ADAPTIVE SYNCHRONIZATION OF HYPER-MINIMUM-PHASE SYSTEMS WITH NONLINEARITIES

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*Abstract.*The problem of synchronizing two systems with models formed of linear and nonlinear parts is considered. The solution for systems with hyper-minimum-phase linear part is given under weakened matching conditions. Application of the proposed technique is demonstrated by example of synchronizing two Chua's circuits.

1. INTRODUCTION

The synchronization of dynamical systems is an interesting phenomenon having numerous applications in mechanics [4,5], communications [31], biology [22]. Some time ago the problem was attacked by control theorists [24,35] who used conventional linear model of systems to synchronize. Of particular interest is the problem of synchronizing two or more systems when not only initial state (phases) but also values of some parameters are not available to the controller designer. This more complicated problem will be refered to as one of adaptive synchronization.

A stream of publications arised quite recently devoted to the problem of synchronizing chaotic oscillations based on strongly nonlinear models, see surveys [8,29,33]. These papers are published mainly in physical or electronical journals treating the corresponding applications. The unified framework for synchronization and control of dynamical (including chaotic) systems based on Lyapunov stability concept is developed in [37]. Another related formulation was suggested in [14] where also the general method of synchronization is described based on the reference model and speed-gradient approaches developed previously in nonlinear afapitve control field [13-18].

The present paper examines the adaptive synchronization further. The problem of synchronizing two systems with models formed of linear and nonlinear parts is considered. The solution for systems with hyper-minimum-phase linear part is given under weakened matching conditions.

The general adaptive and nonadaptive synchronization problems as in [4,14] are reviewed in section 2. Section 3 recalls the speed-gradient method. The main result for the hyper-minimum-phase systems is presented in section 4. An example synchronization of two Chua circuits is given in cection 5.

2. SYNCHRONIZING OSCILLATIONS AND MODEL REFERENCE CONTROL

The problem of (two or more) oscillating processes synchronization is formulated as follows [4,5]. Given equations of r interacting subsystems

$$\dot{x}_i=\overline{F}_i(x_1,\dots x_r,u,t), x\in R_i^{n_i}, i=1\dots r \qquad (2.1)$$

and the equation of connection system

$$\dot{u}=U(x_1,\dots x_r,u,t),\ u\in R^m, \qquad (2.2)$$

find conditions of existence and stability of T-periodic solutions $\overline{x}_i(t)$, $\overline{u}(t)$ of (2.1), (2.2).

In terms of control theory the set of equations (2.1) represent the controlled plant model. Moreover the connection system (2.2) can be regarded as control algorithm while $\overline{x}_i(t)$ is no more than the trajectory of the reference model that is solution of (2.1) for inputs $u_i(t)$ specified as T-periodic functions of time. Therefore the general synchronization problem is formally a special case of the nonlinear reference model control problem for the case when the solution of reference model coincides with the solution of plant model in some T-periodic mode. In synchronization problems the structure of connection system (2.2) is usually given, while some physical parameters ensuring the control aim are to be determined.

Based on the above formulation the adaptive synchronization problem can be posed as follows. Given equations of controlled systems

$$\dot{x}_i=\overline{F}_i(x_1,\dots x_r,u,t,\xi),\ i=1\dots r \qquad (2.3)$$

where $\xi\in\Xi$ is vector of unknown parameters, find the equation of the control algorithm

$$u=U(x_1,\dots x_r,\theta,t)) \qquad (2.4)$$

and adaptation algorithm

$$\dot{\theta}=\Theta(x_1,\dots x_r,\theta,t) \qquad (2.5)$$

ensuring the control goal

$$\|x_i(t)-\overline{x}_i(t)\|\le\Delta \text{ for } t>t_* \qquad (2.6)$$

The problem (2.3)-(2.6) looks very much like the general adaptive control problem [11,15], if the notation is introduced $x=(x_1\dots x_r)\in R^n$, $n=\Sigma n_i$ and the objective $Q_t=\|x(t)-\overline{x}(t)\|$. The difference is that $\overline{x}(t)$,T, and Q_t in the problem (2.3)-(2.6) are not given *a priori.* Note that the periodicity of $\overline{x}(t)$ can be removed without any losses. However the existing model reference adaptive control methods cannot be applied directly to theproblem because they usually require asymptotic stability of reference model. As to oscillatory systems they correspond to the boundary of stability region while chaotic systems are locally unstable (see [36].

