Control of a Noise-Induced Transition in a Nonlinear Dynamical System

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(Dated: December 1, 2007)

Abstract

Feedback control is applied to change conditions of a noise-induced transition in a nonlinear second order dynamic system. The mathematical model used in the analysis is a system of two-component equations describing operation of a semiconductor–gas-discharge image converter. The control algorithm is proposed using the speed-gradient method for a linearized model system. It is found by computer simulations that, under conditions when the noise is effective in determining the destructive dynamics of the system without control, the role of noise can be essentially suppressed by a proper feedback control. The control efficiency depends on the amplitude of control signal in a non-monotonic way, thus demonstrating a resonance-like regularity. Application of the proposed control method can be useful in solving other problems, such as providing survival of endangered species in ecology, improving stability of lasers, etc.

PACS numbers: 05.45.Gg, 07.50.Hp, 42.79.Qx

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I. INTRODUCTION

Behavior of a number of nonlinear dynamical systems is essentially influenced by noise. A characteristic example is the non-linear stochastic resonance [1–4]. It is well known that sensitivity of detection of weak signals by some systems can be notably increased in the presence of noise [4]. In such cases, noise plays the constructive role. In other cases, nonlinear oscillators may undergo a phase transition in the presence of noise, i.e., a qualitative change of the dynamics of a system can be observed when the amplitude of noise reaches some critical value [5]. A well-known example is the emergence of large amplitude oscillations due to noise in a system that is stable at the absence of noise. The oscillations arise as the response of the system to the modulation of a system parameter by noise. In this case, the macroscopic dynamics of the system to a large extent is determined by noise (which in a real system originates from microscopic processes). Such a phenomenon—parametric noise resonance—is, essentially, a variant of nonlinear stochastic resonance.

Appearance of the parametric noise resonance is undesirable if it destroys the normal functioning of a dynamical system. For instance, this can take place in converters of optical images that are used to record high speed processes in the infrared (IR) range of light [6, 7]. The operation of the converters is based on electronic properties of the structure "semiconductor–gas-discharge gap". It has been found that, at a small current density, the intrinsic noise of the device can initiate large amplitude oscillations in current. This can result in spontaneous interruption of the discharge process in the gap [6, 8]. In other words, transition from the conductive to dielectric state of the system can occur. In the dielectric state, the device becomes insensitive to an incoming pulse of IR light and, therefore, is not able to convert an image.

It is of practical importance to investigate whether it is possible to provide a proper control of the considered (and similar) systems by applying a low amplitude action that varies in time. The purpose of the control is to suppress the escape of the dynamic system from the area of the phase space where the main function of the system is ensured. In the present report, we make an attempt to implement such a control while using the non-linear model of the device introduced in [6]. To design a feedback control algorithm, the speed gradient method [9–11] is employed. The control is provided by a proper temporal variation of one of the model parameters. In the real device, it corresponds to feeding voltage. Dynamics of the controlled system is analyzed by solving numerically the corresponding equations.

For the parameters chosen in the present analysis, the positive result of control is obtained at a rather small value of the control amplitude that equals only a small fraction of the main feeding voltage. It is revealed that the efficiency of control depends on the amplitude of the control signal in a non-monotonic way, thus exemplifying a resonance-like regularity. In our opinion, this indicates a relation of the obtained results to the nonlinear stochastic resonance phenomenon, the essence of which is a non-monotonic response of a system to the deterministic signal as dependent on the amplitude of stochastic (noise) input.

II. ANALYZED MODEL

We study the following model that is supposed to describe properly the dynamics of the device [6]:

$$dE/dt = a_1(E_m - E) - b_1 N E \ , \tag{1}$$

$$dN/dt = \frac{N}{\tau}(\frac{E}{E_c} - 1) \ , \tag{2}$$

where E is the electric field strength in the discharge gap of the device and N is the density of free charge carriers in the gap. The first equation describes the charging of capacity of the discharge gap from a source of feeding voltage and its discharging due to the presence of free carriers in the gap. The characteristic time of the charging process is τ_E=$1/a_1$, and b_1 is a coefficient. The maximal value of E in the gap, that can be provided by a source of constant voltage, is E_m. For its value, we have the obvious relationship E_m=U_m/d, where U_m is the voltage of the feeding source and d is the length of the gap in the direction of the electric current.

The second equation describes dynamics of density of free carriers in the gap, which is governed by processes of their generation and decay. It is supposed that the process of carriers generation prevails over their recombination when E is larger than some critical electric field strength E_c. The parameter τ defines the rate of a temporal variation of the charge carriers density when the electric field in the gap is not equal to the critical value.

