SYNTHESIS OF ADAPTIVE SYSTEM OF STABILIZATION OF LINEAR DYNAMIC PLANTS

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The synthesis is considered of an adaptive system of stabilization of linear dynamic plants. Necessary and sufficient conditions of adaptivity are obtained for the existence of a quadratic Lyapunov function in the synthesized system. It is shown that the adaptivity of the system is preserved under the action of plant disturbances that decay in time. Examples are presented that illustrate the use of the obtained results.

1. Introduction

Let us consider a control system described by the equations*

$$dx/dt = Ax + bu + f(t),\ y = L^*x, \tag{1}$$

$$u = c^*y. \tag{2}$$

Here **x**, **y**, **f**, **b**, and **c** are real vectors of dimension n, l, n, n, and l respectively; A and L are matrices of dimension $n \times n$ and $n \times l$; u is a scalar. Equations (1) describe the dynamics of a controlled plant in the interval $0 \le t < \infty$ (u = u(t) is the control, **y** = **y**(t) is the vector of output variables, **x** = **x**(t) is the state of the plant, and **f**(t) is a disturbance); Eq. (2) describes the controller. We shall assume that A, **b**, and L do not vary in time, and that $\int_0^\infty \|f(t)\|^2 dt < \infty$ (i.e., the disturbances decay in time). By $\chi(\lambda)$ we shall denote the $(l \times 1)$ transfer matrix of the plant:

$$\chi(\lambda) = L^*(\lambda I_n - A)^{-1}b,$$

where I_n is a unit matrix of order n, and λ is a complex variable.

It is required to stabilize the system (1)-(2) in indeterminate conditions, i.e., in the case that various parameters of the plant (1) and the disturbance **f**(t) are unknown. Since the initial vector of coefficients of the controller **c**(0) may not ensure the stability of the system, it is necessary to adjust the controller. The adjustment is realized with the aid of a continuous adaptation (self-adjustment) algorithm

$$dc/dt = F(y). \tag{3}$$

As is usual in the adaptive approach [1], the missing information about the required control law can be extracted from observations at the output of the plant during its normal operation. It is evident that "good" performance of the stabilization system signifies that with the passage of time the vector **x**(t) becomes small, whereas the coefficients of the controller **c** (t) are "frozen." Let us give a precise definition.

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*The vectors are column vectors, the asterisk denotes transposition, and $\|\cdot\|$ is the Euclidean norm of a vector or matrix.

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Following [2], we shall assume that $A = A(\xi)$, $\mathbf{b} = \mathbf{b}(\xi)$, $L = L(\xi)$, $\mathbf{f}(t) = \mathbf{f}_\xi(t)$, and $\xi \in \Xi$, where ξ is the set of all unknown parameters, and Ξ is a known set of feasible values of ξ. Let $\mathbf{x}(0)$ be the initial state of the plant.

Definition 1. A stabilization system (1)-(3) is said to be adaptive in the class Ξ, if for any $x(0)$, $c(0)$, and any $\xi \in \Xi$ the solution $\{\mathbf{x}(t), \mathbf{c}(t)\}$ of the differential equations (1)-(3) is defined for any $t \geq 0$ and it satisfies the conditions

1) $\lim_{t\to\infty} \mathbf{x}(t) = \mathbf{0}$,

2) there exists a finite limit $\lim_{t\to\infty} \mathbf{c}(t)$.

Thus the synthesis of an adaptive stabilization systems involves the determination of a function $F(y)$ that does not depend on $\xi \in \Xi$, and such that the conditions 1 and 2 are satisfied.

In this paper we shall find the necessary and sufficient conditions of existence, in the system (1)-(3), of a Lyapunov function

$$V(\mathbf{x}, \mathbf{c}) = \mathbf{x}^* H_0 \mathbf{x} + (\mathbf{c} - \mathbf{c}_0)^* H_1 (\mathbf{c} - \mathbf{c}_0) \tag{4}$$

that has the following properties: a) $V(\mathbf{x}, \mathbf{c}) > 0$ for $\mathbf{x} \neq 0$, $\mathbf{c} \neq \mathbf{c}_0$; b) $\dot{V}(\mathbf{x}, \mathbf{c}) < 0$ for $x \neq 0$, $\mathbf{f}(t) \equiv 0$, where $H_0(H_1)$ is a real symmetric matrix of order $n(l)$, $\mathbf{c}_0$ is an l-dimensional vector, and $\dot{V}(\mathbf{x}, \mathbf{c})$ is the derivative of the function (4) by virtue of the system (1)-(3). These conditions are also conditions of adaptivity in the corresponding class. They are formulated in terms of the transfer matrix $\chi(\lambda)$ of the plant.