For special case of linear controlled plant the high-gain adaptive stabilizer was suggested in [24]. In [28] the identification-based adaptive phase-locked loop design was suggested. The synchronization algorithm for two chaotic systems (drive and response) was suggested [7]. The idea of [7] is to replace the part of the response system state variables by the corresponding state variables of the drive system. The synchronization model is as follows:

$$\dot{v}=f(v,w),\ \dot{w}=f(v,w)\quad \text{(drive)} \tag{2.7}$$

$$\dot{v}'=f(v',w'),\ \dot{w}'=f(v,w')\quad \text{(response)} \tag{2.8}$$

In [30,37] the general convergence (synchronization) conditions are given for scheme with linear synchronizing signal and full state measurement:

$$\dot{\bar{x}}=\bar{f}(\bar{x}) \tag{2.9}$$

$$\dot{x}=f(x)+K(\bar{x}-x) \tag{2.10}$$

Note that (2.9) corresponds to the reference model while (2.10) is just the controlled plant. The terms "drive-response" or "master-slave" systems are also usable. It is shown [30,37] that the distance between solutions of the systems (2.9) and (2.10) is small provided K>0 is sufficiently large. On the other hand the results of [4,14] give conditions of the control aim achievement with arbitrarily small K. Such conditions are important for control of oscillations because the lifetime of oscillatory systems extends usually to large number of its periods and the energy per period of reasonable control should be sufficiently small (so called swinging control, see [17].

In fact both the scheme (2.7),(2.8) and the scheme (2.9),(2.10) are special cases of the system

$$\dot{\bar{x}}=\bar{f}(\bar{x}),\quad \dot{x}=f(x)+g(x)u, \tag{2.11}$$

where $u=U(x,\bar{x})$ is the control/synchronization signal to be determined (In case of (2.7),(2.8) we may assign $x=(v,w)$, $g(x)=1$, $u=f(v,w')-f(v',w')$).

Note that the known applications of Pecora-Carrol's method [7] (for Rössler system [7]) Chua's cirquit [32], Lorenz equation [10]have function $f_2(v,w)$ linear in v. Hence the convergence of the synchronized trajectories in this scheme (as well as in the scheme (2.9),(2.10)) can be established by means of speed-gradient method (see Theorem 1 below for $Q_t=\|v-v'\|^2$ or $Q_t=\|x-\bar{x}\|^2$). It is clear that other choice ofobjective function leads to other algorithms and the extended convergence conditions.

3. SPEED GRADIENT ALGORITHMS

A great deal of existing nonlinear adaptive control and synchronization algorithms can be analyzed and designed in the following framework. Suppose the main loop (2.4) is already chosen and substitute (2.4) into (2.3). The obtained dynamic equations

$$\dot{x}=F(x,\theta,t),\ t\geq 0 \tag{3.1}$$

(dependence on ξ is omitted for simplicity) represent some controlled plant with new input vector θ. Now achieving the control goal

$$Q_t\to 0 \text{ for } t\to\infty \tag{3.2}$$

for the plant (3.1) can be regarded as a new control problem.

