ADAPTIVE SYNCHRONIZATION IN PRESENCE OF NOISE WITH APPLICATION TO TELECOMMUNICATIONS

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Abstract

The problem of synchronization of chaotic systems in the presence of additive bounded noise is considered. Two kinds of adaptive synchronization algorithms are investigated: a) based on parametric feedback; b) using dead zone. The results are illustrated by numerical examples of synchronization of the Chua systems. Appication of proposed adaptive synchronization algorithms to the problem of signal transmission is discussed.

1 Introduction

During recent decade a growing interest was observed in the problem of synchronizing chaotic systems [2, 3, 16]. It was motivated not only by scientific interest, but also by practical applications in different fields particularly in telecommunications [3, 4, 5, 12]. Of practical interest is the problem of synchronizing two or more systems when the designer of the receiver does not know not only initial state but also some parameters. This more complicated problem is referred to as *adaptive synchronization* [1, 8, 9, 10, 15, 18]. Its solution may be used in communications in the case when parameter modulation is used for message transmission, see, e.g. [10] where the solution based on adaptive observers in an idealized setting with neglected noise was proposed.

In this paper the scheme related to that of [10] and [1] is investigated in the case of noisy channel. The novelty of the paper compared to [1] and other papers (e.g. [11]) is that new ways of robustification are examined for noisy transmission channel. In Section 2 two kinds of adaptive observers (using a parametric feedback and a dead zone) are proposed. In Section 3 an example of signal transmission with carrier signals generated by Chua system is considered. Simulation results are given in Section 4.

2 Adaptive synchronization algorithms

Consider the transmitter described by state space equations in Lur'e form:

$$\dot{x}_d = Ax_d + \varphi_0(y_d) + B\sum_{i=1}^{m}\theta_i\varphi_i(y_d), \quad y_d = Cx_d \tag{1}$$

where $x_d \in \mathbb{R}^n$ is the transmitter state vector, $y_d \in \mathbb{R}^l$ is the vector of outputs (transmitted signals), $\theta = \mathrm{col}\,(\theta_1, \ldots, \theta_m)$ is the vector of transmitter parameters (possibly representing a message). It is assumed that the nonlinearities $\varphi_i(\cdot)$, $i = 0, 1, \ldots, m$, matrices A, C and vector B are known.

The receiver is designed as another dynamical system that provides estimates $\hat{\theta}_i$, $i = 1, \ldots, m$ of the transmitter parameters based on the (noisy) observations of the transmitted signal $y_d(t)$. The problem is to design receiver equations

$$\dot{z} = F(z, y_r), \tag{2}$$

$$\hat{\theta} = h(z, y_r) \tag{3}$$

ensuring convergence

$$\overline{\lim_{t\to\infty}} \left\|\hat{\theta}(t) - \theta\right\| \le \Delta, \tag{4}$$

where $y_r(t) = y_d(t) + \xi(t)$ is the received signal, $\xi(t)$ is channel noise, $\hat{\theta}(t) = \mathrm{col}\left(\hat{\theta}_1(t), \ldots, \hat{\theta}_m(t)\right)$ is the vector of parameter estimates, $\Delta \ge 0$ is given accuracy bound.

One of receivers considered below was proposed in [10]. This receiver is a kind of adaptive observer. Its simplest version for the case when A, B, C are known is as follows:

$$\begin{aligned}\dot{x} &= Ax + \varphi_0(y_r) + B\left(\sum_{i=1}^{m}\hat{\theta}_i\varphi_i(y_r) + \hat{\theta}_0 G(y_r - y)\right), \\ y &= Cx,\end{aligned} \tag{5}$$

$$\dot{\hat{\theta}}_i = \psi_i(y_r, y), \quad i = 0, 1, 2, \ldots, m, \tag{6}$$

where $x \in \mathbb{R}^n, y \in \mathbb{R}^l, \theta_0 \in \mathbb{R}$ and $G \in \mathbb{R}^l$ is the vector of weights. The adaptation algorithm (6) is provided by standard gradient/speed-gradient techniques as follows:

$$\dot{\hat{\theta}}_i = -\gamma_i(y - y_r)\varphi_i(y_r), \quad i = 1, \ldots, m, \tag{7}$$

