DESIGN OF AN ADAPTIVE SYSTEM FOR STABILIZING A LINEAR OBJECT WITH DISTRIBUTED PARAMETERS

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We consider the design of an adaptive stabilization system in the case of a linear dynamic object with distributed parameters. The dynamics of the class of objects under discussion is described in the language of semigroups. For the stated problem we give a solution based on frequency conditions for the solvability of certain operator inequalities (an infinite-dimensional analogue of the Yakubovich–Kalman lemma). The results obtained are illustrated by an example.

1. INTRODUCTION

Problems of controlling complicated objects with distributed parameters are encountered more and more frequently in engineering practice [1]. Such objects include chemical and nuclear reactors, industrial plants in which heat, diffusion, or wave processes occur, elastic moving objects, and so on. The design of a system for controlling distributed objects is frequently complicated by the fact that it must operate under conditions of uncertainty, when the values of the object parameters and of the external effects are not known precisely or can change in an unforeseeable way. As is well known, adaptation methods provide an effective means for solving control problems under conditions of uncertainty [2, 3]. Most of the work on adaptive control of objects with distributed parameters (see, e.g., [4-6]) is based on finite-dimensional approximation of the infinite-dimensional phase space of the object (i.e., replacement of the distributed object by a concentrated one) and subsequent use of standard "finite-dimensional" methods [2, 3, 7]. However, to check the effectiveness of the designed system in this approach it is necessary to solve the problem of the stability and accuracy of the finite-dimensional approximation. As simple examples show [8], even in the "nonadaptive" case the solution to this problem is far from trivial. Therefore, a justification for the correctness of the indicated approach requires additional investigation.

On the other hand, in recent years there has been a development of techniques for finding frequency conditions for the existence of solutions of linear and quadratic operator inequalities [9, 10]: infinite-dimensional variants of the Yakubovich–Kalman lemma. The use of these conditions enables us to carry the Lyapunov methods for designing adaptive control systems directly over to objects with distributed parameters [3, 11-13]. Moreover, the infinite dimensionality of the state space of the object leads to a number of analytical difficulties which cannot be correctly overcome without changing both the definitions and statements of the problems and the methods for proving the effectiveness of the systems.

This paper deals with an extension to objects with distributed parameters of the method for designing adaptive stabilization systems that was proposed in [13]. To describe the dynamics of a linear controlled object we choose the language of semigroups [14, 15], which enables us to include a broad class of objects that can be described by partial differential equations with lagging argument, etc. The number of measurable outputs of the object is assumed to be finite, and the controlling signal is assumed (for simplicity) to be a scalar. The specific character of the infinite-dimensional state space made it necessary to impose on the operator of the object a number of requirements of a general character that are listed below. The statement of the problem considered in this paper relates to the case of unmeasurable but damped perturbations and reflects the specific character of problems of "rough" stabilization when there is little a priori information about the object and the perturbations, and when there is no driving signal and the goal of the adaptation is essentially to ensure stability of the system. We remark that, as in the case of a concentrated object [16], the proposed algorithm can be made effective even in the case of undamped perturbations by regularization: the introduction of a negative feedback in the adaptation circuit.

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2. DEFINITIONS AND NOTATION. STATEMENT OF THE PROBLEM

It is assumed that the dynamics of the controlled object is described by the linear differential equation

$$dx/dt = Ax + bu + f, \quad y = L^* x, \tag{1}$$

in the Hilbert space X; $x(t) \in X$ is the state vector of the object, y is the l-dimensional output vector of the object, u is a scalar control, and f(t) is a perturbation vector. The operator A is a linear unbounded (generally speaking) operator with dense domain D(A) in X and generating a semigroup of the class C_0 [14, 10]. We consider the operators A whose resolvents, the operators $R_\lambda(A) = [\lambda I - A]^{-1}$ ($\lambda \in C$, I the identity operator), satisfy additional conditions. Namely, we suppose that $R_\lambda(A)$ is a compact operator and $\|R_{i\omega}(A)\| \le k/|\omega|$, $\|R_\lambda(A)\| \to 0$ ($|\lambda| \to \infty$, $\mathrm{Re}\,\lambda > -\delta$), respectively, for sufficiently large $\omega \in R^1$ and some $k > 0$, $\delta > 0$.

The last conditions mean, in essence, that some neighborhood of the right half-plane contains at most finitely many points of the spectrum of the operator A. Let $\mathscr{A}$ denote the class of operators satisfying these conditions.

We remark that in the finite-dimensional case, when (1) is a system of ordinary differential equations and the operator A is a matrix, the class $\mathscr{A}$ includes all matrices. In the case $X = L_2(\Omega)$, where $\Omega \subset R^n$ is a bounded domain,* n = 1, 2, 3, the class $\mathscr{A}$ contains the strongly elliptic differential operators and thereby embraces a large collection of practically important boundary value problems. Examples (and formulas enabling us to check the conditions of the class $\mathscr{A}$) can be found in [8, 17].