To solve this problem introduce function $\omega(x,\theta,t)$ as a speed of changing Q_t along trajectories of (3.1). Particularily for memoryless functional $Q_t=Q(x(t),t)$ we have $\omega(x,\theta,t)=(\nabla_x Q)^T F(x,\theta,t)$. Then we can build the speed-gradient algorithm in its most general, so called combined form looking as follows [1].

$$\frac{d}{dt}[\theta+\psi(x,\theta,t)]=-\Gamma\nabla\,\omega(x,\theta,t) \tag{3.3}$$

where $\psi(\ )$ satisfies pseudogradient condition $\psi^T\nabla\,\omega\geq 0$ and $\Gamma=\Gamma^T>0$ is m×m gain matrix. The equation (3.3) can be rewritten in integral form

$$\theta=-\psi(x,\theta,t)-\Gamma\int_0^t \nabla\,\dot{Q}_s ds$$

The main special cases of (3.3) are SG-algorithm in differential form

$$\frac{d}{dt}\theta = -\Gamma\nabla\,\omega(x,\theta,t) \tag{3.4}$$

and SG-algorithm in the finite form

$$\theta=-\psi(x,\theta,t) \tag{3.5}$$

The typical forms of algorithm (3.5) are linear and relay ones:

$$\theta = -\Gamma\nabla\,\omega(x,\theta,t), \tag{3.5a}$$

$$\theta = -\Gamma\text{sign}\{\nabla\,\omega(x,\theta,t)\} \tag{3.5b}$$

where components of vector sign{z} are signs of the corresponding components of vector z. The stability theorems for SG-systems (3.1),(3.3) can be found in [Fradkov, 1990]. We recall here just the two typical results.

Theorem 1 *(combined form).* Assume that the right hand sides of the system (3.1),(3.3) are smooth functions in x,θ which are bounded together with derivatives in any region where the function $Q(x,t)$ is bounded. Assume also that $\omega(x,\theta,t)$ is convex in θ and the following *stabilizability condition* is valid:

there exists $\theta_*\in R^m$ such that

$$\omega(x,\theta_*,t)\leq 0. \tag{3.6}$$

for all $x\in R^n$, $t\geq 0$.
Then $Q(x,t)$ is bounded along each trajectory of (3.1),(3.3). Moreover if the *asymptotic stabilizability* condition is valid

$$\omega(x,\theta_*,t)\leq -\rho(Q(x,t)) \tag{3.7}$$

where $\rho(Q)>0$ for $Q>0$, then the goal (3.2) is achieved for all trajectories of (3.1), (3.4).

The proof of the theorem is based on Lyapunov function

$$V(x,\theta,t)=Q(x,t)+(2\gamma)^{-1}\|\theta-\theta_*\|^2 \tag{3.8}$$

In the case when it is difficult to find constant "ideal" control θ_*, satisfying (3.6) or (3.7), SG-algorithms in finite form may be applied. Their applicability conditions are as follows.

Theorem 2 *(finite form).* Assume that function Q(x,t) is smooth and the right hand side of the system (3.1) is smooth function in x,θ , bounded together with its derivatives in any region where function Q(x,t) is bounded. Assume that equation (3.5) is solvable for θ for any $x\in R^n$ and a solution of system (3.1),(3.5) (e.g. Filippov solution) exists locally for any initial $x(0)\in R^n$. Assume also that $\omega(x,\theta,t)$ is convex in θ and satisfies *stabilizability condition* (3.6). Then Q(x,t) is bounded along each trajectory of (3.1),(3.5).

Besides, if the *asymptotic stabilizability* condition (3.7) and the following *strong pseudogradient condition*

$$\psi^T\nabla\ \omega(x,\theta)\geq\beta\|\nabla\ \omega(x,\theta)\|^{\delta} \quad (3.9)$$

are valid for some $\beta>0$, $\delta>0$, then the goal (3.2) is achieved for all trajectories of (3.1), (3.5).

Note that Lyapunov function proving the theorem 2 is just the objective function Q(x,t). Choosing various types of plant equations, input and output vectors, various objective functionals and vector-funtions $\psi(x,\theta,t)$, one can obtain different structures of control algorithms.

In adaptive control problems the SG-method can be used both for the main loop and for adaptation algorithm design. In the latter problem the equation (3.1) represents the generalized controlled plant , obtained by substitution the main loop control law into the plant equation. In this case the inputs of the plant are adjustable parameters (controller parameters , the parameters of adjustable model etc.). Different applications of SG-method to regulation and tracking algorithms design can be found in [1,11-16].