$$\dot{\hat{\theta}}_0 = -\gamma_0 (y - y_r)^2, \tag{8}$$

where γ_i, $(i = 0, 1, \ldots m)$ are positive adaptation gains. In presence of noise in the channel algorithm (7), (8) has to be regularized (robustified) [6, 7, 17]. One way of regularization is introducing the *parametric feedback*. It provides the following form of adaptive algorithm

$$\dot{\hat{\theta}}_i = -\gamma_i (y - y_r)\varphi_i(y_r) - \alpha_i \hat{\theta}_i, \quad i = 1, \ldots, m, \tag{9}$$

$$\dot{\hat{\theta}}_0 = -\gamma_0 (y - y_r)^2 - \alpha_0 \hat{\theta}_0, \tag{10}$$

where $\alpha_i > 0$ $(i = 0, 1, \ldots m)$ are regularization gains. Another way of regularization lies in the usage of the *dead zone* in the adaptation law [6, 7]. In this case the adaptation algorithm (6) takes the form

$$\dot{\hat{\theta}}_i = \begin{cases} -\gamma_i \delta \varphi_i(y_r), & \text{when } |\delta| \geq \Delta_i \\ & \quad i = 1, \ldots, m, \\ 0 & \text{else,} \end{cases} \tag{11}$$

$$\dot{\hat{\theta}}_0 = \begin{cases} -\gamma_0 \delta^2, & \text{when } |\delta| \geq \Delta_0 \\ 0 & \text{else,} \end{cases} \tag{12}$$

where $\delta \triangleq y - y_r$ is an *adaptation error*, $\Delta_i \geq 0$ $(i = 0, 1, 2, \ldots m)$ are choosen parameters of the algorithm (*the dead zones*).

To formulate properties of the proposed algorithms introduce definition.

Definition 1 ([7]). The system $\dot{x} = \bar{A}x + \bar{B}u$, $y = \bar{C}x$ with transfer matrix $W(\lambda) = \bar{C}(\lambda I - \bar{A})^{-1}\bar{B}$, where $u,\ y \in \mathbb{R}^l$ and $\lambda \in \mathcal{C}$ is called *hyper-minimum-phase* if it is *minimum-phase* (i.e. the polynomial $\varphi(\lambda) = \det(\lambda I - \bar{A}) \det W(\lambda)$ is Hurwitz), and the matrix $\bar{C}\bar{B} = \lim_{\lambda \to \infty} \lambda W(\lambda)$ is symmetric and positive definite.

Remark: For $l = 1$ the system of order n is hyper-minimum-phase if the numerator of its transfer function is a Hurwitz polynomial of degree $n - 1$ with positive coefficients.

Properties of the receiver with regularized adaptation algorithm (9), (10) are described by the following theorem.

Theorem 1. Let the noise function $\xi(t)$ be bounded: $|\xi(t)| \leq \Delta_\xi$; all the trajectories of the transmitter (1) be bounded and linear system with transfer function $W(\lambda) = GC(\lambda I - A)^{-1}B$ be hyper-minimum-phase. If $\Delta_\xi > 0$ is sufficiently small, then all the trajectories of the system (1), (5), (9), (10) are bounded and the goal (4) and the *auxiliary goal*

$$\overline{\lim_{t \to \infty}} \|x(t) - x_d(t)\| \leq \Delta_x \tag{13}$$

holds for some $\Delta > 0$, $\Delta_x > 0$. Moreover, the gains $\alpha_i > 0$, $i = 0, 1, \ldots, m$ can be chosen in such a way (sufficiently small) that the values Δ in (4) and Δ_x in (13) are arbitrarily small.

Proof of the Theorem 1 follows from the results of [10] and [6]. Similar result for the algoritm (11), (12) is also valid.

Another variant of the adaptive observer based on the well known Lion's method [14], consists of augmented signals implementation. These signals are generated by the row of filters, therefore the parameter estimates can be based on output measurements only.