The perturbation f(t) is assumed to be damped and smooth, more precisely,

$$f(\cdot) \in L_2(0, +\infty; X) \cap C^1(0, +\infty; X), \quad f(0) \in D(A).$$

In (1) the operators $b: R^1 \to X$, $L: R^l \to X$ are assumed to be bounded and such that $b \in \bigcap_n D(A^n)$, $R(L) \subset \bigcap_n D(A^{*n})$, where R(L) denotes the range of the operator L.

Let $\mathscr{L}(X, Y)$ be the set of bounded linear operators acting from X into Y (X, Y are normed spaces). A vector $a \in X$ obviously generates a linear operator in $\mathscr{L}(R^1, X)$, and we denote it also by a. For any vectors a, b in the Hilbert space X we let a^*b denote their scalar product. Let $\sigma(A)$ denote the spectrum of the operator A; A is said to be Hurwitz if $\sigma(A) \subset \{\lambda \in C: \mathrm{Re}\lambda < -\delta;\ \delta > 0\}$. The symbol A^* denotes the operator adjoint to A.

Let $B \in \mathscr{L}(R^m, X)$, $L \in \mathscr{L}(R^l, X)$. The pair {A, B} is said to be controllable if $R(B) \subset \bigcap_n D(A^n)$ and the set $\bigcup_n A^n R(B)$ is dense in X. The pair {A, L} is observable if $\{A^*, L\}^n$ is controllable. The triple {A, B, L} is said to be nondegenerate if {A, B} is controllable and {A, L} is observable.

The transfer function of the object (1) is defined to be the vector function $W(\lambda) = L^*(\lambda I - A)^{-1} b = L^* R_\lambda(A) b$, $\lambda \in C$. If $A \in \mathscr{A}$, then the elements $W(\lambda)$ are meromorphic, and it can be represented in the form $W(\lambda) = [\delta(\lambda)]^{-1}\Psi(\lambda)$, where the entire function $\delta(\lambda)$ and the vector of entire functions $\Psi(\lambda)$ do not have common zeros [14].

We proceed to the statement of the problem. Suppose that the object is to be stabilized by a linear regulator of the form

$$u = c^* y, \tag{2}$$

where $c \in R^l$ is a vector of adjustable coefficients.

An algorithm for adjusting the coefficients will be found in the class of algorithms of the form

$$dc/dt = F(y). \tag{3}$$

As usual [7], we assume that the "coefficients" A, b, L of the object and the perturbation f(t) depend on a vector ξ of unknown parameters with $\xi \in \Xi$, where Ξ is a known set.

Definition 1. We say that the system (1)-(3) is adaptive in the class Ξ, if for any $\xi \in \Xi$ and on any trajectory of the system the goal of the control is attained: the satisfaction of the relations

*Here and in what follows we use the standard notation for the function spaces: $L_2(\Omega)$ is the Hilbert space of square-integrable functions in the domain Ω; $L_2(0, +\infty; X)$ is the Hilbert space of vector functions x(t) with values in the Hilbert space X and such that $\|x(t)\|$ is square-integrable on the half-line; $W_2^2(\Omega)$ is the Sobolev space (see, e.g., [14]).

$$x(t)\in L_2(0,+\infty;X), \tag{4}$$

$$\lim_{t\to\infty} c(t) \text{ exists.} \tag{5}$$

The problem of designing an adaptive stabilization system consists in determining a function F in (3) that does not depend on $\xi\in\Xi$ and is such that the system has the adaptivity property in the given class Ξ.

We solve the stated problem below.

3. STATEMENT OF THE RESULTS

To state the adaptivity conditions we need the following definition, which coincides with the common one [18] in the finite-dimensional case.

Definition 2. Let $\chi(\lambda)$ be defined by the equality $\chi(\lambda) = L^*(\lambda I - A)^{-1}b$, where $A\in\mathcal{A}$, b, $L\in X$. This $\chi(\lambda)$ is called a minimal phase function if there exist operators $B_1\in\mathcal{L}(R^l, X)$, $L_1\in\mathcal{L}(R^l, X)$, such that the triple $\{A, B_1, L_1\}$ is nondegenerate and $\varphi(\lambda)\neq 0$ for $\mathrm{Re}\,\lambda\geq 0$, where $\varphi(\lambda) = \delta_1(\lambda)\chi(\lambda)$, $\delta_1(\lambda)$ is the denominator of the matrix function $\chi_1(\lambda) \doteq L_1^*(\lambda I - A)^{-1}B_1 = [\delta_1(\lambda)]^{-1}\Psi_1(\lambda)$.

We remark that the triple $\{A, B, L\}$ can be degenerate (see § 2). But if $\{A, b, L\}$ is nondegenerate, then instead of $\varphi(\lambda)\neq 0$ we can require that $\chi(\lambda)\neq 0$ for $\mathrm{Re}\,\lambda \geq 0$.

