A Nonlinear Philosophy for Nonlinear Systems

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Abstract

A framework for system analysis and design is described based on nonlinear system models and nonperiodic signals generated by nonlinear systems. The proposed approach to analysis of nonlinear systems is based on *excitability index* - a nonlinear counterpart of magnitude frequency response of linear system. It can be used for stability analysis of fully nonlinear cascade systems similarly to absolute stability analysis of Lur'e systems. Speed-Gradient algorithms of creating feedback resonance in nonlinear multi-DOF oscillators are described. For strictly dissipative systems bounds of energy and excitability change by feedback are established.

Keywords: Nonlinear systems, stability, excitability, passivity.

1 Introduction

A few last decades of the XXth century were marked by enormous spread of nonlinear models over various fields of natural sciences and engineering, including control theory. The main reason of using nonlinear system description is that complex system behavior in many cases cannot be understood and explained within a linear framework. However, nonlinear theories have often been developed as direct extensions of corresponding linear ones. Treating linear systems as the simplest and the most important ones has become a cornerstone of education in a majority of disciplines. Similarly, harmonic signals are conventionally treated as the simplest and the most important ones.

A good example of such a line of development in control theory is provided by absolute stability theory. It started half a century ago with seminal contributions of A.Lur'e, M.Aizerman and others [1, 2]. Absolute stability study is based on splitting a system model into linear and nonlinear parts. Properties of the overall system are then derived from the properties of the linear part (its frequency response, i.e. reaction to a harmonic signal) and properties of the set, containing the graph of the nonlinearity. Such an approach has been extended later to analysis of properties different from stability: instability, oscillations, robustness, etc., see, e.g. [3, 4] and many texts by Western authors.

In the 1980s a new idea was introduced by C.Byrnes and A.Isidori [5] - to develop a "frequency-domain" philosophy directly for nonlinear systems. Such important notions of linear control theory as canonical forms, left and right half plane zeros, relative degree were extended to nonlinear systems. In a few years the idea turned into a systematic analysis and design methodology [6], based also upon some previous studies of passivity and dissipativity concepts [7, 8, 9, 10]. Such a framework was readily accepted by control community and, along with the feedback linearization approach and iterative designs (backstepping) formed the basis of nonlinear control theory of the 1990s.

However in some real life applications frequency-domain philosophy and feedback linearization may loose their power. It happens, e.g., when studying systems under input constraints. Moreover, frequency-domain characteristics are sometimes helpless for nonlinear systems because of lack of superposition principle and evidence of complex (even chaotic) behavior under harmonic excitation. It means that harmonic signals cannot serve as universal testing signals when studying nonlinear systems.

Perhaps, it is time to change the paradigm and to develop a "totally nonlinear philosophy" where the role of linear nominal systems and harmonic signals will not be as fundamental as before.

The purpose of this paper is to outline one possible "purely nonlinear" framework for system analysis (It was first introduced in [11]). In Section 2 an absolute stability approach and circle stability criterion are recalled. The proposed framework is based on the concepts of *feedback resonance* and *excitability index* first introduced in [12, 13]. These concepts are recalled in Section 3. In Section 4 lower and upper bounds for excitability index and a new interpretation of circle criterion for nonlinear nominal system are given. Section 5 contains a sketch of a "nonlinear" approach to the theory of signals.

2 Excitablity of linear systems

First of all we need to find a substitute for magnitude frequency response for nonlinear systems. To this end recall classical absolute stability problem (Lur'e problem) for a system consisting of a linear part described by transfer function

$$y = W(p)u, \quad p = d/dt \tag{1}$$

and a static nonlinearity

$$u = \varphi(y), \tag{2}$$

with graph lying in a symmetric sector

$$|\varphi(y)| \le K_\varphi |y|. \tag{3}$$

Consider SISO case $(u, y \in \mathbb{R}^1)$ for simplicity and assume that $W(p)$ is stable ($W(p)$ has only left poles) . It follows from the circle criterion that the zero equilibrium of the system (1)-(3) is asymptotically stable if the following inequality holds

$$K_W K_\varphi < 1, \tag{4}$$

where

$$K_W = \max_\omega |W(j\omega)|. \tag{5}$$

Note that in order to examine stability of the system (1)-(3) the knowledge of the maximum K_W of the magnitude frequency plot $A(\omega) = |W(j\omega)|$ rather than the values of $A(\omega)$ for all $\omega \in R^1$ is needed. In other words, stability of the closed loop depends on the resonance properties of the linear part. The depth of the resonance depends on the value K_W which can be called *resonance degree* or *excitability* index of linear system (1). In order to determine K_W for a real world system one needs to apply harmonic input signals $u(t) = \sin \omega t$ and to find resonance frequencies ω, realizing

$$\max_\omega \overline{\lim_{t\to\infty}} |y(t)|. \tag{6}$$

Upper limit is used in (6) in order to eliminate influence of the transients.