Let us describe the scheme in detail. The transfer function $W(\lambda)$ of the linear part in Lur'e form (1) can be written as

$$W(\lambda) = \frac{b_0\lambda^k + b_1\lambda^{k-1} + \cdots + b_k}{\lambda^n + a_1\lambda^{n-1} + \cdots + a_n},$$

so that for output and input signals of the linear part is valid

$$y_r^{(n)} + a_1 y_r^{(n-1)} + \cdots + a_n y_r = b_0 u^{(k)} + \cdots + b_{k-1} u, \tag{14}$$

where $u(t)$ stands for the output of nonlinear part (1), i.e. $u(t) \triangleq \varphi_0(y_r) + B \sum_{i=1}^{m} \theta_i \varphi(y_r)$. Equation (14) leads to the similar interrelation with respect to outputs $\tilde{y}_r(t)$, $\tilde{u}(t)$ of the identical filters, actuated by the signals $y_r(t), u(t)$:

$$\tilde{y}_r^{(n)} + a_1 \tilde{y}_r^{(n-1)} + \cdots + a_n \tilde{y}_r = b_0 \tilde{u}^{(k)} + \cdots + b_{k-1} \tilde{u}, \tag{15}$$

In contrast to (14), all derivatives in (15) can be measured without differentiation of input/output signals. In the case of unknown parameters a_i, b_j in (14), the following *implicit adjustable model* can be written

$$\begin{aligned} &\tilde{y}_r^{(n)} + \hat{a}_1(t)\tilde{y}_r^{(n-1)} + \cdots \\ &+ \hat{a}_n(t)\tilde{y}_r = \hat{b}_0(t)\tilde{u}^{(k)} + \cdots + \hat{b}_{k-1}(t)\tilde{u}, \end{aligned} \tag{16}$$

where the *adjustable parameters* $\hat{a}_i(t), \hat{b}_j(t)$ stand for the estimates of *a priori* unknown parameters a_i, b_j of (14). Signal

$$\begin{aligned} \delta(t) \triangleq\ &\tilde{y}_r^{(n)} + \hat{a}_1(t)\tilde{y}_r^{(n-1)} + \cdots \\ &+ \hat{a}_n(t)\tilde{y}_r - \hat{b}_0(t)\tilde{u}^{(k)} - \cdots - \hat{b}_{k-1}(t)\tilde{u} \end{aligned} \tag{17}$$

can be referred as an *identification error* and its magnitude has to be minimizied by the identification algorithm. Introducing the regressor ϕ and the vector of adjustable parameters $\hat{\theta}$ as

$$\phi \triangleq [\tilde{y}_r^{(n-1)}, \ldots, \tilde{y}_r, -\tilde{u}^{(k)}, - \ldots, -\tilde{u}]^T,$$

$$\hat{\theta}(t) \triangleq [\hat{a}_1(t), \ldots, \hat{a}_n(t), \hat{b}_0(t), \ldots, \hat{b}_{k-1}(t)]^T,$$

one gets (17) in the form

$$\delta(t) = \tilde{y}_r^{(n)} + \phi(t)^T \hat{\theta}(t). \tag{18}$$

Appliyng the speed-gradient techniques and the *matrix square-root algorithm*, one finally obtains the identification law as follows:

$$\dot{\Gamma} = -\gamma \Gamma \phi \phi^T \Gamma + \alpha \Gamma, \tag{19}$$

$$\dot{\theta} = -\gamma \Gamma \phi \delta, \tag{20}$$

where $\Gamma = \Gamma(t)$ is square *gain matrix*, α, γ are algorithm parameters. The particular form of algorithm (17) – (20) is given in the next section.

As an example we consider the problem of synchronizing pair of Chua systems with unknown parameters and incomplete measurements.

3 Example: communication using Chua systems

Consider the example of information transmission where both transmitter and receiver system are implemented as a Chua's circuit, (see [4, 10]. The transmitter model in dimensionless form is as follows:

$$\begin{array}{rcl} \dot{x}_{d_1} & = & p(x_{d_2} - x_{d_1} + f(x_{d_1}) + sf_1(x_{d_1})) \\ \dot{x}_{d_2} & = & x_{d_1} - x_{d_2} + x_{d3} \\ \dot{x}_{d_3} & = & -qx_{d_2} \end{array} \qquad (21)$$

where $f(z) = M_0 z + 0.5(M_1 - M_0) f_1(z)$, $f_1(z) = |z+1| - |z-1|$, M_0, M_1, p, q are the transmitter parameters, $s = s(t)$ is the signal to be reconstructed in the receiver. Assume that the transmitted signal is $y_r(t) = x_{d_1}(t)$, and the values of the parameters p, q are known.