We construct the adaptation algorithm (3) similarly to [13] in the form

$$dc/dt = -(g^*y)Py, \tag{6}$$

where g is some l-dimensional vector, and $P = P^*$ is the positive-definite $l\times l$ matrix of amplification coefficients of the adaptation circuit. The following theorem gives conditions for the effectiveness of a system with the algorithm (6).

Theorem 1. Suppose that for any $\xi\in\Xi$ the parameters A, b, L of the object and the perturbation f(t) satisfy the conditions of § 2. Then the system (1), (2), (6) is adaptive in the class Ξ, if $g^*W^*(\lambda)$ is a minimal phase function and $g^*L^*b > 0$ for any $\xi\in\Xi$

A proof of the theorem is given in the Appendix. For the finite-dimensional case Theorem 1 was proved in [13].

The design of an adaptive stabilization system on the basis of Theorem 1 can be carried out as follows.

1. The operator A of the object is written out (taking into account the differential equation and the boundary conditions) and the inclusion $A\in\mathcal{A}$ is verified.

2. On the basis of the solution to the boundary value problem (see, for example, the tables in [8, 17]) the transfer function $W(\lambda)$ of the object is written out.

3. The class of functions $W_\xi(\lambda)$ corresponding to the given class of uncertainty Ξ is determined.

4. A vector $g\in R^l$ is found which ensures that for any $\xi\in\Xi$ the function $g^*W_\xi(\lambda)$ is a minimal phase function and that the coefficient g^*L^*b is positive.

5. The adaptation algorithm is taken to be (6) with the vector g found.

The central point in the proof of Theorem 1 is the use of the so-called degenerate frequency theorem, which has independent significance and a broad circle of applications [10, 12, 13]. Below, we state Theorem 2, which is a needed variant* of the frequency theorem in [10] that uses the "limiting frequency inequality."

Let $A: D(A)\subset X\to X$ be an infinitesimal generator of a semigroup of the class C_0, $B\in\mathcal{L}(R^m, X)$, $\mathcal{F}(x,u) = x^*F_1x + 2\,\mathrm{Re}\,xF_2u$ a Hermitian form on $X\times R^m$ degenerate with respect to $u\in R^m$; $F_1 = F_1^*\in\mathcal{L}\times(X,X)$, $F_2\in\mathcal{L}(R^m, X)$. By $\Pi(\omega)$ we denote the $m\times m$ matrix of the Hermitian form $\mathcal{F}[R_{i\omega}(A)Bu, u]$, and by $g(\omega)$ the greater of the two numbers: $\|R_{i\omega}(A)B\|^2$, $\|R_{i\omega}(A)F_2\|^2$. Obviously, if A is a Hurwitz operator, then $g(\omega)$ is a bounded positive function.

Theorem 2. Suppose that the following three conditions hold:

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*Theorem 2, in a formulation close to ours, was obtained by L. O. Barsuk. For the case of bounded operators and $g(\omega) = |\omega|^{-2}$ this theorem was published in [19].

I. the operator A is Hurwitz;

II. $\Pi(\omega) > 0 \ \forall \omega \in R^1$ (the frequency inequality);

III. $\lim_{\omega \to \infty} \Pi(\omega)/g(\omega) > 0$ (the limiting frequency inequality).

Then:

1) there exist an operator $H = H^* \in \mathscr{L}(X, X)$ and a number $\delta > 0$ such that

$$HB = F_2, \quad \mathrm{Re}\, x^* HAx + x^* F_1 x \leq -\delta |x|^2 \forall x \in D(A); \tag{7}$$

2) if $F_1 \leq 0$, then $H > 0$.

Below (see the Appendix) we derive Theorem 2 from results in [10, 20]. We remark that if $F_1 = 0$, $F_2 = L$, and $\|R_{i\omega}(A)\| \leq K|\omega|^{-1}$, then $g(\omega) = K\omega^2$, $K > 0$, and, consequently, the conditions of the theorem can be written as follows:

IIa) $\mathrm{Re}\, W(i\omega) > 0 \ \forall \omega \in R^1$ (the frequency inequality);

IIa) $\lim_{\omega \to \infty} \omega^2 \mathrm{Re}\, W(i\omega) > 0$ (the limiting frequency inequality).

Under these conditions there exists an operator $H > 0$ such that (7) holds.

4. EXAMPLE

We consider a linear controlled heat transfer process in a rod of finite length. It is assumed that the heat source adds or absorbs heat at the point z in proportion to the local temperature T(z, t), while the source of the controlled output u(t) adds or absorbs heat uniformly along the length of the rod. The mathematical model of the process has the form

$$\frac{\partial T}{\partial t} = \xi_1 \frac{\partial^2 T}{\partial z^2} + \xi_0 T + \xi_2 u, \tag{8}$$

where ξ_0, ξ_1, ξ_2 are constant numerical parameters. It is assumed that the heat flow through the ends of the rod is equal to zero.

Of the possible variants for observation we choose an observation with constant effectiveness function:

$$y(t) = \xi_3 \int_0^1 T(z, t)\, dz, \tag{9}$$

where ξ_3 is a constant numerical parameter.