Let us note that similarly formulated problems are examined in the theory of searchless self-adjusting systems (SSS) with a reference model [3-7]. Moreover, the idea of using a Lyapunov function (4) for the construction of an adaptation algorithm has been adopted by the author from well-known papers [3-5]. In [3-7], however, the only case under consideration is $l = n$, $L = I_n$, which signifies that all the components of the state vector of the plant are observable. The procedure of synthesis of the adaptation algorithm based on the results of [3-7] involves finding the matrix H_0 in (4) from the relation $H_0 A_0 + A_0^* H_0 = -Q$, where $A_0 = A + \mathbf{b}\mathbf{c}_0^* L^*$, and $Q = Q^* > 0$ is an assigned matrix. Here the matrix A_0 must be known. The procedure proposed by us does not require that the matrix A_0 be known; on the other hand it comprises all the algorithms that can be obtained with the aid of Lyapunov functions of the form (4). Finally, in contrast to [3-7], our algorithms do not require integration of the differential equation of the "reference model," so that their realization can be simplified.

A similar problem was also considered in [8] and [9], where, however, u(t) is a piecewise-constant function and the algorithms of control and adaptation operate in discrete time, and they differ in form from (2) and (3).

The examples presented in Sections 3 and 4 show that in the synthesis of an adaptation algorithm on the basis of the obtained results, the required a priori information about the plant is in many cases very small, i.e., the adaptivity class Ξ is sufficiently large.

2. Formulation of Results

By $\alpha(\lambda)$ we shall denote the numerator of the $(l \times 1)$ transfer matrix of the plant, i.e., of the matrix $\alpha(\lambda) = \chi(\lambda)\delta(\lambda)$, where $\delta(\lambda) = \det(\lambda I_n - A)$. It is evident that the elements of $\alpha(\lambda)$ are polynomials of degree not higher than $n-1$.

For a $V(\mathbf{x}, \mathbf{c})$ of the form (4) we have

$$\frac{1}{2}\dot{V}(\mathbf{x}, \mathbf{c}) = \mathbf{x}^* H_0 (A + \mathbf{b}\mathbf{c}^* L^*)\mathbf{x} + (\mathbf{c} - \mathbf{c}_0)^* H_1 \mathbf{F}(\mathbf{y}) + \mathbf{x}^* H_0 \mathbf{f}(t).$$

As usual, the polynomial $\tau(\lambda) = \tau_0 + \tau_1\lambda + \ldots + \tau_k\lambda^k$ will be called a Hurwitz polynomial of degree k if $\tau_k \neq 0$ and all the roots of the equation $\tau(\lambda) = 0$ have negative real parts. The principal result of this paper can be formulated as a theorem.

Theorem 1. Suppose that the plant transfer matrix $\chi(\lambda) = \alpha(\lambda)/\delta(\lambda)$ does not identically vanish. For the system (1)-(3) to have a Lyapunov function (4) that satisfies the conditions a) $V(\mathbf{x}, \mathbf{c}) > 0$ for $\mathbf{x} \neq 0$, $\mathbf{c} \neq \mathbf{c}_0$; and b) $\dot{V}(\mathbf{x}, \mathbf{c}) < 0$ for $\mathbf{x} \neq 0$ and for $\mathbf{f}(t) \equiv 0$, it is necessary and sufficient that the adaptation algorithm (3) have the form

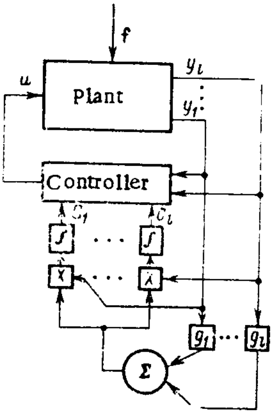

Fig. 1

$$dc/dt = -(g^*y)Py, \tag{6}$$

where $P = P^*$ is a positive-definite matrix of dimension $l \times l$, and the l-dimensional vector g is such that $g^*\alpha(\lambda)$ is a Hurwitz polynomial of degree n−1 with positive coefficients.