Equations (1,2) have been successfully used for interpretation of main peculiarities in dynamics of the experimental devices, such as the appearance of oscillations at low current density and the spontaneous interruption of the discharge glow [6, 8]. The latter effect can be identified as a noise-induced transition. When such a transition does occur, the converter can not process an incoming image. We point out also equations (1,2) have been used earlier to demonstrate an application of the optimal control method to increase the high-speed performance of the converter, see [12].

The stationary solution (E_0, N_0) of system (1,2) in the absence of noise is

$$E_0 = E_c, \qquad N_0 = \frac{a_1}{b_1}\,(\frac{E_m}{E_c} - 1). \tag{3}$$

The linear analysis of stability of this state reveals that it is a stable focus at values of τ_E large enough. When τ_E increases further, the oscillatory properties of the system become more pronounced—that is, its Q-factor grows. In its turn, the value of N_0 decreases. This corresponds to lowering of electrical current in the device, which properties are simulated by equations (1,2).

It should be stressed that generation of free carriers in the gas-discharge gap is provided by the avalanche ionization of gas atoms and molecules. The efficiency of this process is known to fluctuate in time, which serves as a source of intrinsic noise of the experimental non-equilibrium system. In a simple approach, the influence of the noise on dynamics of the system can be simulated by stochastically changing parameter E_c in time in equations (1,2) [6]. It has been found in the cited work that such an approach can give growth of oscillations in time and cause interruption of electric current in the device.

An example of dynamics of the system under the action of noise in parameter E_c is represented by Fig. 1. The data is obtained at the following set of parameters of equations (1,2): b_1=$5 \cdot 10^{-3}$ cm^3/s, τ=$1.5 \cdot 10^{-9}$ s, E_m=$8 \cdot 10^4$ V/cm, E_c=E_0=$4 \cdot 10^4$ V/cm, N_0=$2 \cdot 10^6$ cm^{-3}. The value of the coefficient a_1 is $2 \cdot 10^4$ s^{-1}. The above values of numerical parameters are in correspondence with the physical parameters of the real device [6–8]. The amplitude of the uniformly distributed stochastic noise used to obtain the curve in Fig. 1 equals 1 % of E_c.

Our next step is to elucidate whether it is possible to suppress the tendency of the system to enter the "dangerous domain" of the phase space, where the discharge decays, by applying a proper control. The following Section is devoted to the elaboration of such a control algorithm.

III. DESIGN OF CONTROL ALGORITHM

Physical principles of the device under consideration suggest that the role of controlling action can be played either by the supply voltage or conductance of the semiconductor component. The latter option can be implemented in an experiment due to the photoelectrical effect in the semiconductor, by applying its optical excitation. Using such a method corresponds to variation of the coefficient a_1 in equation (1).

In this study, we consider the first option where the system is controlled by varying the supply voltage in time. It is also assumed that both state variables $E(t)$, $N(t)$ of the model (1,2) are available for measurement. To construct a control algorithm, we use the speed-gradient method [9, 11] suggesting to change control variables along the gradient of some goal function $Q(x)$ which small values correspond to achievement of the control goal.

The first step of the procedure is to choose the goal function properly. Since the control goal is to maintain the system trajectories near the nominal state (3), the goal function can be taken as a positive definite function of the deviation $X(t) - X_0$, where $X(t) = [E(t), N(t)]^T$ and $X_0 = [E_0, N_0]^T$. We choose the goal function as the quadratic form $Q(X) = (X - X_0)^T P(X - X_0)$, where P is a symmetric positive definite 2×2 matrix to be determined. In order to specify the matrix P, we linearize the model (1,2) near X_0:

$$\dot{X} = A(X - X_0), \tag{4}$$

where

$$A = \begin{bmatrix} -a_1 - b_1 N_0 & -\, b_1 E_0 \\ N_0/(\tau E_c) & 0 \end{bmatrix}.$$

Evaluation of the characteristic polynomial $\delta(\lambda)$ of A yields $\delta(\lambda) = \det(\lambda I - A) = \lambda^2 + (-a_1 - b_1 N_0)\lambda + b_1 E_0 N_0/(\tau E_c)$. Since all its coefficients are positive, the eigenvalues of A have negative real parts and the matrix is stable. Hence, the Lyapunov equation $PA + A^T P = -I$, where I is the identity matrix, has a unique positive definite solution. Let this solution be chosen as P.