Thus, in this case the set of unknown parameters of an object $\xi \in R^4$ consists of the numbers $\xi_0, \xi_1, \xi_2, \xi_3$, moreover, from physical considerations the parameter ξ_1 must be positive. We take the class Ξ of possible parameter values to be the set $\Xi = \{\xi\colon \xi_1 > 0,\ \xi_2\xi_3 > 0,\ \xi_0 < \pi^2 \xi_1\}$. Obviously, the class Ξ contains also unstable objects (for positive values of ξ_0), therefore, the necessity of stabilization arises in the solution of the problem.

To use the results presented above, we take $X = L^2[0, 1]$,

$$Ax = \xi_1 \frac{\partial^2 x}{\partial z^2} + \xi_0 x, \quad D(A) = W_2^2[0,1] \cap \left\{ x : \frac{\partial x(0)}{\partial z} = \frac{\partial x(1)}{\partial z} = 0 \right\},$$

$$b = \xi_2, \quad L = \xi_3 \in X.$$

Then Eqs. (8), (9) take the form (1) for $l = 1$. It follows from the general theory of differential operators [14] that for any $\xi \in \Xi$ the operator A is in the class $\mathscr{A}$.

We show that the system (8), (9), complemented according to (2), (6) for g = 1 by the adaptive regulator

$$u = cy, \quad dc/dt = -y^2,$$

is adaptive in the given class Ξ in the sense of Definition 1. To do this we verify the conditions in Theorem 1. Suppose first that $\xi_0 = 0$, $\xi_1 = \xi_2 = \xi_3 = 1$. Then, using the tables given in [8], we have $W(\lambda) = L^*(\lambda I - A)^{-1} b = \lambda^{-1}$.

To verify the minimal phase property we take $L_1 = B_1 = \sum_{k=0}^{\infty} 2^{-k} \cos k\pi z$. Using the controllability criteria in [10], we can show that the triple $\{A, B_1, L_1\}$ is not degenerate, and the denominator of its transfer function is equal to $\sqrt{\lambda}\,\mathrm{sh}(\sqrt{\lambda})$. Thus, $\varphi(\lambda) = \mathrm{sh}(\sqrt{\lambda})/\sqrt{\lambda} = \prod_{k=1}^{\infty}(1+\lambda(k\pi)^{-2})$, i.e., $\varphi(\lambda) = 0$ for $\lambda = -(k\pi)^2$ $(k = 1, 2, \ldots)$. Since for other values of ξ the corresponding function $\varphi_\xi(\lambda)$ has the form $\varphi\left(\frac{\lambda-\xi_0}{\xi_1}\right)$ we have that $\varphi_\xi(\lambda) = 0$ for $(\lambda-\xi_0)/\xi_1 = -(k\pi)^2$ $(k = 1, 2, \ldots)$, i.e., $\varphi\xi(\lambda) \neq 0$ for $\mathrm{Re}\,\lambda \geq 0$ if $\xi \in \Xi$.

Accordingly, the minimal phase condition is satisfied for the function $g^*w(\lambda)$. It is easy to see that $g^*L^*b > 0$ also holds for our choice of $g = 1$ for any $\xi \in \Xi$. Theorem 1 now gives us that the system (8)-(10) is adaptive in the class Ξ.

APPENDIX

We use Theorem 2 to prove Theorem 1. Therefore, we first prove Theorem 2. We precede the proof by two lemmas.

Lemma 1. Let $\Pi(\omega)$, $G(\omega)$ be two Hermitian $m \times m$ matrices that depend continuously on the parameter $\omega \in R^1$. If 1) $\Pi(\omega) > 0$ $\forall \omega \in R^1$, 2) $G(\omega) \leq g(\omega) I_m$ $\forall \omega \in R^1$, 3) $\lim_{\omega\to\infty} \Pi(\omega)[g(\omega)]^{-1} > 0$, then for some $\delta > 0$ we have the inequality

$$\Pi(\omega) \geqslant \delta G(\omega), \quad \omega \in R^1. \tag{A.1}$$

Proof. Suppose the opposite: $\forall k = 1, 2, \ldots, \exists u_k \in R^m, \omega_k \in R^1 : u_k^* \Pi(\omega_k) u_k < (1/k) u_k^* G(\omega_k) u_k$. If the set $\{(u_k, \omega_k)\} \subset R^{m+1}$ is bounded in R^{m+1}, then there exists a limit point $(u_\infty, \omega_\infty)$ of this set, $(u_{kj}, \omega_{kj}) \xrightarrow[j\to\infty]{} (u_\infty, \omega_\infty)$. Then the continuity condition implies that $u_\infty^* \Pi(\omega_\infty) u_\infty = \lim_{j\to\infty} u_{kj}^* \Pi(\omega_{kj}) u_{kj} \leq \lim_{j\to\infty} \frac{1}{k_j} u_{k_j}^* G(\omega_{k_j}) \cdot u_{k_j} = 0$, which contradicts 1. But if the set $\{(u_k, \omega_k)\}$ is not bounded in R^{m+1}, then without loss of generality we may assume that $u_k \to u_0$, $|u_k| = |u_0| = 1$, $|\omega_k| \to \infty$. Consequently,

$$\lim_{k\to\infty} u_k^* \Pi(\omega_k) u_k \frac{1}{g(\omega_k)} \leqslant \lim_{k\to\infty} \frac{1}{kg(\omega_k)} u_k^* G(\omega_k) u_k \leqslant \lim_{k\to\infty} \frac{g(\omega_k)}{kg(\omega_k)} = 0,$$

which contradicts the condition 3.