The proof of Theorem 1 is presented in Appendix I. It is based on frequency conditions of existence of a solution of matrix inequalities (the so-called "Yakubovich–Kalman lemma" [11]) which appeared for the first time in investigations of the absolute stability of nonlinear control systems [10, 12].

From Theorem 1 it is easy to obtain the following theorem which gives us the conditions of adaptivity of the stabilization system (1)-(3) in the sense of Definition 1.

Theorem 2. Suppose that the adaptation algorithm (3) has the form (6), where $P = P^*$ is a positive-definite $(l \times l)$ matrix, and g is a given l-dimensional vector. Then the system (1)-(3) will be adaptive in a given class Ξ, if for any $\xi \in \Xi$ the polynomial $g^*\alpha_\xi(\lambda)$ is Hurwitzian of degree n−1 with positive coefficients and $\int_0^\infty \|f_\xi(t)\|^2 dt < \infty$. In this case, $\int_0^\infty \|x(t)\|^2 dt < \infty$.

The proof of Theorem 2 is presented in Appendix II. By taking P as a diagonal matrix, it is possible to simplify the adaptation algorithm. In particular, for $P = I_l$ the algorithm (6) assumes the form

$$dc/dt = -(g^*y)y. \tag{7}$$

A block diagram of an adaptive stabilization system that realizes the algorithm (7) is presented in Fig. 1.

Remark 1. The system (1), (2), (6) is structurally stable in the sense that the adaptivity of the system is not violated by small variations of the parameters of the plant (1) (with preservation of order of magnitude), or by small variations of the parameters of the adaptation algorighm (6).

Remark 2. Let us note that if the conditions of Theorem 1 and 2 are satisfied, the original control system (1)-(2) will become stable if the controller coefficients are taken in the form $c = -\varkappa g$, where $\varkappa > 0$ is sufficiently large. Thus the system (1)-(2) will belong to the class of configurations that are stable for farily large feedback gains (this class of systems has been considered in detail in [13]). In spite of this it is often less preferable to use in this case rigid ("nonadaptive") feedback. Indeed, the minimum value of the controller gain that would ensure system stability is not known, since it depends on unknown plant parameters. On the other hand to choose the gain "with a margin" is often inacceptable in the practical realization of a control system. An adaptive control system finds automatically the controller gains that ensure stability and adjusts them if the plant parameters vary.

Now let us consider examples of solution of the problem of synthesis of an adaptive stabilization system on the basis of the obtained results.

3. Particular Case

Suppose that the control system is described by the equations

$$A(p)\sigma = B(p)u + \varphi(t), \tag{8}$$

$$u = C(p)\sigma, \tag{9}$$

where

$$A(p) = p^n + \sum_{i=1}^{n-1} a_i p^i, \quad B(p) = \sum_{i=0}^{k} b_i p^i, \quad C(p) = \sum_{i=0}^{l-1} c_i p^i, \quad p = d/dt$$

is the differentiation operator ($p^i = d^i/dt^i$), and u, σ, and φ are scalars with $\int_0^\infty \varphi^2(t)\,dt<\infty$. Here the observable output of the plant is the vector $y = (\sigma, \dot\sigma, \ldots, \sigma^{(l-1)})^*$. The transfer matrix has the form $\chi(\lambda) = B(\lambda)/A(\lambda) \times (1, \lambda, \ldots, \lambda^{l-1})^*$.

It is well known that for $k+l \leq n$ the system (8)-(9) can be reduced to the form (1)-(2). The adaptation algorithm (6) assumes the form

$$dc/dt = -[G(p)\sigma]Py, \tag{10}$$

where $c = (c_0, \ldots, c_{l-1})^*$, $G(p) = \sum_{i=0}^{l-1} g_i p^i$, and $P = P^*$ is a positive-definite matrix of dimension $l \times l$. In the case under consideration we have $g^*\alpha(\lambda) = G(\lambda)B(\lambda)$, i.e., the polynomial $g^*\alpha(\lambda)$ is Hurwitzian if $G(\lambda)$ and $B(\lambda)$ are Hurwitzian.