Let the control variable be $u = E_m - E_{m\,nom}$. The next step is to evaluate the derivative dQ/dt, which is the speed of varying Q along trajectories of (1,2), and then to find the partial derivative of dQ/dt in control u. This procedure yields

$$dQ/dt = (X - X_0)^T P \begin{bmatrix} a_1(E_m - E) - b_1 N E \\ \frac{N}{\tau}(\frac{E}{E_c} - 1) \end{bmatrix}.$$

Taking into account that X_0 depends on the control $u(t)$, we obtain for the partial derivative of dQ/dt in control the expression

$$\frac{\partial \dot{Q}}{\partial u} = \mu - \eta u,$$

where

$$\mu = a_1 p_{11}(E - E_0) + a_1 p_{21}(N - N_0) -$$
$$\frac{a_1 p_{21}}{b_1 E_0} a_1 (E_m - E) - b_1 N E,$$

$\eta = 2\frac{a_1^2}{b_1 E_0} p_{21}$, while p_{11}, p_{21} are the elements of the first column of the matrix P.

Choosing the speed-gradient algorithm in the finite form [10, 11], we arrive at the following control algorithm:

$$u = -\frac{\gamma \mu}{1 - \gamma \eta}, \tag{5}$$

where $\gamma > 0$ is a design parameter of the algorithm. For the purpose of analysis it is convenient to modify (5) by introducing the saturation of the control intensity and fixing its maximal value:

$$u = -\overline{u}\,\mathrm{sat}\left(\frac{\gamma \mu}{\overline{u}(1 - \gamma \eta)}\right), \tag{6}$$

where $\overline{u} > 0$ is a new design parameter,

$$\mathrm{sat}(z) = \begin{cases} z, & |z| \le 1, \\ \mathrm{sign}\, z, & |z| \ge 1. \end{cases}$$

If the deviation $X - X_0$ is small and the noise is absent, the achievement of the control goal (the convergence $X(t) - X_0(t) \to 0$ takes place) follows immediately from the stability of the linearized system, since Q plays the role of the Lyapunov function of the system. For large deviations of $X - X_0$, the behavior of the closed control loop system (1,2,6) is studied by means of computer simulation.

IV. APPLICATION OF THE CONTROL ALGORITHM

The main goal of the analysis is to look into a possibility of suppressing the noise-induced transition through the control mechanism introduced above. Given the control is successful, our next task is to investigate quantitatively dependence of the efficiency of the control on the amplitude of the control action. The analysis has been performed in a series of simulations in the MATLAB environment.

Our main interest is to apply control to a case where without control the system is rather sensitive to noise, i.e., the noise is able to destroy its normal functioning. This problem is analyzed for the same fixed parameters as those used to get data of Fig. 1. The value of parameter a_1 is varied within the range $1 \cdot 10^3$–$2.5 \cdot 10^4$ s^{-1}.

The first stage of the analysis is to study dynamics of the uncontrolled system at the presence of noise, while varying the parameter a_1. Recall that this parameter can be adjusted in experiments by photoelectrical excitation of the semiconductor detector of the device [7]. Similarly to calculations made to obtain the curve of Fig. 1, the amplitude of the uniformly distributed stochastic noise is taken to be 1% of E_c.

We wish to define how the value of a_1 influences the time t_c of the system transition to the nonconductive (dielectric) state, when it evolves from the initial state (3) under the action of noise. In the calculations, this time is specified as the time of the first crossing of the level $N_* = 10^2$ cm^{-3} by the trajectory $N(t)$. Remark that this density of free carriers is supposed to correspond to the minimal value of the discharge current in the physical device—observing the state with carriers density lower than $N_* = 10^2$ cm^{-3} means finding less than *one* free charge carrier in the gap [6], which actually corresponds to the non-conducting case. Therefore, the discharge is interrupted when the state $N_* = 10^2$ cm^{-3} is reached.

Observed realizations of the events where the "discharge interruption" takes place are shown on Fig. 2. The data are calculated for the range of parameter a_1, where dynamics of the system manifests strong oscillations due to the noise. The set of points indicates times of transition to the non-conducting state. The data refer to three temporal realizations of noise, which gives scattering of points at a given value of a_1. It can be seen that the behavior of the system is weakly statistically reproducible. However, the tendency of t_c to increase can be seen as a_1 grows. Based on the obtained results, the value $a_1 = 10^4$ s^{-1} has been taken in further calculations. In the absence of control, the time of the system stay in the conducting state is rather short for this value of a_1.

At the next step of study, the possibility to maintain the discharge by means of control has been investigated. Since the system trajectories are random functions, the minimal value of the free carriers density $N_* = \min N(t)$ is chosen as a measure of the discharge stability (reliability). The minimum is determined inside the time interval $0 \leq t \leq T$ for T

significantly exceeding the typical time t_c.