Note that usage of frequency-domain response (Bode plot) in classical absolute stability criteria in fact does not require knowledge of the whole frequency-domain response as function of frequency. E.g. to apply circle stability criterion (4) one needs only to calculate maximum value of Bode plot over frequency range measuring resonance properties of the linear nominal system.

Now turn to fully nonlinear cascades where the linear part (1) is replaced by

$$\dot{x} = F(x, u), y = h(x) \tag{7}$$

where $x \in R^n$ is state vector. It seems intuitively clear that stability of the overall system (7), (2), (3) should depend on resonance properties of the nonlinear system (7). However, there is well known obstacle for studying of nonlinear resonance: frequency of the forced oscillations depends on the amplitude of the input signal. Therefore, the question arises: how to describe resonance behavior of nonlinear system?

3 Feedback resonance and excitability index

The concept of *resonance* is of utmost importance for physics and mechanics. It has numerous applications in spectroscopy, optics, mechanical engineering, laser and communication technologies, etc. Its essence is that small resonant force applied to a system leads to significant changes in system behavior. First clear description of resonance phenomenon was given by Galileo Galilei in "Discorsi a Dimostrazioni Matemache" (1638) [14]: "...Pendulum at rest although very heavy, can be put into motion, and very significant if we stop our breath when it is coming back and blow again at the instance, corresponding to its swing".

The resonance phenomenon is well understood and perfectly studied for linear systems. If, however, the dynamics of the system is nonlinear, the resonance is much more complicated because interaction of different harmonic signals in nonlinear system may create complex and even chaotic behavior [15]. The reason is, roughly speaking, in that the natural frequency of a nonlinear system depends on the amplitude of oscillations.

In [12, 13] the idea was pursued to create resonance in a nonlinear oscillator by changing the frequency of external action as a function of oscillation amplitude. To implement this idea $u(t)$ should depend on the state of the system $x(t)$ or on the current measurements $y(t)$, which exactly means introducing a state feedback

$$u(t) = U\left(x(t)\right). \tag{8}$$

or output feedback

$$u(t) = U\left(y(t)\right). \tag{9}$$

Now the problem is: how to find the feedback law (8) or (9) in order to achieve the maximum limit amplitude of output?

In [13] this problem was formulated as that of optimal control. We may pose it as finding

$$Q(\gamma) = \limsup_{\substack{|u(s)|\le\gamma,\\ 0\le s\le t,\\ x(0)=0,\\ t\ge 0}} |y(t)|^2. \tag{10}$$

We assume that the system (7) is BIBO stable and $x = 0$ is equilibrium of the unforced system ($F(0,0) = 0, h(0) = 0$) in order to ensure $Q(\gamma)$ to be well defined. Apparently, the signal providing maximum excitation should depend not only on time but also on system state, i.e. input signal should have a feedback form. Note that for linear systems the value of the problem (10) depends quadratically on γ. Therefore it is naturally to introduce the *excitability index* (EI) for the system (7) as follows:

$$E(\gamma) = \frac{1}{\gamma}\sqrt{Q(\gamma)}, \tag{11}$$

where $Q(\gamma)$ is the optimum value of the problem (10). It is clear that for linear asymptotically stable systems $E(\gamma) = \text{const}$. For nonlinear systems $E(\gamma)$ is a function of γ that characterizes excitability properties of the nonlinear system. It was introduced in [13] with respect to the energy-like output. The concept of EI is related to the concepts of *input-to-output* stability and *input–output (I-O) gain* pursued by E.Sontag [17, 18]. E.g. if I-O gain exists, it provides an upper bound for EI. Conversely, if EI is finite, it estimates the minimal value of I-O gain. EI is also related to the hyperstability of V.M.Popov [19] and absolute stability in sense of V.A.Yakubovich [20].