4. ADAPTIVE SYNCHRONIZATION OF TWO NONLINEAR SYSTEMS WITH HYPER-MINIMUM-PHASE LINEAR PART

Consider the problem of synchronizing oscillations of the controlled system

$$\dot{x}=Ax + f(x,t)+Bu,\quad y=Cx \quad (4.1)$$

with the trajectories of the reference model

$$\dot{x}_m=f_m(x_m,t),\qquad y_m=Cx_m. \quad (4.2)$$

where $x\in R^n$, $x_m\in R^n$ are state vectors, $y\in R^l$, $y_m\in R^l$ are measurable outputs, $u\in R^m$ is control variable. The synchronization aim is formalized as follows:

$$\lim_{t\to\infty} e(t)=0 \quad (4.3)$$

where $e=x-x_m$.

Suppose that both the parameters of linear part A,B,C and the nonlinearities f(.), $f_m(.)$ are unknown to the system designer. In other words they depend on some vector of unknown parameters $\xi\in\Xi$. The problem is to determine the control law using only measurable variables and perhaps some information about nonlinearities such that the aim (4.3) is achieved for all $\xi\in\Xi$.

To solve the problem write down the error equation:

$$\dot{e}=Ae+\Phi(x,x_m,t)+Bu \quad (4.4)$$

where $\Phi=Ax_m+f(x,t)-f_m(x_m,t)$. Now impose the main restriction on the class of the problems: suppose that the following representation is valid:

$$\Phi=\sum_{i=1}^{m} B_i[\xi_i^T z_i(x,x_m,t)+v_i(x.x_m,t)], \quad (4.5)$$

where B_i are the columns of matrix B, $\xi_i\in R^N$ are vectors of unknown parameters and the values of vector-functions $z_i(.)\in R^N$ and scalar functions $v_i(.)$ are measurable. Assumption (4.5) means that all the nonlinearities and uncertainties act in span of the control. It does mean however (unlike the standard matching conditions) neither that the unknown parameters appear linearly in the model, nor that all the uncertainties can be cancelled by the proper choice of control (because the term with A in right hand side of (4.4) may not be cancellable). So (4.5) may be called *weakened matching condition.*

To solve the posed problem choose the following natural structure of adaptive controller:

$$u_i=\theta_{0i}^T(y-y_m)+\theta_{1i}^T z_i(x,x_m,t)-v_i(x,x_m,t)\ , \quad (4.6)$$

where $\theta_{0i}\in R^l$, $\theta_{1i}\in R^N$, i=1,...m are vectors of adjustable parameters. The adaptation algorithm can be chosen by SG-method using objective function $Q=e^TPe$, where $P=P^T>0$ is a positive definite matrix. Acting along lines of section 3 we obtain the following adaptation algorithm in integral form

$$\theta_{ji}(t)=-\psi_{ji}(w_{ji}(t))-\Gamma_{ji}\int_0^t w_{ji}(s)ds \quad (4.7)$$

where j=0,1; i=1,...m; $w_{0i}=(B_i^TPe)(y-y_m)$; $w_{1i}=(B_i^TPe)z_i$, $\Gamma_{ji}=\Gamma_{ji}^T\geq0$ are gain matrices and $\psi_{ji}(w)^Tw\geq0$ for all w. However algorithm (4.7) is not applicable because it requires nonmeasurable state error e(t). To obtain its realizable form the feedback Kalman-Yakubovich lemma can be employed. Start with the following definition.

Definition 1 ([11], see also [2]). System $\dot{x}=Ax+Bu$, $y=Cx$ where $u\in R^m$, $y\in R^m$ is called *hyper-minimum-phase* if it is *minimum-phase* (i.e. the polynomial $\varphi(\lambda)=\det(\lambda I-A)\det W(\lambda)$, where $W(\lambda)=C(\lambda I-A)^{-1}B$ is stable) and the matrix $CB=\lim_{\lambda\to\infty}\lambda W(\lambda)$ is symmetric and positive definite.