Typical time histories of the system with and without control are shown in the left part of Fig. 3. The maximal amplitude of the control signal $\overline{u} = 0.2E_m$ is applied in the calculations. The corresponding trajectories in the space of state variables are represented in the right side of Fig. 3. The data unambiguously illustrates that the implemented control can be quite efficient in improving the stability of the system.

Since the main purpose of control is to maintain a value of N_* large enough, it is of interest to investigate how this value may depend on the amplitude of the control signal. An example of such a calculation is shown on Fig. 4 (a). The data shows that the noise-induced phase transition (the discharge failure in the physical device) does not occur yet for the control amplitude about 5% of E_m. The subsequent increase of the amplitude $\overline{u}$ leads to an increase of the value of N_*. However, when $\overline{u}$ reaches approximately 0.2 of E_m, the growth of N_* is slowing down. The further increase of $\overline{u}$ is accompanied by the decrease of N_*.

The sensitivity of the system behavior to the amplitude of the controlling signal may be expressed in terms of the relative efficiency of the control process $\chi = N_*/\overline{u}$. The dependence of this parameter on the control amplitude is represented on Fig. 4 (b). We point out that the value of χ can be interpreted as a stochastic version of the excitability index. This figure of merit has been introduced in [11, 13] as a measure of ability of a system to absorb the energy from an external control.

It follows from the data of Fig. 3 that the application of control can shrink essentially the area of the phase space occupied by the system trajectories. As a measure of statistical reproducibility of the effect of control, the ratio R=max {Nmin}/min {Nmin} can be adopted. The corresponding characteristic calculated as a function of $\overline{u}$ is shown on Fig. 5, where max {Nmin} and min {Nmin} are determined over 10 realizations of noise for the fixed $\overline{u}$. We see that the effect of control is practically statistically reproducible for $\overline{u} \geq 1 \cdot 10^4$.

It is assumed in the above simulations that the state of the system is available for measurement at each instant of numerical integration of equations (1,2), i.e., the system is considered as being continuous in time. Since the integration step is Δt=$2.5 \cdot 10^{-9}$ s and the duration of one cycle of the phase trajectory is about $2 \cdot 10^{-6}$ s, it means that about 10^3 measurements per a cycle of the orbit should be performed.

Yet, an implementation of so frequent measurements in a real experiment would be a rather difficult task. Therefore, it is interesting to determine whether one can perform a more rare sampling of the state of the system. The results obtained for samplings that are 20, 30, 40, and 50 times more rare as compared to that used in the above simulations are collected in Table I.

As follows from these data, even 50 times less frequent measurements do not influence significantly the efficiency of control for the system under consideration. Namely, for this case the minimal value of $N(t)$ decreases by 16%, while the measure of randomness R=max {Nmin}/min {Nmin} increases only by 15%. We point out also that the graph of the controlling function for rare measurements resembles that observed for frequent measurements, see Fig. 6, where examples of corresponding curves are presented.

V. CONTROL FOR INCOMPLETE MEASUREMENTS

We note that in real experiments it is difficult to measure the two components of the system dynamical state, which are $E(t)$ and $N(t)$. Therefore, it might be of interest to establish whether an efficient control can be implemented if only one component of the state vector of the system is available for measurements. From the experimental point of view, measuring the variable $N(t)$ is preferable, because this quantity is directly related to the electric current in the system.

We study the possibility to replace the other variable $E(t)$ by its estimate $\hat{E}(t)$ obtained from available measurements. Such estimation algorithms for nonlinear systems (so-called *partial nonlinear observers*) have been actively studied recently [14, 15]. In this paper, we propose to evaluate the estimate $\hat{E}(t)$ according to the following equation:

$$d\hat{E}/dt = a_1(E_m - \hat{E}) - b_1 N\hat{E}. \tag{7}$$

The equation for the estimation error $e(t)$=$E(t)$–$\hat{E}(t)$ is obtained by subtracting (7) from (1):

$$de/dt = -a_1 e(t) - b_1 N(t)e(t) = -(a_1 + b_1 N(t))e(t). \tag{8}$$

Though the coefficient in linear equation (8) varies in time, the value of the error $e(t)$ converges to zero. Indeed, evaluating the rate of change of the squared error $e(t)^2$ with respect to (8), we obtain

$$de(t)^2/dt = 2e(t)de(t)/dt = -2(a_1 + b_1 N(t))e(t)^2.$$

Since $N(t) \geq 0$, the inequality

$$de(t)^2/dt \leq -2a_1 e(t)^2$$

holds. Hence, the exponential decay rate of $e(t)$ for a_1=10^4 s^{-1} may be estimated as 10^4 s^{-1}.