The solution to the problem (10) is quite complicated in most cases. However we can use approximate locally optimal or speed-gradient solution

$$u(x) = \gamma \,\text{sign}\, \left(g(x)^T \nabla h(x) h(x)\right), \tag{12}$$

where $g(x) = \left.\frac{\partial F(x,u)}{\partial u}\right|_{u=0}$, obtained by maximizing the principal part of instant growth rate of $|y(t)|^2$. It follows from the results of [21, 22] that for small γ the value of $|y(t)|$ achievable with input (12) for sufficiently large $t \geq 0$ differs from the optimal value $Q(\gamma)$ by the amount of order γ^2. An important consequence is that excitability index can be estimated directly by applying input (12) to the system. For real world systems it can be done experimentally. Otherwise, if a system model is available, computer simulations can be used. The typical plots of excitability index for pendulum

$$\ddot{\varphi} + \varrho\dot{\varphi} + \omega_0^2 \sin\varphi = u(t), \tag{13}$$

and Duffing system

$$\ddot{\varphi} + \varrho\dot{\varphi} + \omega_0^2(\varphi - (\varphi)^3) = u(t) \tag{14}$$

are shown in Fig.1, Fig.2:

4 Properties of excitability index

Since stability of a closed loop system depends on resonance properties of the nominal system (7), excitability

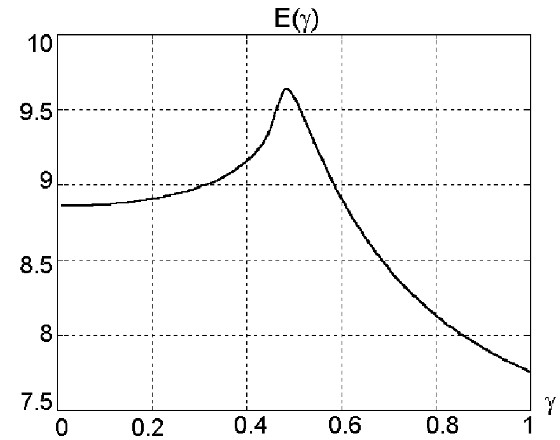

Figure 1: Excitability plot for pendulum ($\varrho = 0.1$, $\omega_0^2 = 10$).

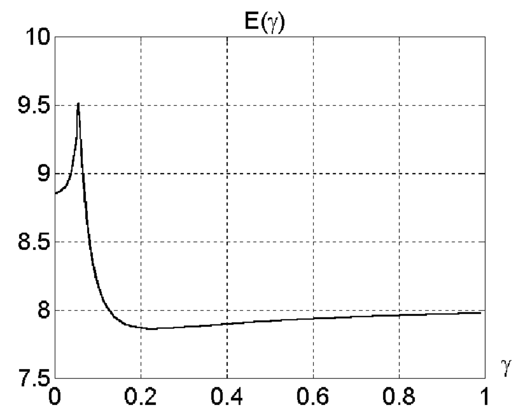

Figure 2: Excitability plot for Duffing system ($\varrho = 0.1$, $\omega_0^2 = 10$).

plots can be used for formulating stability criteria for closed loop and cascade systems. For example we reformulate analogue of circle criterion for systems consisting of a nonlinear nominal part (7) and a static nonlinearity

$$u = \varphi(y), \tag{15}$$

with graph lying in a symmetric sector

$$|\varphi(y)| \leq K_\varphi |y|. \tag{16}$$

Using definition (10), (11), the stability criterion based on passivity theorem (see [7, 10]) is reformulated as follows.

Proposition 1. The system (7), (15), (16) is asymptotically stable if the following inequality holds:

$$K_F K_\varphi < 1. \tag{17}$$

where

$$K_F = \sup_\gamma E(\gamma). \tag{18}$$

Comparing (17) with standard circle criterion we see that the *maximum excitability index* K_F plays the role of maximum frequency-domain response for linear systems.

In the second statement the bounds of excitability index for strictly passive systems (systems with full dissipation) are established. To simplify formulations we will study the averaged excitability index defined as $\overline{E}(\gamma) = \frac{1}{\gamma}\sqrt{\overline{Q}(\gamma)}$, where

$$\overline{Q}(\gamma) = \sup_{\substack{|u(s)|\leq\gamma \\ 0\leq s\leq t,\, t\geq 0 \\ x(0)=0}} \frac{1}{t}\int_0^t y^2(s)ds. \qquad (19)$$