The parameters M_0, M_1 are assumed to be *a priori* unknown. The receiver designed according to the results of Section 2 is modeled as

$$\begin{array}{rcl} \dot{x}_1 & = & p(x_2 - x_1 + f(y_r) + c_1 f_1(y_r) + c_0(x_1 - y_r)), \\ \dot{x}_2 & = & x_1 - x_2 + x_3, \\ \dot{x}_3 & = & -qx_2, \end{array} \qquad (22)$$

where c_0, c_1 are the adjustable parameters. The adaptation algorithm (9), (10), takes the form

$$\begin{array}{rcl} \dot{c}_0 & = & -\gamma_0(y_r - x_1)^2 - \alpha_0 c_0, \\ \dot{c}_1 & = & -\gamma_1(x_1 - y_r) f_1(y_r) - \alpha_1 c_1, \end{array} \qquad (23)$$

where γ_0, γ_1 are the adaptation gains, α_0, α_1 are the regularization gains.

The adaptation algorithm (11), (12), takes the form

$$\dot{c}_0 = \begin{cases} -\gamma_0 \delta^2, & \text{when } |\delta| \geq \Delta_0, \\ 0, & \text{else}, \end{cases} \qquad \dot{c}_1 = \begin{cases} \gamma_1 \delta f_1(y_r), & \text{when } |\delta| \geq \Delta_0, \\ 0, & \text{else}, \end{cases} \qquad (24)$$

where $\delta = y_r - x_1$, $\Delta_0 \geq 0, \Delta_1 \geq 0$.

In order to examine the ability of the system (22), (23) to receive and to decode messages we verify the conditions of the Theorem 1 assuming that $s(t) = \text{const}$. Clearly, if $s(t)$ is a time-varying (e.g. piecewise constant) signal, the Theorem 1 can be used only if the parameter estimation is fast enough, at least much faster than the actual parameter modulation. Writing the error equations yields

$$\left\{ \begin{array}{rcl} \dot{e}_1 & = & p(e_2 - e_1 + (c_1 - s) f_1(y_r) + c_0 e_1) \\ \dot{e}_2 & = & e_1 - e_2 + e_3 \\ \dot{e}_3 & = & -qe_2, \end{array} \right. \qquad (25)$$

where $e_i = x_i - x_{d_i}$, $i = 1, 2, 3$. The system (25) is, obviously in Lur'e form with

$$A = \begin{bmatrix} -p & p & 0 \\ 1 & -1 & 1 \\ 0 & -q & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad C = [1\ 0\ 0],$$

$\hat{\theta}_1 = c_1, \theta_1 = s, \theta_0 = c_0$.

The transfer function of the linear part is

$$W(\lambda) = \frac{\lambda^2 + \lambda + q}{\lambda^3 + (p+1)\lambda^2 + q\lambda + pq} \qquad (26)$$

We see that the order of the system is $n = 3$, while the numerator polynomial is Hurwitz and has degree 2 for all $q > 0$ and all real p. Therefore the hyper-minimum-phase condition holds for $q > 0$ and any p, M_0, M_1. Thus, Theorem 1 yields the boundedness of all receiver trajectories $x(t)$ and small limit value of the observation error if the intensity of the noise and the value of gains α_i are sufficiently small.