Numerical study has been done for the same parameters values as in the previous Section. It is found that the proposed algorithm provides good control efficiency. In general, the result of control depends on the initial value of the electric field strength $E(0)$—an increase in deviation of $E(0)$ leads to an increase of time until the control becomes efficient, see Fig. 7. The following values of $E(0)$ were used in calculations: 0.9 E_0, 0.5 E_0, and zero. Data at a given value of $\hat{E}(0)$ were calculated for three different realizations of noise, which practically did not influence the results.

The corresponding convergence time of the method, where the estimation algorithm is applied, is found to be $(2-3) \cdot 10^{-4}$ s for $\hat{E}(0)$=0 and $(0.5-1.5) \cdot 10^{-4}$ s for $\hat{E}(0)$=0.5 E_0. After this initial stage, the realizations of $N(t)$ practically coincide with those evaluated for the case where both state variables are measured. It is also found that for the range of the initial electric field from 0.9 E_0 to 0.5 E_0 the minimal value of the variable N during the transient time is not less than 10^3 cm^{-3}. That is, it is well above the critical carriers density for the discharge interruption (10^2 cm^{-3}) discussed in the previous Section.

In the above examples, the set of values of initial electric field is chosen in a rather broad range somewhat arbitrarily. However, the obtained data unambiguously demonstrate the capability of the method to provide the robust control even in cases, where the initial value of electric field strongly deviates from the fixed point of the system. We suppose that estimation-based algorithms can be applied to other systems which models are close to (1,2).

VI. CONCLUSION

In summary, in the present work a new feedback algorithm is suggested to control a nonlinear two-component dynamical system. The physical realization of the mathematical model is the semiconductor–gas-discharge gap image converter. The control variable is added to the voltage feeding the system. The designed control algorithm is based on the speed-gradient method. According to the obtained results, the control keeps the studied system in the domain of the phase space where its tendency to make a noise-induced transition to the non-conducting state is suppressed. It has been found that the positive result of control can be observed at a rather low amplitude of the control variable that can be less than 5 % of the nominal voltage feeding the device.

It seems plausible that speed-gradient-based control algorithms can be employed to "correct" dynamics of other systems which behavior becomes undesirable due to the influence of stochastic forces. We suppose that the algorithms can be applied, e.g., to control ecological systems with an intention to develop a technology the providing the survival of threatened species. Among other applications, improving of the operation stability of lasers can be mentioned. In this relation, it is worth noting that some models that are close to equations (1,2) were discussed in literature, e.g., models of species interaction [16] or models of laser dynamics [17].

It might be also of interest to extend the proposed approach to the problems of controlling spatially extended systems which dynamics can produce spatiotemporal structures due to the noise-induced resonance phenomena. As an experimental system which dynamics might be simulated, the "semiconductor–gas-discharge gap" device could be employed again. Being spatially extended, this device exemplifies a number of scenarios of self-organized behavior at some sets of experimental parameters [18]. Some patterns that arise in the device can be interpreted with a relatively simple theoretical model [19, 20] which, in essence, is an extension of equations (1,2). This model seems to be promising for examination as an object of spatially extended control.

Finally, at present the growing tendency to diminish dimensions of electronic devices is observed, and the so-called "one-electron" devices are the natural physical limitation of this development. In general, this is accompanied by the increasing role of intrinsic noise in their operation. On the other hand, active electronic devices function in far from equilibrium nonlinear modes, where noise may critically determine macroscopic behavior of a device. Therefore, suppressing the role of noise in a device operation may become crucial. In a more general setting, the control of noisy behavior of small electronic systems is related to an interesting physical problem to control microscopic systems while varying in time some macroscopic parameters.

Acknowledgments

This work is supported by the President Grant of Russia for Scientific Schools (Grant 5920.2006.2), Research Program No. 22 of Presidium of Russian Academy of Sciences (project 1.8) and Russian Foundation for Basic Research (Grants 07-2-00168-a and 05-01-00869-a).

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[1] V. S. Anishchenko, A. B. Neiman, F. Moss, and L. Schimansky-Geier, Physics-Uspekhi **169**, 7 (1999).

[2] Yu. L. Klimontovich, Physics-Uspekhi **169**, 37 (1999).

[3] P. S. Landa, J. Commun. Technol. Electron. **46**, 1069 (2001).

[4] W. Horsthemke and R. Lefever, *Noise-Induced Transitions. Theory and Applications in Physics, Chemistry, and Biology*, (Springer, Berlin 1984).

[5] V. A. Vasiliev, Y. M. Romanovskii, D. S. Chernavskii, and V. G. Yakhno, *Autowave Processes in Kinetic Systems*, (VEB Deutscher Verlag der Wissenschaften, Berlin, 1987).