It follows from Theorem 2 that the system (8)-(10) is adaptive for any minimum-phase plant (8); if $k+l = n$, the polynomial $G(\lambda)$ will be Hurwitzian and the coefficients of $B(\lambda)$ and $G(\lambda)$ will have the same sign. Let us note that the condition $k+l = n$ signifies that it is necessary to observe the maximum possible number of derivatives of $\sigma(t)$ which is equal (with σ also taken into account) to the difference of the orders of the numerator and denominator of the transfer function of the plant from u to σ.

Thus for the synthesis of an adaptive system of stabilization of the plant (8) on the basis of Theorem 2, it is not necessary to know the order of the differential equation of the plant. It suffices to know the difference between the degrees of the polynomials $A(\lambda)$ and $B(\lambda)$, i.e., the quantity $n-k$.

For $P = I_l$, the algorithm (10) can be written in simple form, convenient for analog-computer realization:

$$dc_i/dt = -[G(p)\sigma]p^i\sigma \quad (i=0, 1, \ldots, l-1). \tag{11}$$

4. Examples of Synthesis of Adaptive Stabilization Algorithms

Example 1. On the basis of Theorem 2 let us synthesize an adaptive system of stabilization of a flying vehicle with respect to the pitch channel. With a number of simplifying assumptions it is possible to describe the motion of a flying vehicle with respect to the center of gravity, for the pitch angle, by the following equation [14]:

$$(p^2+a_1p+a_2)p\vartheta = -(b_1p+b_2)\delta_e+\varphi(t), \tag{12}$$

where ϑ is the pitch angle, δ_e is the angle of deviation of the elevator, a_1, a_2, b_1, and b_2 are constant coefficients, with $b_1, b_2 > 0$, and $\varphi(t)$ is a disturbance. Suppose that the control law has the form

$$\delta_e = c_1\vartheta + c_2p\vartheta, \tag{13}$$

where c_1 and c_2 are feedback factors. In this case $y = (\vartheta, p\vartheta)^*$, and the algorithm (11) can be written in the form

$$dc_1/dt = \gamma(\vartheta + \alpha p\vartheta)\vartheta, \qquad dc_2/dt = \gamma(\vartheta + \alpha p\vartheta)p\vartheta, \tag{14}$$

where $\gamma > 0$ and $\alpha > 0$.

It follows from Theorem 2 that the synthesized system (12)-(14) is adaptive for any values of the unknown coefficients a_1, a_2, b_1, b_2 ($b_1, b_2 > 0$) and for $\varphi(t) \in L_2(0, \infty)$.

For checking the operability of the proposed method, we simulated the system (12)-(14) on a BESM-4 computer under the following conditions: $b_1 = 100$, $b_2 = 250$, $\alpha = 4$, $\gamma = 10$, $c_1(0) = c_2(0) = 0$, and $\varphi(t) = \exp(-3t)$.

We carried out experiments for various values of a_1 and a_2 corresponding to unstable plants (12), and for various initial conditions of the plant. The simulation results showed that the process of adaptation of the controller coefficients converges must faster than the transient process in the plant. In all the experiments the transient time varied in an interval from 2 to 10 seconds, whereas the controller coefficients varied between 0.2 and 1 seconds. The experiments also showed that after the completion of the process of adaptation of the controller, the output variables of the plant satisfy with great accuracy the equation $\vartheta(t) + \alpha p\dot\vartheta(t) = 0$. Thus the equation $\vartheta + \alpha p\dot\vartheta = 0$ (in the general case the equation $G(p)\sigma = 0$) can be interpreted as the equation of the "reference model." Its coefficients must be selected on the basis of the desired performance of the control system after adaptation.