Lemma 1 [12]. Assume rank(B)=m. Then there exist positive definite n*n matrix $P=P^T>0$ and the m*l matrix θ_*such that

$$PA_*+A_*^TP<0,\ PB=C^T,\quad A_*=A+B\theta_*C \qquad (4.8)$$

if and only if the system $\dot{x}=Ax+Bu$, $y=Cx$ is hyper-minimum-phase.

Now we are in position to formulate the main result of the paper.

Theorem 3. Let function $x_m(t)$ be bounded, and functions $z_i(x,x_m,t)$, $v_i(x,x_m,t)$ $i=1,..m$ be bounded in any region $\{(x,t): \|x\|\le r,\ t\ge 0\}$. Choose $l*m$-matrix G with columns $g_i, i=1,...m$ such that the system with transfer function $W(\lambda)=G^TC(\lambda I-A)^{-1}B$ is hyper-minimum-phase for all $\xi\in\Xi$ and take the adaptation algorithm (4.7) where

$$w_{0i}=g_i^T(y-y_m)(y-y_m);\quad w_{1i}=g_i^T(y-y_m)z_i,$$

$\Gamma_{ji}=\Gamma_{ji}^T\ge 0$ are gain matrices, $\psi_{ji}(w)^Tw\ge 0$ for all w,

$j=0,1;\quad i=1,...m.$

Then all the trajectories of the system (4.1), (4.2), (4.6), (4.7) are bounded and the aim (4.3) is achieved.

To prove the theorem consider the Lyapunov function candidate

$$V(x,\theta,t)=\tfrac{1}{2}e^TPe+\tfrac{1}{2}\sum_{i=1}^{m}\left[\|\theta_{0i}-\theta_{0i}^*\|^2_{\Gamma_0^{-1}}+\|\theta_{1i}-\xi_i\|^2_{\Gamma_1^{-1}}\right] \qquad (4.9)$$

where matrices $P=P^T>0$ and θ_{0i}^* to be determined later. Standard calculations of $\dot V$ give

$$\dot V=e^T(PA_*+A_*^TP)e+\sum_{i=1}^{m}(\theta_{0i}-\theta_{0i}^*)^T(y-y_m)[e^TPB_i-g_i^T(y-y_m)] \qquad (4.10)$$

where $A_*=A+B\theta_0^*C$. Choosing P and θ_{0i}^* mentioned in Lemma 1 yields inequality

$$\dot V\le -e^TQe \qquad (4.11)$$

where $Q=Q^T>0$. From (4.11) and boundedness of $x_m(t)$ we conclude that $V(x(t),\theta(t),t)$ is bounded. Hence $e(t)$, $\theta_{0i}(t)$ and $\theta_{1i}(t)$ are bounded too and the first statement of the theorem is proved. Since $z_i(t)$, $v_i(t)$ are bounded $\dot e(t)$ is also bounded. Now the last statement of the theorem follows from $\int_0^\infty e^2(t)dt<\infty$.

Remark 1. Reference model (4.2) as well as the controlled plant need not to be stable. It is important for control of oscillations and chaotic motions.

Remark 2. For the case when both plant and reference model are linear stable systems the algorithm (4.7) generalizes the algorithms of papers [27,3] (see also the book [34]). To transform the algorithms of [27] to the form (4.7) take $\psi(w)=w$, $f(x,t)=0$, $f_m(x_m,t)=Ax_*+Bu_*$, where

$$x_m(t)=S_{11}\bar{x}_m(t)+S_{12}\bar{u}_m(t),\ u_m(t)=S_{21}\bar{x}_m(t)+S_{22}\bar{u}_m(t),$$

and $\bar{x}_m(t)$, $\bar{u}_m(t)$ are state and input of the reduced order reference model, S_{21}, S_{22} are appropriate constant matrices, $z_{11}=\bar{x}_m(t)$, $z_{12}=\bar{u}_m(t)$.

Theorem 3 applies to design of both conventional nonlinear adaptive control problems and adaptive control of oscillatory and chaotic systems, as it is shown in the next section.