[6] Yu. A. Astrov, *Dynamic Properties of Discharge Glow in a Device with Resistive Electrode*, Ioffe Physico-Technical Institute Report No. 1255, 1988 (in Russian).

[7] Yu. A. Astrov, L. M. Portsel, S. P. Teperick, H. Willebrand, and H.-G. Purwins, J. Appl. Phys. **74**, 2159 (1993).

[8] Yu. A. Astrov and S. P. Teperick, Sov. Phys. Tech. Phys. **34**, 550 (1989).

[9] A. L. Fradkov and A. Yu. Pogromsky, *Introduction to control of oscillations and chaos*, (World Scientific, Singapore, 1998).

[10] A. L. Fradkov, Physics-Uspekhi **48**, 103 (2005).

[11] A. L. Fradkov, *Cybernetical physics: From control of chaos to quantum control*, (Springer, 2007).

[12] J.-H R. Kim, H. Maurer, Yu. A. Astrov, M. Bode, and H.-G. Purwins, J. Comput. Phys. **170**, 395 (2001).

[13] A. L. Fradkov, Physica D **128**, 159 (1999).

[14] H. Nijmeijer, Physica D **154**, 219 (2001).

[15] G. Santoboni, A. Yu. Pogromsky, and H. Nijmeijer, Intern. J. Bifurc. Chaos **13**, 453 (2003).

[16] S. Nee, Nature **367**, 123 (1994).

[17] G. L. Lippi, S. Barland, and F. Monsieur, Phys. Rev. Lett. **85**, 62 (2000).

[18] Yu. A. Astrov, E. Ammelt, S. P. Teperick, and H.-G. Purwins, Phys. Lett. A **211**, 184 (1996).

[19] Yu. A. Astrov and Yu. A. Logvin, Phys. Rev. Lett. **79**, 2983 (1997).

[20] Yu. A. Astrov, Phys. Rev. E **67**, 035203(R) (2003).

TABLE I: Minimal density of carriers and ratio R versus resampling coefficient

Resampling coefficient	$N_{min} \cdot 10^5$	R
1:1	2.2471	1.6534
1:20	2.2004	1.7453
1:30	2.1668	1.8253
1:40	1.9828	1.8971
1:50	1.8822	1.9151

Figure captions

Fig. 1. Typical realization of a process without control. For details of calculations, see the text.

Fig. 2. Time elapsed before the "discharge interruption" as dependent on parameter a_1. Data are obtained for different temporal realizations of noise of the same intensity, which gives scattering of points at a given value of the parameter a_1.

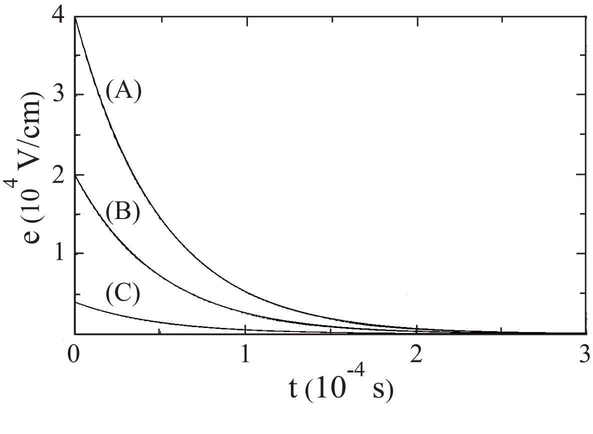
Fig. 3. Examples of dynamics of the system without control, plots (a) and (a′), and with control, plots (b) and (b′). The amplitude of control signal $\overline{u} = 0.2E_m$. Pay attention to different scales on axes of plots for uncontrolled and controlled cases.

Fig. 4. Dependence of minimal density of carriers (a) and the excitation index of the system (b) on the relative value of maximal amplitude of control.

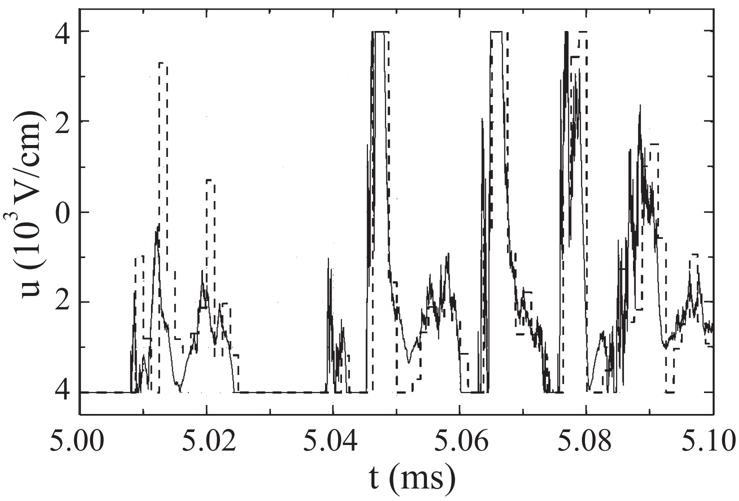
Fig. 5. Dependence of ratio R=max {Nmin}/min {Nmin} on the maximal value of control signal.