Lemma 2. Suppose that $J(h) = h^*Rh + 2\,\mathrm{Re}\,r^*h + \rho$, a continuous quadratic functional on the Hilbert space $\mathscr{H}$: $R = R^* \in \mathscr{L}(\mathscr{H}, \mathscr{H})$, $r, h \in \mathscr{H}$, $\rho \in R^1$. The functional $J(h)$ is lower semibounded,

$$\exists \gamma \in R^1 : J(h) \geqslant \gamma \quad \forall h \in \mathscr{H} \tag{A.2}$$

if and only if $|r^*h| \leq \varkappa^2 h^*Rh$ $\forall h \in \mathscr{H}$. The proof of the lemma is elementary.

Proof of Theorem 2. Let us consider the set $\mathscr{U}_+ = \{u(\cdot) \in L^2(R^1; R^m) : u(t) \equiv 0 (t < 0)\}$. Let $x(t)$: $\dot{x} = Ax + bu$, $x(0) = x_0$, $x(t) \equiv 0 (t < 0)$. Then it is clear that $\hat{x}(\omega) = R_{i\omega}(A) B \hat{u}(\omega) + R_{i\omega}(A) x_0$, where $\hat{x}(\omega)$, $\hat{u}(\omega)$ are the Fourier transforms of the functions $x(t)$, $u(t)$. On the Hilbert space $\mathscr{U}_+$ we consider the functional $J(u) = \int_0^\infty \mathscr{F}[x(t), u(t)]\,dt$.

Parseval's equality gives us that

$$J(u) = \int_{-\infty}^{+\infty} \mathscr{F}[\hat{x}(\omega), \hat{u}(\omega)]\,d\omega = \int_{-\infty}^{+\infty} \{\hat{u}^*(\omega)\Pi(\omega)u(\omega) + 2\,\mathrm{Re}\,x_0^* K(\omega)\hat{u}(\omega)\}\,d\omega + \rho,$$

where $K(\omega) = R^*_{i\omega}(A)[F_2 + F_1 R_{i\omega} B] \in \mathscr{L}(R^m; X)$,

$$\rho = \int_{-\infty}^{\infty} x_0^* R_{i\omega}^*(A) F_1 R_{i\omega}(A) x_0\,d\omega.$$

The functional J, thus, has the form indicated in Lemma 2. We verify that the condition in Lemma 2 holds.

Letting $G(\omega)$ be the matrix $\|K(\omega)u\|^2$, we get from the conditions II, III, and the inequality $G(\omega) \leq Cg(\omega)$, according to Lemma 1, that $u^*\Pi(\omega)u \geq \delta |K(\omega)u|^2$, $\delta > 0$, $\forall u$, $\forall \omega$. Then

$$\int_{-\infty}^{\infty} \hat{u}^*(\omega)\Pi(\omega)\hat{u}(\omega)d\omega \geq \delta \int_{-\infty}^{\infty} |K(\omega)\hat{u}|^2 d\omega \geq \frac{\delta}{|x_0|^2} \left| \int_{-\infty}^{\infty} x_0^* K^*(\omega)\hat{u}(\omega)d\omega \right|^2.$$

Setting $r = K^*(\cdot)x_0 \in L^2(R_1; R^m)$, $\varkappa = |x_0|^2/\delta$, we obtain the condition of Lemma 2.

Accordingly, the functional $J(u)$ is lower semibounded on $\mathcal{U}_+$.

By Theorem 5 in [10] and Lemma 1, we get assertion 1) in Theorem 2. Assertion 2) follows from the statement of Theorem 2 in [19]. Theorem 2 is proved.

The proof of Theorem 1 will be preceded by two lemmas.

Lemma 3. Let $A \in \mathcal{A}$, $B \in \mathcal{L}(R^m; X)$, $L \in \mathcal{L}(R^l; X)$. Suppose that the triple $\{A, B, L\}$ is nondegenerate. Then at each point of the spectrum of the operator A the transfer matrix $\chi(\lambda) = L^*R_\lambda(A)B$ has a pole of the same order as the resolvent $R_\lambda(A)$.