Proposition 2. Let the controlled system (7) be output strictly passive, i.e. there exist functions $V(x) \geq 0, \varrho(y) \geq 0$ and positive numbers $\alpha, \varrho_0, \varrho_1$ such that $\varrho_0|y|^2 \leq \varrho(y) \leq \varrho_1|y|^2, V(x) \geq \alpha|y|^2, V(0) = 0$ and

$$V(x(t)) - V(x(0)) = \int_0^t [u(s)y(s) - \varrho(y(s))]\, ds. \qquad (20)$$

Then the following inequality holds for $x(0) = 0$ and all $t \geq 0$:

$$\int_0^t [y(s)^2]\, ds \leq \varrho_0^{-2} \int_0^t [u(s)^2]\, ds. \qquad (21)$$

If, additionally, $|u(t)| \leq \gamma$, then

$$\frac{1}{t}\int_0^t [y(s)^2]\, ds \leq \left(\frac{\gamma}{\varrho}\right)^2 \qquad (22)$$

and, therefore, $\overline{E}(\gamma) \leq \varrho_0^{-1}$. Besides,

$$\underline{\lim}_{t\to\infty} \sup_{\substack{|u(s)|\leq\gamma \\ 0\leq s\leq t,\, x(0)=0}} V(x(t)) \geq \alpha\left(\frac{\gamma}{\varrho_1}\right)^2. \qquad (23)$$

We see that the action (12) creates a sort of resonance mode in a nonlinear system: for weakly damped systems even a small action having form (12) leads to large oscillations of the output and can insert a substantial amount of energy into the system.

5 Nonlinear theory of signals

Important fields where harmonic signals, linear models and frequency-domain (spectral) methods still play key role are circuits, signals and communications. However, "nonlinear philosophy" may offer some alternatives for those fields as well. Consider a harmonic signal

$$y(t) = a\sin(\omega t + \alpha) \qquad (24)$$

which is treated as the simplest one in conventional theories. The modulation of the signal (24) is understood as changing its parameters a (amplitude), ω (frequency) and α (initial phase). E.g. in case of frequency modulation the parameter ω is changing: $\omega = \omega(t)$. It means that frequency should be called a signal rather than a parameter and may represent a coded message. If the modulated signal $y(t)$ containing coded message is transmitted along a communication channel, it may be demodulated and decoded in the receiver. Extracting the message from the received signal $\hat{y}(t) = y(t) + \xi(t)$, where $\xi(t)$ is channel noise is the main problem of the communication theory. [24].

A lot of discussion in the communication theory in the 90s was related to using broadband signals, e.g. chaotic signals as carriers instead of harmonic signals (24) [25, 26]. In order to develop a general communication theory incorporating both chaotic and conventional harmonic signals (24) we may replace explicit signal description (24) by representing signals as output functions of model systems (signal generators). For example, the signal (24) can be generated by a linear model

$$\ddot{y}(t) + \omega^2 y(t) = 0. \qquad (25)$$

More general nonlinear models (7) allow to generate more general nonperiodic, and even chaotic signals. Important that the input vector $u = u(t) \in \mathbb{R}^m$ can represent both real input signals influencing the system (7) and changes of system parameters, i.e. modulating signals.

The receiver is also represented as a nonlinear dynamical system

$$\dot{z} = \Phi(z, \hat{y}), \quad \hat{u} = \chi(z), \qquad (26)$$

wher $\hat{y} = y(t) + \xi(t)$ is transmitted signal (input of the receiver), $z = z(t) \in \mathbb{R}^n$ is state of the receiver, $\hat{u} = u(t)$ is estimate of the message signal (output of the receiver).

In case of bounded noise the problem of receiver design can be posed as search for a model (26), ensuring the goal

$$|u(t) - \hat{u}(t)| \leq \Delta_u. \qquad (27)$$

If relation (27) cannot be ensured at the initial stage of the process (e.g. in case of uncertain parameters of the transmitter and environment) then the asymptotic goal is posed as follows:

$$\overline{\lim_{t\to\infty}} |u(t) - \hat{u}(t)| \leq \Delta_u. \qquad (28)$$

One possible approach to the receiver design is based on using observers. Then state vector $z(t)$ may contain estimates of the transmitter state, e.g. $z(t) = \hat{x}(t) \in \mathbb{R}^n$. In this case the receiver should generate estimates of both state and parameters of the transmitter, i.e. it should be adaptive observer. Some methods of adaptive observer design for general nonlinear systems can be found in [27, 28, 29].