Fig. 6. Fragments of dependencies of the control signal on time for frequent (solid line) and rare (dash line) measurements of variable N. The two curves are obtained for the same variation of noise in time. For details, see the text.

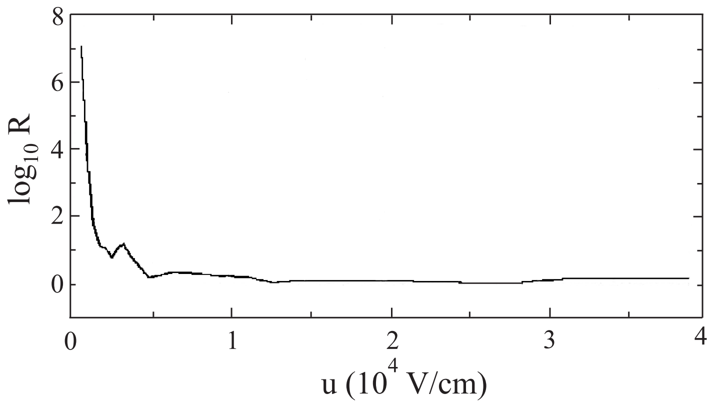
Fig. 7. Dependencies of the estimation error on time. Curves are obtained for the following values of the initial (at t=0) electric field: (A): $\hat{E}(0)$=0; (B): $\hat{E}(0)$=0.5 E_0, (C): $\hat{E}(0)$=0.9 E_0.



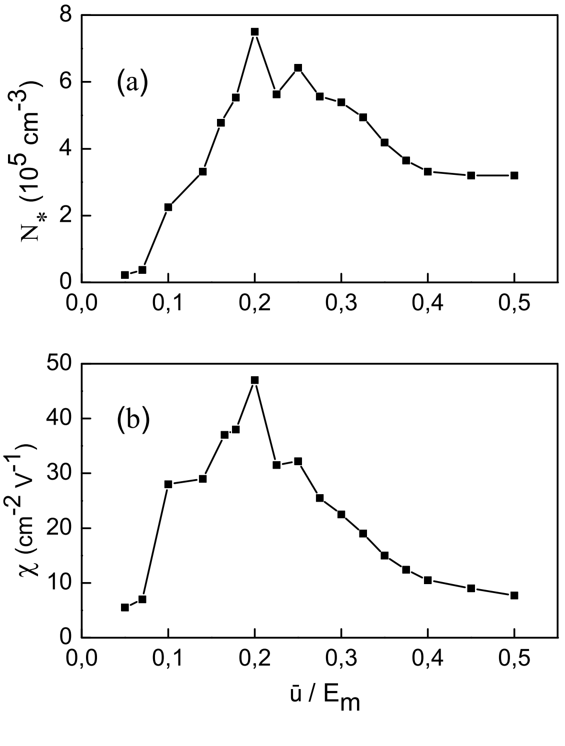
e (10^4 V/cm)
t (10^-4 s)
(A)
(B)
(C)
0
1
2
3
4