Proof. It is known (see [14]) that the operators in the class $\mathcal{A}$ have a spectrum consisting of isolated points that are poles of the resolvent. In a neighborhood of a pole λ_0 of it of order r the resolvent can be expanded in a Laurent series

$$R_\lambda(A) = \sum_{-\infty}^{\infty} (\lambda - \lambda_0)^n A_n, \quad A_n = 0 \quad \text{for} \quad n < -r, \quad A_{-r} \neq 0, \tag{A.3}$$

where the A_n are bounded operators that commutate with A, and [14]

$$(A - \lambda_0 I)A_n = A_{n-1} \quad \text{for} \quad n \neq 0. \tag{A.4}$$

We assume that at the point λ_0 the matrix $\chi(\lambda)$ has a pole of order $r' \geq 0$, $r' < r$. Then it follows from (A.3) that $L^*A_nB = 0$ for $n < -r'$. With the help of (A.4) it is easy to get from this that $L^*A^kA_nA^jB = 0$ for $n < -r'$, k, $j = 0, 1, 2, \ldots$.

Consequently, for any $y \in R^l$, $u \in R^m$ we have $y^*L^*A^kA_{-r}A^jB_nu = 0$ for $k, j = 0, 1, 2, \ldots$. Since the triple $\{A, B, L\}$ is nondegenerate, we get $A_{-r} = 0$, a contradiction. The lemma is proved.

Lemma 4.* We consider $A \in \mathcal{A}$, $b \in \cap D(A^*)$, $d \in \cap D(A^{*n})$, $L_1 \in \mathcal{L}(R^1, X)$, $B_1 \in \mathcal{L}(R^1, X)$ and write $A_\varkappa = A - \varkappa bd^*$, $\chi_\varkappa(\lambda) = d^*R_\lambda(A_\varkappa)b$, $\varkappa \in R^1$. Suppose that the triple $\{A, B_1, L_1\}$ is nondegenerate, $d^*b > 0$, and $\varphi(\lambda) \neq 0$ for $\mathrm{Re}\,\lambda \geq 0$, where $\varphi(\lambda) = \delta_1(\lambda)\chi(\lambda)$, $\chi(\lambda) = d^*R_\lambda(A)b$, $\chi_1(\lambda) = L_1^*R_\lambda(A)B_1 = \Psi_1(\lambda)[\delta_1(\lambda)]^{-1}$.

Then $A_\varkappa \in \mathcal{A}$ and for sufficiently large $\varkappa$ there exist an operator $H \in \mathcal{L}(X, X)$, $H = H^* > 0$, and a number $\varepsilon > 0$ such that

$$Hb = d, \quad \mathrm{Re}\, x^*HA_\varkappa x \leq -\varepsilon |x|^2 \quad \text{for} \quad x \in D(A). \tag{A.5}$$

Proof. We use Theorem 2. For this we show that for sufficiently large $\varkappa$ we have that

$$A_\varkappa \in \mathcal{A}, \; A_\varkappa \text{ — a Hurwitz operator.} \tag{A.6}$$

$$\mathrm{Re}\,\chi_\varkappa(i\omega) > 0 \quad \text{for} \quad \omega \in R^1, \tag{A.7}$$

$$\lim_{\omega \to \infty} \omega^2 \chi_\omega(i\omega) > 0. \tag{A.8}$$

Let us first see that $A_\varkappa \in \mathcal{A}$. Indeed, by a perturbation theorem in [14], $A_\varkappa$ generates a semigroup of the class C_0, because $A - A_\varkappa = \varkappa bd^* \in \mathcal{L}(X, X)$.

We consider the set $\Omega_\delta = \{\lambda: \mathrm{Re}\,\lambda \geq -\delta, \; \delta > 0\}$. Since $\|R_\lambda(A)\| \to 0$ $(\lambda \in \Omega_\delta)$, all points of Ω_δ sufficiently large in modulus are regular for the operator A and (as is not hard to see) for the operator $A_\varkappa$. This follows from the fact that a regular point λ of A is a regular point of $A_\varkappa$ if $\|\varkappa bd^*R_\lambda(A)\| = q < 1$. Since, moreover, $\|R_\lambda(A) -$

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*Lemma 4 is the basis for the proof of Theorem 1. It gives sufficient conditions for the existence of solutions of the operator inequalities (A.5), which signify that the derivative of the Lyapunov function of the system (1)-(3) is negative. We remark that in the finite-dimensional case these conditions are necessary and sufficient [13].

$\|R_\lambda(A_\varkappa)\| \le q/(1-q)\|R_\lambda(A)\|$, we have that $R_\lambda(A_\varkappa) \to 0$ as $|\lambda| \to \infty$ if $\lambda \in \Omega_\delta$ and $\|R_{i\omega}(A)\| \le c/|\omega|$ for sufficiently large $|\omega|$. Finally, by the second resolvent identity,* the resolvent of $A_\varkappa$ is compact, and, consequently, $A_\varkappa \in \mathscr{A}$. We now show that for large $\varkappa$ the operator $A_\varkappa$ is Hurwitz. To do this we consider the functions $\chi(\lambda)$ and $\chi_\varkappa(\lambda)$ and convince ourselves that for large $\varkappa$ the function $\chi_\varkappa(\lambda)$ does not have singuliarities in the right half-plane. Indeed, it follows from the second resolvent identity