Now let us turn to application to the posed problem the algorithm (18) – (20). In accordance with the transfer function of the linear part (26), equation (17) in the considered example can be written as

$$\delta(t) = \tilde{y}^{(3)}(t) + (1+p)\tilde{y}^{(2)}(t) + q\tilde{y}^{(1)}(t) + pq\tilde{y}(t) - \tilde{u}^{(2)}(t) - \tilde{u}^{(1)}(t) - q\tilde{u}, \qquad (27)$$

where $\tilde{y}^{(i)}, \tilde{u}^{(i)}$ stand for outputs, inputs and their time derivatives of two third order Butterworth low-pass filters with input signals $y_r(t), u(t)$ respectively; $y_r(t)$ is the received signal (in the case of an ideal channel, $y_r(t) \equiv x_{d_1}(t)$ in (21)); the "input of the model linear part" $u(t)$ is determined as $u = p\tilde{f}(y_d) + p\tilde{s}\tilde{f}_1(y_d)$, where $\tilde{s} = \tilde{s}(t)$ stands for the transmitter parameter estimate, the nonlinear part is described by functions $\tilde{f}_1(y_d) = |y_r + 1| - |y_r - 1|$, $\tilde{f}(y_r) = M_0 y_r + 0.5(M_1 - M_0)\tilde{f}_1(y_r)$, p, q, M_0, M_1 are given constants. Signal $\tilde{f}_1(y_r(t))$ excites the next one Butterworth filter to obtain the regressor $\phi(t)$ as $\phi = -p(\tilde{f}_{1_f}^{(2)} + p\tilde{f}_{1_f}^{(1)} + q\tilde{f}_{1_f})$, where $\tilde{f}_{1_f}^{(i)}$ stand for output filter signal and its derivatives. The adaptive algorithm (17), providing estimate $\tilde{s}(t)$ of the received signal $s(t)$ takes the form

$$\begin{array}{l} \dot{\Gamma} = -\gamma\Gamma^2\phi^2 + \alpha\Gamma, \\ \dot{\tilde{s}}(t) = -\gamma\Gamma\phi\delta. \end{array} \qquad (28)$$

In practice the channel is subjected to noise, i.e. the investigation of the signal transmission in the case of noisy channel is very important. In our simulations it was assumed that the white noise $\xi(t)$ is added to the transmitter output, so that the received signal $y_r(t)$ is modeled as

$$y_r(t) = y_d(t) + \xi(t), \qquad (29)$$

where $\xi(t)$ is a Gaussian white noise with zero average value and intensity σ. [1]

Although boundedness of the noise is important for theoretical investigations, it is interesting how the proposed schemes work in case of Gaussian noise which is not bounded. The problem was examined by computer simulations.

[1] More precisely, $\xi(t)$ is modeled as a piecewise constant random process with sample time Δ_t and $\xi(t_k) = \zeta_k\sqrt{\Delta_t}$, $(k = 0, 1, 2, \ldots,\ t_k = k\Delta_t)$, where ζ_k are Gaussian random numbers, having zero mean and the standard deviation σ.

4 Simulation results

We carried out simulations for the above schemes. Parameter values were selected as $p = 9; q = 14.286; M_0 = 5/7; M_1 = -6/7$. For these parameter values the system (21) possesses a chaotic attractor, resembling that of the system used in [4] (after some rescaling of space and time variables).

The initial conditions for the transmitter were taken as $x_d(0) = [0.3\ \ 0.3\ \ 0.3]$. For the receiver zero initial conditions were chosen for the state x_0 as well as for the adjustable parameters $c_0(0),\ c_1(0)$. In order to eliminate the influence of initial conditions no message was transmitted during the first 20 time units ("tuning" or "calibration" of the receiver), i.e. $s(t) \equiv 1$ for $0 \leq t \leq 20$. The time history of observation errors and parameter estimates during tuning show that all observation errors and parameter estimation error $c_1(t) - s$ tend to zero rapidly. The value $c_0(t)$ tends to some constant value.

After the tuning period the square wave message

$$s(t) = s_0 + s_1 \operatorname{sign} \sin\left(\frac{2\pi t}{T_0}\right), \qquad (30)$$

where $s_0 = 1.005$, $s_2 = 0.005$ was sent.

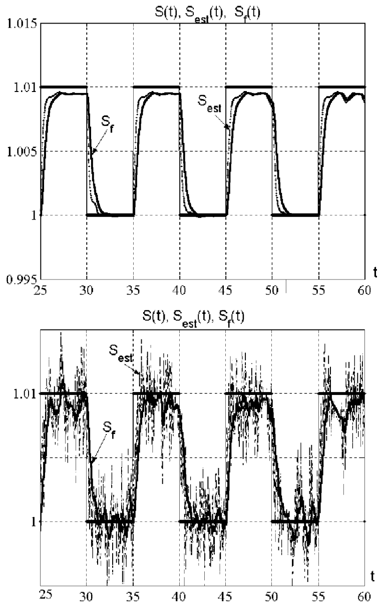

Figure 1: Parameter estimation by means of the adaptive observer (22), (23).