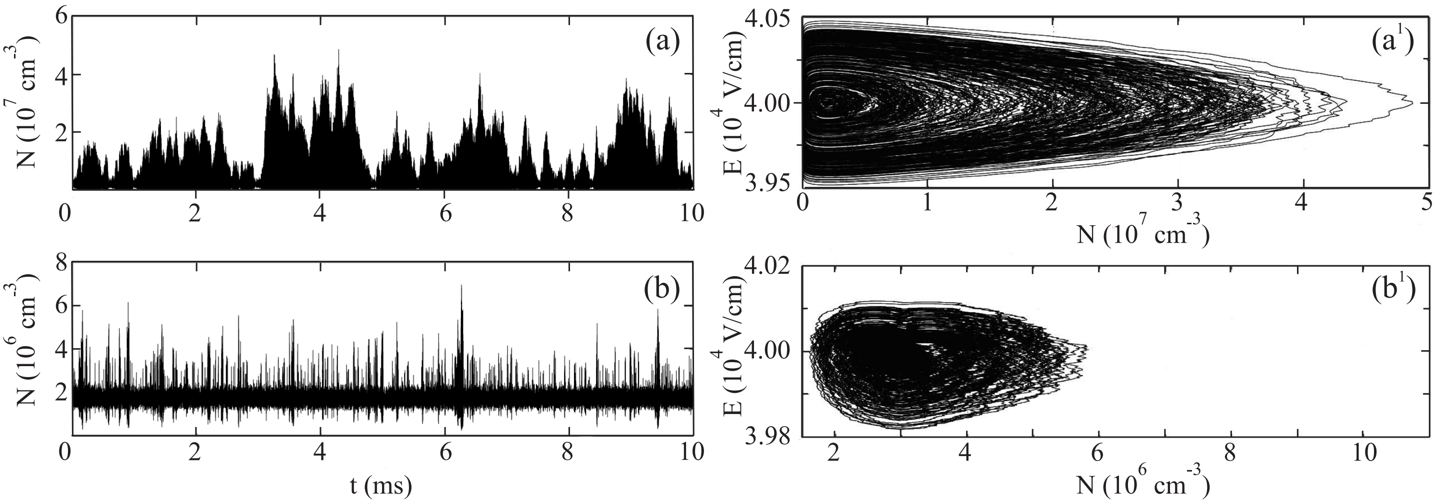
u (10³ V/cm)
4
2
0
2
4
5.00
5.02
5.04
5.06
5.08
5.10
t (ms)



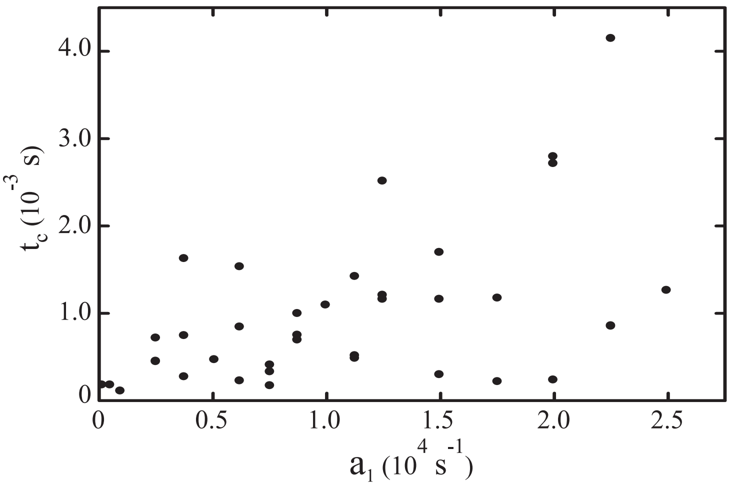
8
6
4
2
0
0
1
2
3
4
log10 R
u (10^4 V/cm)



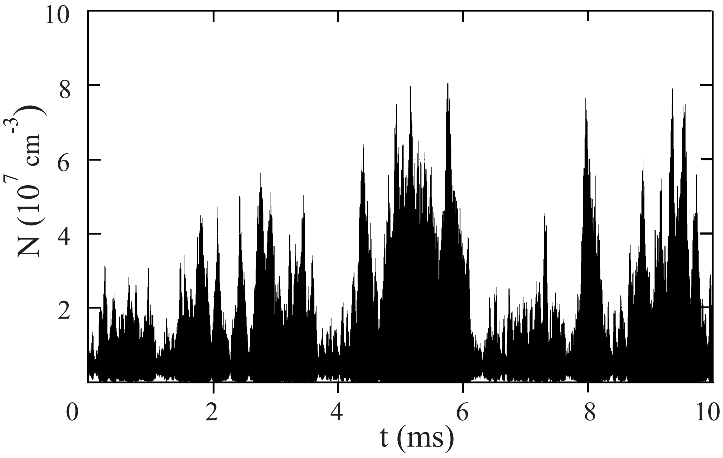
(a)
N_* (10^5 cm^{-3})
(b)
χ (cm^{-2} V^{-1})
$\bar{u}$ / E_m



(a)
N (10^7 cm^{-3})
(b)
N (10^6 cm^{-3})
t (ms)
(a^1)
E (10^4 V/cm)
N (10^7 cm^{-3})
(b^1)
E (10^4 V/cm)
N (10^6 cm^{-3})



4.0
3.0
2.0
1.0
0
t_c (10^{-3} s)
0
0.5
1.0
1.5
2.0
2.5
a_1 (10^4 s^{-1})



10
8
6
4
2
N (10^7 cm^{-3})
0
2
4
6
8
10
t (ms)