$$R_\lambda(A) - R_\lambda(A_\varkappa) = R_\lambda(A)\varkappa b d^* R_\lambda(A_\varkappa) \tag{A.9}$$

that

$$\chi_\varkappa^{-1}(\lambda) - \chi^{-1}(\lambda) = \varkappa. \tag{A.10}$$

We recall that $\chi(\lambda) = \varphi(\lambda)[\delta_1(\lambda)]^{-1}$, where $\delta_1(\lambda)$, $\varphi(\lambda)$, by Lemma 1, are entire functions, and $\varphi(\lambda) \neq 0$ for $\operatorname{Re}\lambda \ge 0$. From this it follows that $\chi^{-1}(\lambda)$ is a function holomorphic in the right half-plane. If we show that $\operatorname{Re}\chi^{-1}(\lambda)$ is bounded below on the right half-plane, then it follows at once from the equality (A.10) that for large $\varkappa$ the function $\chi_\varkappa^{-1}$ does not have zeros in the right half-plane, and, hence $\chi_\varkappa(\lambda)$ does not have singularities. To prove that $\operatorname{Re}\chi^{-1}(\lambda)$ is bounded, we write two equalities which are easily verified with the help of the second resolvent identity:

$$\lambda\chi(\lambda) = d^*b + d^*R_\lambda(A)Ab, \tag{A.11}$$

$$\lambda^2\chi(\lambda) = \lambda d^*b + d^*Ab + d^*R_\lambda(A)A^2b. \tag{A.12}$$

It follows from (A.12) that

$$\operatorname{Re}\frac{1}{\chi(\lambda)} = \operatorname{Re}\frac{\chi(\lambda)}{|\chi(\lambda)|^2} \geqslant \frac{d^*b \operatorname{Re}\lambda}{|\lambda\chi(\lambda)|^2} - \frac{|d^*Ab + d^*R_\lambda(A)A^2b|}{|\lambda\chi(\lambda)|^2}.$$

The first term on the right-hand side of this inequality is nonnegative, and the second term has a limit as $|\lambda| \to \infty$, $\operatorname{Re}\lambda \ge 0$, by (A.11). It follows from this that $\chi^{-1}(\lambda)$ is bounded below on the right half-plane, and, hence, for sufficiently large $\varkappa$ the function $\chi_\varkappa(\lambda)$ does not have singularities in the right half-plane.

Since the triple $\{A_\varkappa, b, d\}$ is not, generally speaking, nondegenerate, the continuity of $\chi_\varkappa(\lambda)$ for $\operatorname{Re}\lambda \ge 0$ does not now imply directly the Hurwitz property of $A_\varkappa$, and to prove this fact we need some additional constructions which we omit because of their awkwardness.

It remains to verify the conditions (A.7) and (A.8), and we do this. By (A.11), (A.12) we have that $\operatorname{Re}[\chi(i\omega)]^{-1} = \operatorname{Re}\chi(i\omega)|\chi(i\omega)|^2 = -(d^*Ab + d^*R_{i\omega}(A)A^2b)\cdot|i\omega\chi(i\omega)|^{-2} \underset{|\omega|\to\infty}{\to} -\frac{d^*Ab}{|d^*b|^2}$. Therefore, it follows from (A.10) that $\operatorname{Re}[\chi_\varkappa(i\omega)]^{-1} > 0$ and $\lim_{|\omega|\to\infty} \operatorname{Re}[\chi\ (i\omega)]^{-1} > 0$, if $\varkappa$ is sufficiently large. For such $\varkappa$ we obviously have that $\operatorname{Re}\chi_\varkappa(i\omega) > 0$ and $\lim_{\omega\to\infty}\omega^2 \operatorname{Re}(\chi_\varkappa(i\omega) = \lim_{\omega\to\infty}\operatorname{Re}\frac{|i\omega\chi_\varkappa(i\omega)|^2}{\chi_\varkappa(i\omega)} = \lim_{\omega\to\infty}\operatorname{Re}|i\omega\chi_\varkappa(i\omega)| \lim_{\omega\to\infty}\operatorname{Re}[\chi_\varkappa(i\omega)]^{-1}$, since both limits exist and are positive.

The existence of the desired operator H now follows from Theorem 2. Lemma 4 is proved.