Example 2. Let us consider a second-order plant with two outputs ($n = l = 2$). Its transfer matrix is

$$\chi(\lambda) = \frac{1}{\lambda^2 + a_1\lambda + a_2}\begin{pmatrix} b_{11}\lambda + b_{12} \\ b_{21}\lambda + b_{22} \end{pmatrix}. \tag{15}$$

Suppose that the class Ξ is defined by the conditions $b_{ij}^- \le b_{ij} \le b_{ij}^+$, where the $b_{ij}^\pm \neq 0$ are known ($i, j = 1, 2$). This means that the denominator of the transfer function is completely unknown, whereas for the amplifier coefficients we have only some estimates. For this case it is easy to write down the adaptation algorithm (6); it follows from Theorem 2 that the condition of adaptivity of the system in the class Ξ is the fulfillment of the inequalities

$$g_1 b_{11} + g_2 b_{21} > 0, \qquad g_1 b_{12} + g_2 b_{22} > 0 \tag{16}$$

for any $b_{ij}^- \le b_{ij} \le b_{ij}^+$ ($i, j = 1, 2$), where $g = \begin{pmatrix} g_1 \\ g_2 \end{pmatrix}$ is determined from formula (6). It is easy to show that it suffices to solve the inequality (16) for $b_{ij} = b_{ij}^\pm$; a solution of (16) will exist if and only if the convex hull of eight two-dimensional vectors of the form

$$\begin{pmatrix} b_{11}^\pm \\ b_{21}^\pm \end{pmatrix}, \quad \begin{pmatrix} b_{12}^\pm \\ b_{22}^\pm \end{pmatrix}$$

does not contain the origin of coordinates. Thus the synthesis of the adaptation algorithm reduces to solving a system of eight linear inequalities.

In the same way we can consider the case $n = l = 3$. In this case the number of inequalities in the system will be equal to 24.

APPENDIX I

Proof of Theorem 1. Let us note first of all that the condition $V(\mathbf{x}, \mathbf{c}) > 0$ for $\mathbf{x} \neq 0$ and $\mathbf{c} \neq \mathbf{c}_0$ is equivalent* to $H_0 > 0$ and $H_1 > 0$. For $\mathbf{f}(t) \equiv 0$, it is possible to write formula (5) for $\dot{V}(\mathbf{x}, \mathbf{c})$ in the form

$$\dot{V}(\mathbf{x}, \mathbf{c}) = \mathbf{x}^*(H_0A_0 + A_0^* H_0)\mathbf{x} + 2(\mathbf{c} - \mathbf{c}_0)^*[H_1\mathbf{F}(\mathbf{y}) + \mathbf{x}^* H_0 \mathbf{b}\mathbf{y}], \tag{A.1}$$

where $A_0 = A + \mathbf{b}\mathbf{c}_0^* L^*$. By virtue of the linearity, in $\mathbf{c}$, of the right-hand side of (A.1), the condition $\dot{V}(\mathbf{x}, \mathbf{c}) < 0$ for $\mathbf{x} \neq 0$ is equivalent to the relations $H_0A_0 + A_0^*H_0 < 0$, $\mathbf{F}(\mathbf{y}) = -\mathbf{x}^* H_0 \mathbf{b} H_1^{-1}\mathbf{y}$. Since $\chi(\lambda) \not\equiv 0$, it follows that $\mathbf{y} \not\equiv 0$, and hence $\mathbf{x}^* H_0\mathbf{b}$ = const for $\mathbf{y}$ = const, which means that the equation $H_0\mathbf{b} = Lg$ is valid for an l-dimensional vector g. Thus the conditions (a) and (b) of Theorem 1 are equivalent to the fact that the adaptation algorithm is specified by Eq. (6), where $P = H_1^{-1} > 0$, and the vector g satisfies the relations

$$A_0 = A + \mathbf{b}\mathbf{c}_0^* L^*, \qquad H_0A_0 + A_0^* H_0 < 0, \qquad H_0\mathbf{b} = Lg. \tag{A.2}$$

Thus we obtain the following algebraic problem. We are given matrices and vectors A, b, L, and g. It is required to find the conditions of existence of the matrix $H_0 = H_0^* > 0$ and of an l-dimensional vector $\mathbf{c}_0$ that satisfy (A.2). The solution of this problem is given by the next lemma which yields directly the assertion of the theorem.

Lemma 1. Suppose that the matrix $\chi(\lambda) = L^*(\lambda I_n - A)^{-1}\mathbf{b}$ of dimension $l \times 1$ is expressed in the form $\chi(\lambda) = \alpha(\lambda)/\delta(\lambda)$, where $\delta(\lambda) = \det(\lambda I_n - A)$, and $\alpha(\lambda)$ is a matrix of dimension $l \times 1$ consisting of polynomials of degree not higher than $n-1$. For the existence of a matrix $H_0 = H_0^* > 0$ and of a vector $\mathbf{c}_0$ that satisfy (A.2), it is necessary and sufficient that the polynomial $g^*\alpha(\lambda)$ be a Hurwitz polynomial of degree $n-1$ with a positive leading coefficient.