Parameter estimation results by means of the adaptive algorithm with the dead zone (22), (24) under the similar conditions are shown in the Fig. 4, 2. The dead zone $\Delta_0 = \Delta_1 = 2.5 \cdot 10^{-3}$.

The simulation shows that the reconstructed signal $y(t)$ coincides with the transmitted signal $y_r(t)$ with very good accuracy. However both observation errors and estimation errors do not decay completely during the interval when $s(t)$ is constant. Nevertheless, reliable reconstruction of the signal $s(t)$ was demonstrated. The achievable information transmission rate is comparable with the highest frequencies in the carrier

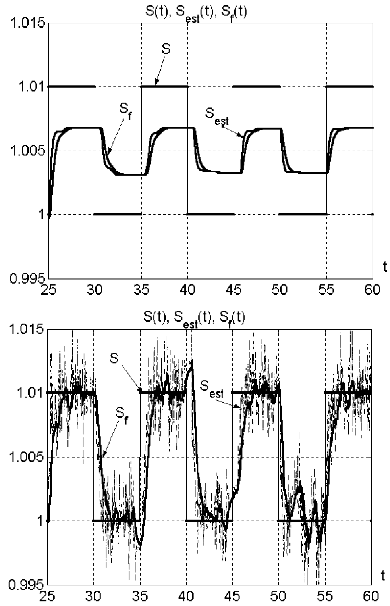

Figure 2: Parameter estimation by means of the adaptive observer (22), (24).

spectrum. Fig. 1 *a* shows the message signal $s(t)$ (with the period $T_0 = 10$), its estimate via adaptive observer algorithm (22), (23) $\hat{s}(t)$ and output of the first-order low-pass filter $s_f(t)$. This filter is used for separation the message in the case of the noisy channel. In the simulation the following parameters of the algorithm were taken: $\gamma_0 = 10$, $\alpha_0 = 1$ $\gamma_1 = 5$, $\alpha_1 = 0.2$, filter pass band is equal to 3.5. Fig. 1 *b* illustrates the influence of the noise with $\sigma = 10^{-3}$ in the transmission channel. One can notice, that the message can be recovered in this case too.

The more complicated algorithm (27), (28) is more noise-immune, as it can be seen from the Fig. 3 *b*, where is taken $\sigma = 0.1$ and no one post-filter is used. Algorithm (28) parameters are taken as: $\gamma = \operatorname{sgn}(t - t_0)$, (where $\operatorname{sgn}(\cdot)$ denotes the unit step function, $t_0 = 5$) $\alpha = 5$, $\Gamma(t_0) = 10^{-5}$, band pass of Butterworth filters is equal to 5.

5 Conclusions

The proposed adaptive observer-based synchronization scheme demonstrates good signal and parameter reconstruction abilities in presence of noise. It allows to achieve sufficiently high information transmission rate. Both methods of regularization of adaptation algorithms – the parametric feedback and the dead zone – provide reasonable values of synchronization error in noisy case.

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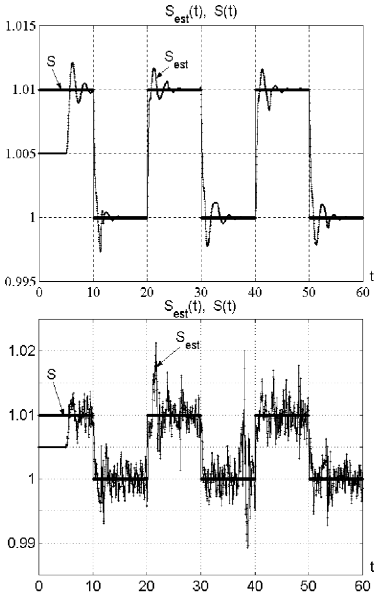

Figure 3: Parameter estimation by means of the identification algorithm (27), (28).

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