Proof of Theorem 1. First of all, we remark that the operator A, the operators B_1, L_1 (from the definition of the minimal phase property of the function $g^*W(\lambda)$), and the vectors b, d = Lg satisfy the conditions of Lemma 4. By this lemma, there exist a number $\varkappa$ and an operator $H \in \mathscr{L}(X, X)$, such that $H = H^* > 0$, $Hb = Lg$, $\operatorname{Re} x^*HA_\varkappa x \le -\varepsilon|x|^2$ for $x \in D(A)$, where $A_\varkappa = A - \varkappa bg^*L^*$. We consider the following function on the phase space of the system (1), (2), (6):

$$V(x, c) = x^*Hx + (c - \varkappa g)^*P^{-1}(c - \varkappa g). \tag{A.13}$$

This function is a Lyapunov function for the system (1), (2), (6), since for its derivative we have, by the system, that†

$$\dot{V}[x(t), c(t)] = 2\operatorname{Re} x^*HA_\varkappa x + x^*Hf(t) \leqslant -\varepsilon|x(t)|^2 + |x(t)|\cdot\|H\|\cdot|f|.$$

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*By the second resolvent identity we mean the following fact [14]: if λ is a common regular point for the two closed linear operators A and B and $D(B) \supset D(A)$, then $R_\lambda(A) - R_\lambda(B) = R_\lambda(B)(A - B)R_\lambda(A)$.

†For smooth perturbations f(t) the solution $x(t) \in D(A)$ $(t > 0)$. We can avoid such restrictions as was done in [20].

Integrating this inequality from zero to $t > 0$ and writing $\rho_t^2 = \int_0^t |x(s)|^2 ds$, $\eta^2 = \int_0^\infty |f(t)|^2 dt$, we obtain

$$\varepsilon\rho_t^2 - \|H\|\eta\rho_t - V[x(0), c(0)] \leq -V[x(t), c(t)] \leq 0.$$

From this it follows, by obvious estimates [13], that the quantity $\rho^2 = \int_0^\infty |x(t)|^2 dt$ is finite. Since the right-hand side of (6) is a quadratic form in x(t), the limit $\lim_{t\to\infty} c(t)$ exists and is finite.

The theorem is proved.

LITERATURE CITED

1. A. G. Butkovskii, Methods of Control of Systems with Distributed Parameters [in Russian], Nauka (1975).
2. Ya. Z. Tsypkin, Adaptation and Learning in Automatic Systems [in Russian], Nauka (1968).
3. B. N. Petrov, S. Yu. Rutkovskii, I. N. Krutova, and S. D. Zemlyakov, Principles of the Construction and Design of Self-Adjusting Control Systems [in Russian], Mashinostroenie (1972).
4. E. P. Zhivoglyadova (editor), Adaptive Systems and Estimation of Parameters of Distributed Plants [in Russian], Ilim, Frunze (1970).
5. V. A. Brusin, "Construction of adaptive control systems using methods in the theory of absolute stability," in: Systems Dynamics [in Russian], No. 12, GGU, Gorki (1977), pp. 67-73.
6. D. P. Derevitskii and A. M. Tsykunov, "Adaptive control of a dynamic object with distributed parameters," in: Problems in Cybernetics. Adaptive Systems [in Russian], Nauchn. Sov. Kompleks. Probl. Kibern., Moscow (1976), pp. 134-136.
7. V. A. Yakubovich, "The method of recursive goal inequalities in the theory of adaptive systems," in: Problems in Cybernetics, Adaptive Systems [in Russian], Nauchn.Sov.Komplesk. Probl.Kibern., Moscow (1976), pp. 32-63.
8. A. G. Butkovskii, Structure Theory of Distributed Systems [in Russian], Nauka (1977).
9. V. A. Yakubovich, "A frequency theory in control theory," Sib. Mat. Zh., No. 2, 384-419 (1973).
10. A. L. Likhtarnikov and V. A. Yakubovich, "A frequency theorem for continuous one-parameter semigroups," Izv. Akad. Nauk SSSR, Ser. Mat., 41, No. 4, 895-911 (1977).
11. D. P. Lindorff and R. L. Carroll, "Survey of adaptive control using Lyapunov design," Int. J. Control, 18, No. 5, 897-914 (1973).
12. R. V. Monopoli, "The Kalman–Yacubovitch lemma in adaptive control system design," IEEE Trans. AC-18, No. 5, 526-529 (1973).
13. A. L. Fradkov, "Design of an adaptive system for stabilizing a linear dynamic object," Avtom. Telemekh., No. 12, 96-103 (1974).
14. S. G. Krein (editor), Functional Analysis (SMB) [in Russian], Nauka (1972).
15. A. Balakrishnan, Introduction to the Theory of Optimization in Hilbert Space [Russian translation], Mir (1974).
16. A. L. Fradkov, "The velocity gradient scheme and its use in adaptive control problems," Avtom. Telemekh., No. 9, 90-101 (1979).
17. A. G. Butkovskii, Characteristics of Systems with Distributed Parameters [in Russian], Nauka (1979).
18. V. Ya. Katkovnik and R. A. Poluéktov, Multidimensional Discrete Control Systems [in Russian], Nauka (1966).
19. L. O. Barsuk and V. A. Brusin, "Investigation of stability of nonlinear distributed systems when the limiting inequality is satisfied. I," in: Systems Dynamics [in Russian], No. 9, GGU, Gorki (1976), pp. 3-14.
20. A. L. Likhtarnikov, "Criteria for absolute stability of nonlinear operator equations," Izv. Akad. Nauk SSSR, Ser. Mat., 41, No. 5, 1064-1083 (1977).