For proving Lemma 1, we shall need the following assertion which can be easily verified with the aid of the Routh-Hurwitz determinant criterion.

Lemma 2. Let $P(\lambda)$ be a polynomial of degree n, and let $Q(\lambda)$ be a Hurwitz polynomial of degree $n-1$. Then the polynomial $P(\lambda) + \varkappa Q(\lambda)$ will be Hurwitzian for all sufficiently large positive $\varkappa$ if the signs of the leading coefficients of $P(\lambda)$ and $Q(\lambda)$ coincide.

*The notation $H > 0$ signifies that the symmetric matrix H is positive definite, i.e., $z^*Hz > 0$ for $z \neq 0$.

Proof of Lemma 1. Let us introduce the notation $\delta_0(\lambda) = \det(\lambda I_n - A_0)$, $\chi_0(\lambda) = L^*(\lambda I_n - A_0)^{-1}\mathbf{b}$. It is easy to show that $\delta_0(\lambda) = \delta(\lambda) - c_0^*\alpha(\lambda)$, $\chi_0(\lambda) = \alpha(\lambda)/\delta_0(\lambda)$. It follows from the results of [10] (Theorem 2) that with fixed A_0 it is necessary and sufficient for the existence of a matrix $H_0 = H_0^* > 0$ satisfying (A.2) that the following conditions hold:

1) $\delta_0(\lambda)$ is a Hurwitz polynomial,

2) $\operatorname{Re} g^*\chi_0(i\omega) > 0$ for any $\omega \in (-\infty, +\infty)$,

3) $\lim\limits_{\omega\to\infty} \omega^2 \operatorname{Re} g^*\chi_0(i\omega) > 0$.

At first we shall show that from the fulfillment of conditions 1, 2, and 3 it follows that the polynomial $\tau(\lambda) = g^*\alpha(\lambda)$ is Hurwitzian of degree n−1, and it has a positive leading coefficient. Indeed, $\tau(\lambda) = g^*\chi_0(\lambda)\delta_0(\lambda)$ therefore $\Delta \operatorname{Arg} \tau(i\omega) = \Delta \operatorname{Arg} g^*\chi_0(i\omega) + \Delta \operatorname{Arg} \delta_0(i\omega)$, where $\Delta \operatorname{Arg} \psi(\omega)$ is the increment of the argument of the function $\psi(\omega)$ for a ω that varies from $-\infty$ to $+\infty$. By virtue of Mikhailov's criterion it follows from condition 1 that $\Delta \operatorname{Arg} \delta_0(i\omega) = n\pi$. From condition 2 it follows that $|\Delta \operatorname{Arg} g^*\chi_0(i\omega)| \le \pi$, i.e., $\Delta \operatorname{Arg} \tau(i\omega) \ge (n-1)\pi$. But $\tau(\lambda)$ is a polynomial of degree not higher than n−1; hence $|\Delta \operatorname{Arg} \tau(i\omega)| \le (n-1)\pi$.

Thus, $\Delta \operatorname{Arg} \tau(i\omega) = (n-1)\pi$. But this signifies that the polynomial $\tau(\lambda)$ is Hurwitzian, its degree is equal to n−1, and all its coefficients are positive, i.e., the necessary of the conditions of the lemma has been proved.

