \tag{17}$$

where $P=P^T>0$ is positive definite matrix and $\varkappa>0$.

5. Adaptive PI-controller with implicit reference model

The above approach can be used for tuning of standard controllers. For instance, consider PI-law in the main loop

$$u(t)=k_P(t)e(t)+k_I(t)\int_0^t e(\tau)d\tau, \qquad (18)$$

where $e(t)=r(t)-y(t)$ is tracking error, $k_P(t)$, $k_I(t)$ are tunable parameters. Take implicit reference model (4) in the form of the second order equation

$$T^2p^2y(t)+2\xi Tpy(t)+y(t)=r(t), \qquad (19)$$

where T, ξ are the model parameters, $p=d/dt$. T and ξ describe desirable closed-loop system behavior.After integration and filtration the error signal can be obtained as follows:

$$\delta(t)=T^2y(t)/\tau+(2\xi-T/\tau)Ty_f(t)-\int_0^t e_f(\tau)d\tau, \qquad (20)$$

where $y_f(t)$, $e_f(t)$ are outputs of first order filters with time constant τ and inputs $y(t)$, $e(t)$ correspondingly. Adaptation algorithm (6) in this case takes the form

$$dk_P/dt=\gamma\delta(t)e(t) - \alpha(k_P- k_P^*), \quad k_P(0)=k_P^*,$$
$$dk_I/dt=\gamma\delta(t)\int_0^t e(\tau)d\tau - \alpha(k_I- k_I^*), \quad k_I(0)=k_I^*, \qquad (21)$$

where coefficients $\alpha > 0$, $\gamma > 0$, k_P^*, k_I^* are prior estimates of the desired values of tunable parameters.

6. Example: the distillation column control

Consider the multivariable distillation column control problem [19]. Let $y_i(t)$ be the heavy component concentrations of the output distillate product, $u_i(t)$ be the distillate flow rates, i=1,2,3. Using the input-output form for plant equation we obtain $y(t)=W(p)u(t)$, where (3×3) transfer matrix W(p) has the following nominal value [19]

$$W(p) = \begin{bmatrix} \frac{0.7}{9p+1} & 0 & 0 \\ \frac{2.0}{8p+1} & \frac{0.4}{6p+1} & 0 \\ \frac{2.3}{10p+1} & \frac{2.3}{8p+1} & \frac{3.1}{7p+1} \end{bmatrix}$$

(all the time constants are given in minutes). Note, that the mutual influence of the different control loops is significant for this plant.

The results of numerical simulation for both non-adaptive and proposed adaptive systems are showed below. Fig. 1 shows the transient processes of the outputs $y_1(t)$, $y_2(t)$ when initial conditions are equal to zero, $r_1(t)=1$, $r_2(t)=r_3(t)=0$ and time-invariant PI-controller is used with the parameters $k_P=2.0$, $k_I=1.0$ min^{-1}. These values were found out accordingly with the nominal mode. The true values of the plant model were taken with the coefficient of variation equal to 0.3. The simulation results for the adaptive control algorithm (18),(20),(21) under the similar conditions are shown at the Fig.2. The desired closed-loop time constant T= 5 min, ξ=0.7, $k_P^*=2$, $k_I^*=1.0$ min^{-1}. One can see that the usage of IMRAC for this problem not only ensures the desired closed-loop system behavior, but also reduces the mutual channels influence.

This example demonstrates also the important property of IMRAC algorithm to provide the adaptation time less than the closed-loop system transient time.

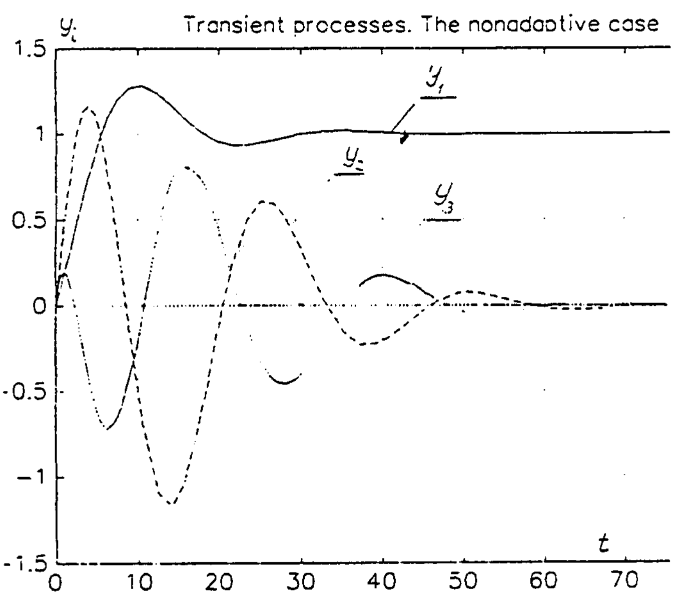

Fig.1

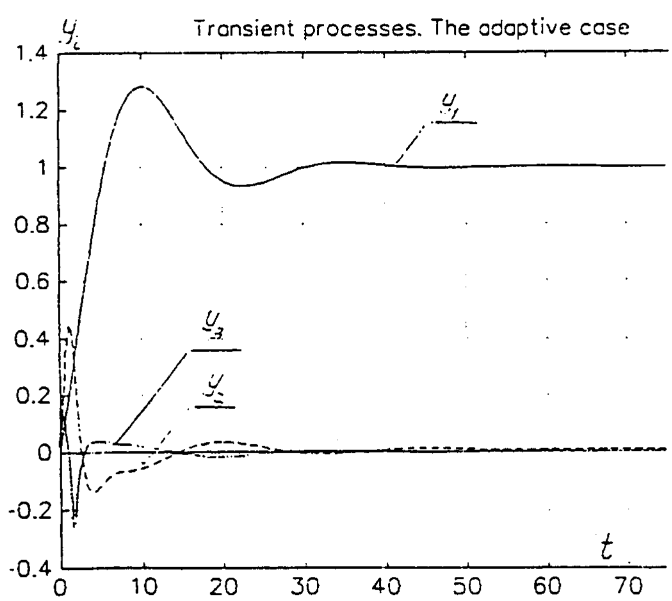

Fig.2

6. Conclusions

The adaptive controller described in this paper incorporates the reference model not as dynamic element of the system, but implicitly, as some "reference equation". It provides the desired performance of adaptive system without complication of its structure. It allows also to weaken the matching conditions.

The idea of relative degree reducing is known in adaptive control [21, 10-12]. The proposed in this paper "small" augmenting transforms any minimum phase system into system with relative degree one. It can be extended onto MIMO plant [20].

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