5. ADAPTIVE CONTROL OF CHUA'S CIRCUIT

The so called Chua's circuit became popular recently as a benchmark example of simple 3rd order nonlinear system exhibiting various forms of chaotic behavior. Its mathematical model in normalized form (see e.g.[8,37]) is as follows

$$dx/dt=a1(y-f(x)),$$
$$dy/dt=x-y+z, \qquad (5.1)$$
$$dz/dt=-a2*y,$$

where $f(x)=M_0*x+0.5(M_1-M_0)*(|x+1|-|x-1|)$. The equations of controlled Chua's circuit can be written as follows:

$$dx/dt=a1(y-f(x))+b1*u,$$
$$dy/dt=x-y+z+b2*u, \qquad (5.2)$$
$$dz/dt=-a2*y+b3*u.$$

Suppose that the value of a1 is unknown. Introduce the reference model ("driving circuit"):

$$dx_m/dt=a1_m(y_m-f(x_m)),$$
$$dy_m/dt=x_m-y_m+z_m, \qquad (5.3)$$
$$dz_m/dt=-a2*y_m,$$

where $a1_m$ is a given value and the control aim

$$e(t)\to 0 \text{ when } t\to\infty \qquad (5.4)$$

where $e=(x-x_m,y-y_m,z-z_m)^T$ is state error vector.

Defining the objective function $Q(e)=e^TPe$, where $P=P^T>0$ is positive-definite matrix and applying the speed-gradient method twice (for the main loop and for the adaptation loop design) we obtain the following adaptive control law:

$$u=\theta_0Ce+\theta_1(y_m-f(x))+a1_m(f(x)-f(x_m))/b1 \qquad (5.5)$$

where θ_0,θ_1 are tunable parameters, and $C=(c_1,c2,c3)$ is a raw vector of output error weights. According to the Theorem 1 the adaptation algorithm in the differential form is as follows.

$$d\theta_0/dt=-\gamma_0(Ce)^2,$$

$$d\theta_1/dt=-\gamma_1 Ce(y_m-f(x)), \quad (5.6)$$

The theorem 3 implies that the aim (5.4) is achieved when CB>0, where $B=(b1,b2,b3)^T$.

The simulation was performed for three values of parameter a1: a1=9, a1=10.6, a1=7, the first two corresponding to chaotic behavior of the system, while the third one showing periodic motions. Choosing different values of reference model parameter $a1_m$ we achieve different behavior of the controlled system, modifying its periodic and chaotic attractors and synchronizing the two chaotic motions. See Fig. 1 - 3.

6. CONCLUSIONS

The general adaptive synchronization problem statement allows to unify and analyze different results in the field. The speed-gradient method and Feedback Kalman-Yakubovich Lemma give a simple powerful approach to adaptive synchronization algorithms design for nonlinear systems.

The author is grateful to A.Markov for performing simulations of adaptive controller for Chua's circuit.

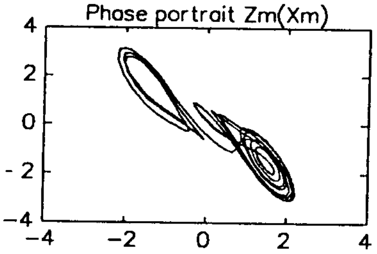

Fig. 1

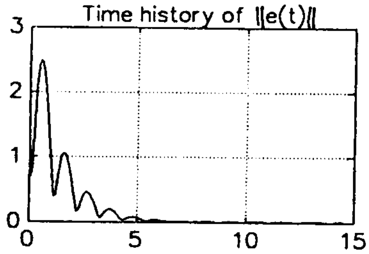

Fig. 2

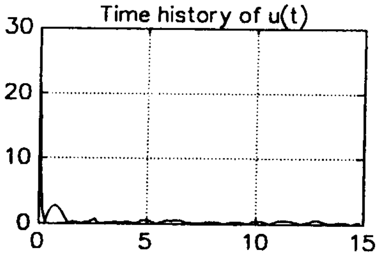

Fig. 3.

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