Now let us prove the sufficiency of the conditions of Lemma 1. Let $g^*\alpha(\lambda) = \tau(\lambda) = \sum_{i=0}^{n-1} \tau_i\lambda^i$ be a Hurwitz polynomial, with $\tau_{n-1} = g^*Lb > 0$. For proving the sufficiency it suffices to take the vector $\mathbf{c}_0$ in such a way that the conditions 1-3 are satisfied. Let us show that such a vector can be taken in the form $\mathbf{c}_0 = -\varkappa g$, where $\varkappa > 0$ is sufficiently large. Indeed, from Lemma 2 and from the equation $\delta_0(\lambda) = \delta(\lambda) + \varkappa\tau(\lambda)$ we obtain condition 1. For proving condition 2, let us note that $\tau(i\omega) \ne 0$ and $\delta(i\omega) + \varkappa\tau(i\omega) \ne 0$ for sufficiently large $\varkappa$ for any $\omega \in (-\infty, +\infty)$. Therefore condition 2 will be equivalent to the inequality $\operatorname{Re}[g^*\chi_0(i\omega)]^{-1} > 0$. It is evident that $\operatorname{Re}[g^*\chi_0(i\omega)]^{-1} = \operatorname{Re}[1 + \varkappa g^*\chi(i\omega)][g^*\chi(i\omega)]^{-1} = \varkappa + \operatorname{Re}(\delta(i\omega)/\tau(i\omega))$; therefore it suffices to show that the quantity $\operatorname{Re}(\delta(i\omega)/\tau(i\omega))$ is bounded for $\omega \to \pm\infty$. But for $\omega \to \pm\infty$ we have $\operatorname{Re}(\delta(i\omega)/\tau(i\omega)) = \operatorname{Re}(i\omega/\tau_{n-1} + O(1)) = O(1)$, i.e., the condition 2 is satisfied. Finally, the fulfillment of condition 3 follows from the equations $\lim\limits_{\omega\to\infty} \omega^2 \operatorname{Re} g^*\chi_0(i\omega) = -g^*L^*A_0b = -g^*L^*Ab + \varkappa\tau_{n-1}$.

This completes the proof of Lemma 1, and hence also of Theorem 1.

APPENDIX II

Proof of Theorem 2. Let $\int_0^\infty \|\mathbf{f}(t)\|^2\,dt = \eta^2 < \infty$. From the condition of the theorem and from Theorem 1 it follows that $\dot V(\mathbf{x}, \mathbf{c}) = -\mathbf{x}^*Q\mathbf{x} + \mathbf{x}^*H_0\mathbf{f}(t)$, where $Q = Q^* > 0$ and $H_0 = H_0^* > 0$ are matrices of dimension $n \times n$. Therefore $\dot V(\mathbf{x}(t), \mathbf{c}(t)) \le -\varepsilon\|\mathbf{x}(t)\|^2 + \|H_0\|.\|\mathbf{x}(t)\|\,\|\mathbf{f}(t)\|$ for a positive ε, and hence $V(\mathbf{x}(t), \mathbf{c}(t)) - V(\mathbf{x}(0), \mathbf{c}(0)) \le -\varepsilon\rho_t^2 + \|H_0\|\rho_t\eta$, where $\rho_t^2 = \int_0^t \|\mathbf{x}(s)\|^2\,ds$.

Since $V(\mathbf{x}, \mathbf{c}) \ge 0$, it follows that

$$\varepsilon\rho_t^2 - \|H_0\|\eta\rho_t \le V(\mathbf{x}(0), \mathbf{c}(0)). \tag{A.3}$$

By solving (A.3) for ρ_t, we obtain $\rho_t \le \eta\|H_0\|/\varepsilon + \sqrt{V(\mathbf{x}(0), \mathbf{c}(0))/\varepsilon}$, i.e., $\int_0^\infty \|\mathbf{x}(t)\|^2 dt < \infty$. But the right-hand side of differential equation (6) is a quadratic form of the vector $\mathbf{x}$; hence there exists $\lim\limits_{t\to\infty} \mathbf{c}(t)$ and the vector function c(t) is bounded in the interval [0, ∞). Next, for any positive t we have

$$\|\mathbf{x}(t)\|^2 = \frac{1}{2}\int_0^t \mathbf{x}^*(s)\frac{d\mathbf{x}(s)}{ds}\,ds = \frac{1}{2}\int_0^t x^*(s)\,[A\mathbf{x}(s) + bc^*(s)L\mathbf{x}(s) + \mathbf{f}(s)]\,ds. \qquad (A.4)$$

Since the integral in the right-hand side of (A.4) is convergent, there exists $\lim_{t\to\infty}\|\mathbf{x}(t)\|^2=\rho$. But $\int^{\infty}\|\mathbf{x}(t)\|^2\,dt < \infty$, so that $\rho = 0$, and we have proved that the synthesized system is adaptive.

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