For example consider the case (popular in the texts devoted to using chaos for information transmission) when both transmitter and receiver systems are implemented as Chua's circuits. Then the transmitter model in dimensionless form is given as:

$$\begin{aligned} \dot{x}_{d_1} &= p(x_{d_2} - x_{d_1} + f(x_{d_1}) + s f_1(x_{d_1})) \\ \dot{x}_{d_2} &= x_{d_1} - x_{d_2} + x_{d3} \\ \dot{x}_{d_3} &= -q x_{d_2} \end{aligned} \quad (29)$$

where $f(z) = M_0 z + 0.5(M_1 - M_0) f_1(z)$, $f_1(z) = |z+1| - |z-1|$, M_0, M_1, p, q are the transmitter parameters, $s = s(t)$ is the signal to be reconstructed in the receiver. Assume for simplicity that (a) transmitted signal is $y_d(t) = x_{d_1}(t)$, (b) values of the parameters p, q are known and (c) channel noise can be neglected. Then the receiver designed by adaptive synchronization is as follows [30]:

$$\begin{aligned} \dot{x}_1 &= p(x_2 - x_1 + f(y_d) + c_1 f_1(y_d) + c_0(x_1 - y_d)), \\ \dot{x}_2 &= x_1 - x_2 + x_3, \\ \dot{x}_3 &= -q x_2, \end{aligned} \quad (30)$$

where c_0, c_1 are the adjustable parameters with the adaptation algorithm [30]

$$\begin{aligned} \dot{c}_0 &= -\gamma_0 (y_d - x_1)^2 - \alpha_0 c_0, \\ \dot{c}_1 &= -\gamma_1 (x_1 - y_d) f_1(y_d) - \alpha_1 c_1, \end{aligned} \quad (31)$$

where γ_0, γ_1 are the adaptation gains. The case of noisy channel is considered in [31].

6 Open problems

Introduction of excitability index for general nonlinear systems opens a broad avenue of research. Let us list only a few unsolved problems on this way.

1. A number of open problems is related to evaluation of EI for different cases. E.g. for Hamiltonian systems with dissipation it is unknown, if the EI with respect to energy is finite when potential energy function is unbounded.

2. Another set of problems is related to systems with disturbances. E.g. when the EI for system under bounded disturbance $v(t)$ ($|v(t)| \leq \delta$) is finite? The answer is not trivial for conservative systems or for systems with incomplete dissipation even for 1-DOF case, see [23]. For systems with bounded disturbances it is interesting to estimate a counterpart of EI - *damping index* $D(\gamma, \delta)$ which may be defined if we replace $\sup_{u(\cdot)}$ by $\inf_{u(\cdot)} \sup_{v(\cdot)}$ in (10) or (19).

3. An important problem is evaluation of error caused by replacing optimal control in (12) with locally optimal one and evaluation of higher order approximations for optimal controller.

4. Perhaps the most exciting line of research is studying relations between dynamical properties of the system and its EI graph. An example of such a relation was observed in the case when the system in question is Hamiltonian system with dissipation and its output is its total energy. Then each peak of EI graph corresponds to escape from some potential well (or crossing of some critical level of the potential energy). It was confirmed by computer simulations [32].

5. Although elements of the "nonlinear signals theory" are already used in various fields, many problems need further study: modulation and demodulation for nonlinearly parametrized systems, generation of chaotic signals with given correlation properties, etc.

Conclusions

The introduced concept of *excitability index* characterizes the depth of the resonance in the nonlinear system achievable by feedback forcing. Excitability index can be used for study of stability and other properties of systems. Its role for nonlinear systems is analogous to that of magnitude frequency response for linear ones. The difference is that the maximum magnitude frequency response is measured by scanning over frequency range, while the excitability index can be measured or calculated by scanning over a range of input amplitudes. Note that the notion of excitability index is most useful for weakly damped (close to conservative) systems, where $E(\gamma)$ can be well determined by measuring a system output response as response to small $u(t)$. The value of $E(\gamma)$ characterizes the damping properties of nonlinear systems.

Examination of the system properties by means of nonperiodic testing signals may become a strong new instrument in the nonlinear systems theory and design. Related instrument in the theory of signals is under development. It is based on replacement of harmonic signal models (generated by linear oscillators) by nonperiodic bounded signals (generated by nonlinear oscillatory systems of form (7)). Then input variables $u(t)$ in (7)) play the role of signal parameters, i.e. modulation becomes a control problem. Modulation, estimation and other operations with signals are described by appropriate parameter adjusting algorithms.

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